

Answers to Review Exercise 2

1. Opening up of capital flows will in either of the two cases “immediately” (within the one-generation time period of the model) equalize the rate of return on capital in the two countries. In our classroom discussion, we assumed the same parameters $(\mathbf{a}, \mathbf{b}, A, n, \mathbf{d})$ for all economies, concluding that equalization of rates of return on capital implied equalization of wages also. But this depended on the fact that, with all countries sharing the same production function parameters (\mathbf{b}, A) , they all must have the same K/L in order to have the same rate of return to capital, and thus must also all have the same wage. This same reasoning then applies to the case in this problem where the initial differences in income are due to differences in \mathbf{a} . The production functions are in this case the same, so equalization of rates of return to K implies equalization of $k = K/L$ and thus equalization of wages. So for this case the impact of the opening up is clear. The young in the poorer country have a higher wage; the old in the poorer country face a lower market value for their savings. The reverse is true in the richer country. The welfare effects are then unambiguous for the old in both countries: welfare of the old is determined entirely by the purchasing power of their savings. For the young, it is more complicated. The higher wage for the young in the poor country is accompanied by a lower rate of return on savings, and the reverse is true for the young in the rich country. Since in this model the young will choose to make $(1+r(t))(1-\mathbf{a})C_1(t) = \mathbf{a}C_2(t+1)$, welfare of the generation born at t is

$$C_1(t) \left(\frac{(1-\mathbf{a})(1+r(t))}{\mathbf{a}} \right)^{1-\mathbf{a}}. \quad (1)$$

Using the fact that C_1 is a fixed fraction of W , the usual expressions relating W and r to k , and the fact that $(1+n)k_{t+1} = (1-\mathbf{a})(1-\mathbf{b})W_t$ (next period’s capital is this period’s saving) we can convert (1) to

$$\begin{aligned} & \mathbf{a} \cdot (1-\mathbf{b})A k_t^b \left(\frac{(1-\mathbf{a})(\mathbf{b}A k_{t+1}^{b-1} + 1-\mathbf{d})}{\mathbf{a}} \right)^{1-\mathbf{a}} \\ & = \mathbf{a} \cdot (1-\mathbf{b})A k_t^b \left(\frac{(1-\mathbf{a}) \left(\mathbf{b}A \left(\frac{(1-\mathbf{a})(1-\mathbf{b})A k_t^b}{1+n} \right)^{b-1} + 1-\mathbf{d} \right)}{\mathbf{a}} \right)^{1-\mathbf{a}} \end{aligned} \quad (2)$$

If we bring the k_t^b term on the left inside the large bracket on the right-hand side of (2), we end up with two terms in k inside the bracket. The one on the right, which multiplies $1-\mathbf{d}$, has the exponent $\mathbf{b}/(1-\mathbf{a}) > 0$. The one on the left has the exponent $\mathbf{b}^2 - \mathbf{b} + \mathbf{b}/(1-\mathbf{a}) > 0$. (That these two inequalities hold can be shown using the fact that both \mathbf{a} and \mathbf{b} are between 0 and 1.) Thus welfare is increasing in k . In other words, the decline in the return on capital cannot offset the positive effect on

welfare of the increased wage. Thus the welfare of the young in the two countries moves in the same direction as the wage, up for the poor country and down for the rich country.

If the initial difference between the two countries arose from their being in steady state, differing only in their A 's, however, the story is quite different. Since steady state k is

$$\left(\frac{(1-a)(1-b)A}{1+n} \right)^{\frac{1}{1-b}}, \quad (3)$$

steady state r is

$$bA\bar{k}^{b-1} + 1 - d = bA \frac{1+n}{A \cdot (1-a)(1-b)} + 1 - d = \frac{b \cdot (1+n)}{(1-a)(1-b)} + 1 - d, \quad (4)$$

which does not depend on A . Thus the two countries would start out with the same r , though the poorer country would have a lower wage. Opening up capital flows between the two countries would have no effect, because the rates of return were already equalized between the two countries.

In the case where the a 's differed, a quick reversal of the capital inflow to the poor country would approximately return the economies to the original situation. If the opening lasted one full period, then was unexpectedly reversed, the generation young at the time of the opening in the poor country would benefit even more. Not only are their wages be higher, but, because the foreign capital disappears when they are ready to sell their capital to firms, they get a higher return to their capital than they would if the opening were sustained. The young at the time the international capital markets are closed would be worse off than if the opening were sustained, because their wages have dropped and aggregate k is therefore lower. But they are still better off than if the opening had never occurred. Domestic savings of those young when the opening occurs is higher because these people have higher wage and save more. Therefore even without the foreign capital, the wage of those young the next period is higher.

2. There is actually a fairly simple solution to this problem, despite what I said in our review session. I should have remembered how the simple answer is set up during the review session, but even more important, I should have given you a stronger hint in the problem statement as to how to specify the taxes to make the model tractable. The trick is to let the lump sum taxes change over time so that they stay the same magnitude relative to the size of the (shrinking) economy and to make the tax on the resource use not a fixed tax per unit of Z (an uncommon sort of tax in the real world) but instead an *ad valorem* excise tax on Z . That is, make it a fixed proportion of the amount *spent* on Z , so that the tax revenues are $qQ_t Z_t$, with q a constant. We derived correctly in the review session, based on the fact that saving is a fixed proportion of total resources and equals the value of purchased resources, the result that

$$Q_t S_t = (1-a) \left(AbL_t^{b-1} Z_t^{1-b} + t_1 - t_2 \frac{Q_t}{Q_{t+1}} \right), \quad (5)$$

where t_1 is per capita lump-sum transfers to the young and t_2 is per capita lump-sum taxes on the old. Let

$$z_t = Z_t / L_t = \frac{N_{t-1}S_{t-1} - N_t S_t}{N_t} = \frac{S_{t-1}}{1+n} - S_t. \quad (6)$$

Then $Q_t = (1+f)(1-b)Az_t^{-b}$, where f is the excise tax rate on purchases of Z , and we can rewrite (5) as

$$A(1-b)z_t^{-b}(1+f)S_t = (1-a) \left(Abz_t^{1-b} + t_1 - t_2 \frac{z_t^{-b}}{z_{t+1}} \right). \quad (7)$$

The easiest case to consider is (iii), where $t_2 = 0$. Then budget balance requires that

$$t_{1t} = fQ_t z_t = f \cdot (1-b)Az_t^{1-b}. \quad (8)$$

Using (8) in (7), then multiplying the whole equation through by z_t^{-b} , produces

$$A \cdot (1-b)(1+f)S_t = (1-a)Az_t(\mathbf{b} + f \cdot (1-b)). \quad (9)$$

Therefore here, as in the case considered in class with no taxes, the ratio z/S is constant, but here it is affected by f . In particular, if we solve for the rate of shrinkage of the aggregate stock of the resource, we get using (6) and (9)

$$\frac{S_{t-1}}{S_t} = (1+n) \left(1 + \frac{z_t}{S_t} \right) = (1+n) \left(1 + \frac{(1-b)(1+f)}{(1-a)(\mathbf{b} + f \cdot (1-b))} \right). \quad (10)$$

This expression depends on f in general, but whether it is increasing or decreasing in f depends on whether $\mathbf{b} > .5$ or not. If, as might be thought to be most realistic, $\mathbf{b} > .5$ (so that less than half of output is a return to the natural resource), then the right-hand side is increasing in f , so that the resource is exhausted faster if its use is taxed. When $\mathbf{b} = .5$, the rate of resource exhaustion is unaffected by f . With $\mathbf{b} < .5$, policy to slow resource exhaustion involves subsidizing resource use, returning the proceeds to the young as a lump sum transfer. Despite the use of an *ad valorem* excise tax, which might generally be thought to be distorting, there is no loss of efficiency. This is because the only way an efficiency loss can occur in this model is for individuals to allocate consumption between the first and second periods of life so that the relative value to them of consumption in the two periods does not reflect the true rate of transformation between goods available in the two periods. But this rate of transformation as seen by consumers is Q_t/Q_{t+1} , which, because the tax distorts both Q_t and Q_{t+1} away from the marginal product of Z by the same factor $1+f$, is itself not distorted by f .

Next easiest to consider is case (i), where $f = 0$. In order to get a case that's easy to analyze we can suppose, in this perfect-foresight world, that fiscal authorities always choose $t_1 = t_2/(1+n)$ (because of the budget balance requirement) and so that

$$t_1 - t_2 \frac{z_t^{-b}}{z_{t+1}} = gQ_t S_t, \quad (11)$$

i.e. the present value of lump sum taxes and transfers is a fixed fraction of wealth. Then (7) becomes

$$(1 - (1 - \mathbf{a})\mathbf{g})(1 - \mathbf{b})S_t = (1 - \mathbf{a})\mathbf{b}z_t . \quad (12)$$

This again implies z_t/S_t is constant, and it can easily be seen that it implies that increasing \mathbf{g} reduces the rate of resource exhaustion.

Finding a solution for case (ii), with $\mathbf{t}_1 = 0$, requires a still higher level of ingenuity. The natural assumption of constant \mathbf{f} implies as a budget constraint

$$\frac{\mathbf{t}_2}{1+n} = \mathbf{f}Q_t z_t = \mathbf{f} \cdot (1 + \mathbf{f}) \mathbf{A} \mathbf{b} z_t^{1-b} .$$

Substituting this into (7) produces an equation involving S at three dates: $t, t+1$, and $t-1$. This implies that current z depends on expected future z 's, and as we discussed in class makes solution messy.

However, having seen the other two solutions, we might guess that this version of the problem also has a solution in which z/S is constant and (therefore) z_t/z_{t+1} is constant. So let's assume z_t/z_{t+1} is constant at the value \mathbf{g} and see if under this assumption (7) generates no contradiction. With this assumption for case (ii), (7) becomes

$$(1 - \mathbf{b})(1 + \mathbf{f})(1 - (1 + n)\mathbf{g}^{-b}\mathbf{f} \cdot (1 - \mathbf{a}))S_t = (1 - \mathbf{a})\mathbf{b}z_t . \quad (13)$$

This again implies constant z/S , so it is consistent with our assumption of a fixed \mathbf{g} . In tracing out the effects of \mathbf{f} here we must be careful to remember that \mathbf{f} determines \mathbf{g} via the first equality in (10). Making the appropriate substitution for \mathbf{g} in (13) produces

$$(1 - \mathbf{b})(1 + \mathbf{f}) \left(1 - (1 + n)^{1-b} \left(1 + \frac{z_t}{S_t} \right)^{-b} \mathbf{f} \cdot (1 - \mathbf{a}) \right) S_t = (1 - \mathbf{a})\mathbf{b}z_t . \quad (14)$$

Solving this explicitly for z/S is not possible except for special values of \mathbf{b} , but one can check that at least for small values of \mathbf{f} the relation between z and \mathbf{f} is monotone increasing, as in case (i).

Note that the result that emerges here is that tax schemes that give the young less wealth, thereby reducing their consumption, reduce the rate of resource exhaustion. The excise tax on resource use, which one might think would encourage conservation, has no such effect in this model, because it has to be offset with transfer-payment spending that encourages consumption.