Lump Sum Taxes and Savings

1. Adding Lump Sum Taxes to the Model

We add to our original model: lump-sum taxes of t_1 per capita on the young, t_2 per capita on the old. The government uses the proceeds of the taxes only to give transfers, and it taxes all members of a given age group uniformly. It is not allowed to borrow or lend. The government's constraint therefore is

$$N_t t_1(t) + N_{t-1} t_2(t) = 0 \ . \tag{1}$$

If we assume constant growth rate of population (as will for the rest of these notes), this becomes

$$t_1(t) + \frac{t_2(t)}{1+n} = 0 \ . \tag{2}$$

An individual's two budget constraints are now

$$C_1(t) + B(t) + t_1(t) = W(t) \tag{3}$$

$$C_2(t+1) + t_2(t+1) = \left(1 + r(t)\right)B(t) \ . \tag{4}$$

When we combine these by eliminating $B(t)$, we arrive at

$$C_1(t) + \frac{C_2(t+1) + t_2(t+1)}{1+r(t)} + t_1(t) = W(t) \ . \tag{5}$$

Though these taxes are lump-sum, so that they don't directly affect perceived prices, they do affect behavior in general. From (5) and the original form of the budget constraint (equation (5) in the previous notes) we see that the new budget constraint differs from the old by an additive term on the left-hand side of the form

$$\frac{t_2(t+1)}{1+r(t)} + t_1(t) \ . \tag{6}$$

This could turn out to be zero in special cases, but generally it does not. Because, according to the government budget constraint (1), t_1 and t_2 are of opposite sign at a given date, we may expect (6) to be small, but it may be either positive or negative in general.

If the economy is in steady state with zero taxes, and the steady state is inefficient, everybody can be made better off by policies that decrease savings and reduce the steady state capital stock. Suppose we were in such a steady state, and we contemplate implementing a policy in which t_1 and t_2 are kept constant. Because of our Cobb-Douglas preference assumption, individuals consume a fixed fraction a of their total resources in the first period of life. In the original model this means they consume $aW(t)$. In the model with taxes this means they consume

$$a \cdot \left(W - \frac{t_2}{1+r} - t_1 \right) \ . \tag{7}$$

What they save is in some sense the rest of their resources, i.e.

$$(1-a)\cdot\left(W-\frac{t_2}{1+r}-t_1\right)\ . \tag{8}$$

However, some of this "savings" is in the form of an artificial asset, the present value of taxes and subsidies. For example, if t_2 is negative, a subsidy is arriving in the second period of life. This makes the consumer feel richer and want to consume more in the second period at a given interest rate. It will not result in higher *B*, though, unless the increase in desired period-2 consumption exceeds t_2. We are interested in how much actual output they put aside, i.e. in *B*, which from (3) and (2) is given by

$$B = W - t_1 - C = W - t_1 - a\cdot\left(W - t_1 - \frac{t_2}{1+r}\right)$$
$$= (1-a)(W-t_1) + a\,\frac{t_2}{1+r} = (1-a)W + t_2\left(\frac{1-a}{1+n}+\frac{a}{1+r}\right)$$

So capital accumulation *B* is strictly increasing in t_2, the lump-sum tax on the old. If steady state is inefficient, that means $\bar{k}$ is too high and welfare is improved by reducing investment. We should therefore make t_2 negative, i.e. tax the young to provide transfers to the old. When it is first implemented, this policy involves giving to the current old, who were not planning on this government generosity, while taxing the young, but promising them that they will get a subsidy when old that is of higher present value than their current tax. Thus both generations alive at the time the policy is implemented are pleased. This is possible because the economy starts from a position of inefficiency. Every one can be made better off, and the tax policy succeeds in doing this.

Note that this inefficiency-reducing policy is a version of what is known as a "pay-as-you-go" social security scheme. Transfers to the retired old people are financed by taxes on the working young. So such a scheme can make everyone better off, under certain conditions. However note also that if the economy is not in an inefficient state to start with, such a scheme, by reducing investment, will reduce capital and thereby lower steady-state welfare. It discourages investment in capital because consumers see their retirement as financed by the government tax scheme, reducing the need for them to provide for themselves by accumulating capital.

The right policy if we wish to increase investment (e.g. if we are at an efficient steady state and wish to increase $\bar{k}$ to the golden rule level) is the opposite of social security: tax the old to provide transfers to the young. As this policy is first implemented, the young are pleased. They receive a subsidy now, and the announced future taxes on them are of lower present value than the current subsidy. But the current old simply pay unanticipated higher taxes. The current young and future generations benefit, but at the expense of the current old.

2. General Equilibrium Analysis

The preceding section is only suggestive, because it does not consider what happens to *W* and *r* as taxes change. To analyze completely the steady state we solve for it in the model with taxes. We have as in the previous version

$$B(t) = k(t+1)\cdot(1+n) \ , \tag{9}$$

and thus

$$k(t+1) = \frac{(1-a)W + t_2\left(\frac{1-a}{1+n} + \frac{a}{1+r}\right)}{1+n} \ . \tag{10}$$

Recognizing the dependence of r on k, we recall that

$$r(t) = bAk(t+1)^{b-1} - d \ . \tag{11}$$

Substituting (11) in (10) gives us a dynamic equation for k like (22) in the previous notes, "Overlapping Generations",

$$k(t+1) = \frac{(1-a)(1-b)Ak(t)^b + t_2\cdot\left(\frac{1-a}{1+n} + \frac{a}{1+bAk(t+1)^{b-1}-d}\right)}{1+n} \ . \tag{12}$$

This equation has $k(t+1)$ on both sides, and except for some specific values of b cannot be solved analytically for $k(t+1)$. We could, however, plot its implications for dependence of $k(t+1)$ on $k(t)$ by solving it for $k(t)$.

This makes the equation for steady-state capital stock

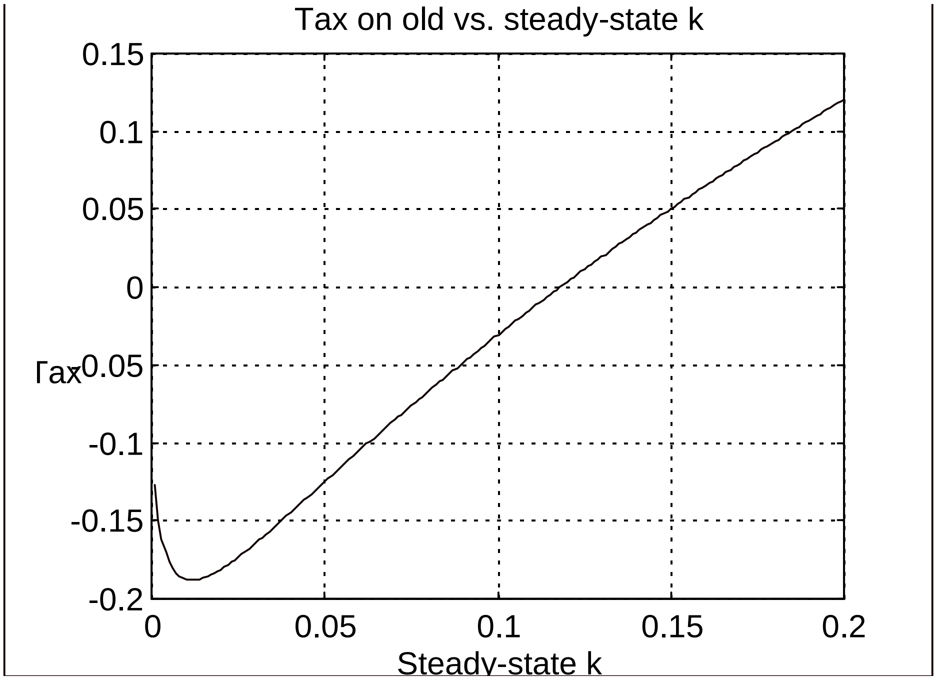
$$\bar{k} = \frac{(1-a)(1-b)A\bar{k}^b + t_2\left(\frac{a}{1+bA\bar{k}^{b-1}-d} + \frac{1-a}{1+n}\right)}{1+n} \tag{13}$$

This equation cannot be solved analytically for $\bar{k}$ except for special values of the parameters. It can be solved for t_2, though, to produce

$$t_2 = \frac{\left(1+bA\bar{k}^{b-1}-d\right)(1+n)}{1+na+(1-a)\left(bA\bar{k}^{b-1}-d\right)}\left((1+n)\bar{k} - (1-a)(1-b)A\bar{k}^b\right) \ . \tag{14}$$

Even checking the derivative of this is painful.

Rather than handle this equation analytically, we just plot it. It could be done in a spreadsheet, though I've used a different program. We can start with "realistic" values of the parameters: $1+n = 1.015^{30}$, $1-d = .93^{30}$, $b = .3$, $a = .5$. We set $A = 1$. Then the no-tax steady state has $\bar{k} = .1179$, $r = .4532$. Since $n = .5631$, this steady state is above golden rule level and thus inefficient. The golden rule $\bar{k}$ is .1053. A plot of (14) for this case is shown below.



Note that, while our partial equilibrium argument that B increases in t is largely borne out, the slope does reverse at very low levels of $\bar{k}$. This is because at such low k's, the rate of return on capital changes very rapidly as k increases, undermining our partial equilibrium reasoning based on holding r fixed initially. Reading across the graph for a given value of t_2, we see that when the retirement benefits to the old are large in absolute value (low on the graph), they may correspond to two values of $\bar{k}$. The lower of the two values of $\bar{k}$ in such a case is an unstable equilibrium. If k starts out below the lower of the two $\bar{k}$ values for a given t_2, it tends to shrink toward zero, while if it starts out above it tends to grow toward the higher $\bar{k}$. This can be seen by examination of the behavior of (12) near the steady states. [Test of understanding: How would you verify the instability of the lower steady state by using (12)?]

Note also that the level of subsidy to the old required to achieve the golden rule level of $\bar{k}$ is around .02 in absolute magnitude. This may seem small, but since the level of $\bar{k}$ itself is .1035 at the golden rule, this means the pay-as-you-go social security scheme is replacing 20% of private saving for retirement. [How does the graph tell you that the transfer to the old is about .02 at the golden rule, given that we know golden rule $\bar{k}$ is .1035?]. {}