

1. Consider the following dictionary which arises in solving a problem using the primal-dual one-phase simplex method:

$$\begin{array}{rccccccc}
 \zeta & = & -3 & & - & (-1 + 2\mu)x_1 & - & (3 - \mu)x_3 \\
 \hline
 x_2 & = & -1 & + & \mu & + & & x_1 & - & & x_3 \\
 x_4 & = & -4 & + & 3\mu & + & & 3x_1 & - & & 2x_3 \\
 x_5 & = & 2 & & & + & & x_1 & + & & x_3 .
 \end{array}$$

- (a) For which values of μ is the current dictionary optimal?
 (b) For the next pivot in the primal-dual one-phase simplex method, identify the entering and the leaving variable.
2. Let P_μ denote the perturbed primal problem (with perturbation μ). Show that if P_μ is infeasible, then $P_{\mu'}$ is infeasible for every $\mu' \leq \mu$. State and prove an analogous result for the perturbed dual problem.
3. Use the primal-dual simplex method to solve the following problem:

$$\begin{array}{ll}
 \text{maximize} & 3x_1 - x_2 \\
 \text{subject to} & x_1 - x_2 \leq 1 \\
 & -x_1 + x_2 \leq -4 \\
 & x_1, x_2 \geq 0.
 \end{array}$$

4. Consider the following one parameter family of linear programming problems (parametrized by μ):

$$\begin{array}{rcccccc}
 \max & (4 - 4\mu)x_0 & - & 2x_1 & - & 2x_2 & - & 2x_3 & - & 2x_4 \\
 \text{s.t.} & x_0 & - & x_1 & & & & & & \leq 1 \\
 & x_0 & & & - & x_2 & & & & \leq 2 \\
 & x_0 & & & & & - & x_3 & & \leq 4 \\
 & x_0 & & & & & & & - & x_4 \leq 8 \\
 & & & & & & & & & x_0, x_1, x_2, x_3, x_4 \geq 0.
 \end{array}$$

Starting from $\mu = \infty$, use the parametric simplex method to decrease μ until you get to $\mu = -\infty$. *Hint: the pivots are straight forward and, after the first couple, a clear pattern should emerge which will make the subsequent pivots easy.* Clearly indicate the range of μ values for which each dictionary is optimal.

5. Consider a linear programming problem that has an optimal dictionary in which exactly k of the original slack variables are nonbasic. Show that, by ignoring feasibility preservation of intermediate dictionaries, this dictionary can be arrived at in exactly k pivots. *Hint: Don't forget to allow for the fact that some pivot elements might be zero.*