

1. Consider the following linear programming problem:

$$\begin{array}{rcl}
 \text{maximize} & x_1 + 2x_2 + 4x_3 + 8x_4 + 16x_5 & \\
 \text{subject to} & x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 & \leq 2 \\
 & 7x_1 + 5x_2 - 3x_3 - 2x_4 & \leq 0 \\
 & & x_1, x_2, x_3, x_4, x_5 \geq 0.
 \end{array}$$

Consider the situation in which  $x_3$  and  $x_5$  are basic and all other variables are nonbasic. Write down:

- (a)  $B$ ,
  - (b)  $N$ ,
  - (c)  $b$ ,
  - (d)  $c_B$ ,
  - (e)  $c_N$ ,
  - (f)  $B^{-1}N$ ,
  - (g)  $x_B^* = B^{-1}b$ ,
  - (h)  $\zeta^* = c_B^T B^{-1}b$ ,
  - (i)  $y_N^* = (B^{-1}N)^T c_B - c_N$ ,
  - (j) the dictionary corresponding to this basis.
2. (a) Let  $A$  be a given  $m \times n$  matrix,  $c$  a given  $n$ -vector, and  $b$  a given  $m$ -vector. Consider the following max-min problem:

$$\max_{x \geq 0} \min_{y \geq 0} (c^T x - y^T A x + b^T y).$$

By noting that the inner optimization can be carried out explicitly, show that this problem can be reduced to a linear programming problem. Write it explicitly.

- (b) What linear programming problem do you get if the min and max are interchanged?