

# Higher Order Expectations

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“...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one’s judgement, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees.” Keynes (1936), page 156.

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## Late Rational Expectations Era

Phelps (1983)

- Rational Expectations with Heterogeneous Expectations
- Monetary Policy depends on higher order expectations of monetary policy
- Individual  $i$  sets a price  $a_i$ , money supply is  $\theta$
- $a_i = (1 - r) E_i(\theta) + r E_i(\bar{a})$  where  $\bar{a} = \frac{1}{N} \sum_{i=1}^N a_i$

- Now

$$\bar{a} = (1 - r) \bar{E}(\theta) + r \bar{E}(\bar{a})$$

where  $\bar{E}$  represents average expectations.

- Progressive substitution of  $a_i = (1 - r) E_i(\theta) + r E_i(\bar{a})$  implies

$$\begin{aligned} \bar{a} &= (1 - r) \bar{E}(\theta) + r \bar{E}(\bar{a}) \\ &= (1 - r) \bar{E}(\theta) + r \bar{E}((1 - r) \bar{E}(\theta) + r \bar{E}(\bar{a})) \\ &= \dots \\ &= (1 - r) \left( \bar{E}(\theta) + r \bar{E}^2(\theta) + r \bar{E}^3(\theta) + \dots \right) \end{aligned}$$

- Townsend (1983), Lucas (1982, 1983).

## Distinctive Role of Public Information

"If the leaders of a country's economic policy are determined to embark on a program of disinflation, then they ought... to devise a way to convey the idea that expectation of disinflation should be presumed, that it should be taken to be a fact. The need is for a "signal" that everyone can and will assume to be interpreted by others in a like-minded way. If the mere announcement of the new disinflationary policy by the central bank constitutes a *public* signal, then well and good! Unfortunately, historical research has increasingly suggested that more visible and palpable signals are needed to do the trick. The German hyperinflation was ended in 1923 when the government fired tens of thousands of postal workers - a useful symbol....In any case, there seems to be a need for a symbol, a catalyst, that will precipitate the expectation on the part of all persons." Phelps (1983)

## History?

- MACRO: The "Higher Order Expectations" work of late 70s / early 80s had limited impact...
- FINANCE: Keynes' beauty contest ideas played limited role in finance despite new asset pricing models with asymmetric information (e.g., Grossman (1976), Hellwig (1980), Diamond and Verrecchia (1981)). Asset pricing models that do focus on higher order expectations do not fit into the mainstream (e.g., Allen, Morris and Postlewaite (1993), Biais and Boessaerts (1998)).
- Hypothesis: Applied economists are adept at finding tools to tame higher order beliefs (in the name of tractability).

- MACRO. Woodford (2003) on Lucas (1983): "(the) rejection of the Phelpsian insight that information imperfections play a crucial role in the monetary transmission mechanism may have been premature. For the unfortunate predictions (of low persistence) relate to the specific model presented by Lucas (1972), but not necessarily to alternative versions of the imperfect-information theory."
- FINANCE. Asset pricing theorists find tricks to combine asymmetric information with versions of martingale asset pricing, e.g., a most informed trader (Wang (1993)), a risk neutral market maker (Kyle (1995)).

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## This Talk

- Higher order beliefs in simple static example
  - discussion of transparency
- Simple static example helps us understand what is going on in dynamic models
  - Finance
  - Macro

## The Logic of Higher Order Beliefs

- Return to Phelps' beauty contest game....

$$a_i = (1 - r) E_i(\theta) + r E_i(\bar{a})$$

- Let  $\theta \sim N(y, \frac{1}{\alpha})$ :  $y$  is public signal about  $\theta$
- Let  $x_i \sim N(\theta, \frac{1}{\beta})$  in continuum population
- Now

$$E_i(\theta) = \frac{\alpha y + \beta x_i}{\alpha + \beta}$$

- So

$$\bar{E}(\theta) = \int_0^1 E_i(\theta) di = \frac{\alpha y + \beta \theta}{\alpha + \beta}$$

- Now  $i$ 's expectation of the average expectation of  $\theta$  is

$$\begin{aligned} E_i(\bar{E}(\theta)) &= E_i\left(\frac{\alpha y + \beta \theta}{\alpha + \beta}\right) \\ &= \frac{\alpha y + \beta \left(\frac{\alpha y + \beta x_i}{\alpha + \beta}\right)}{\alpha + \beta} \\ &= \frac{\left((\alpha + \beta)^2 - \beta^2\right) y + \beta^2 x_i}{(\alpha + \beta)^2} \end{aligned}$$

- ...and av. exp. of av. exp. of  $\theta$  is

$$\begin{aligned}\overline{E}^2(\theta) &= \overline{E}(\overline{E}(\theta)) \\ &= \frac{\left((\alpha + \beta)^2 - \beta^2\right) y + \beta^2 \theta}{(\alpha + \beta)^2}\end{aligned}$$

- ...more generally

$$\begin{aligned}\overline{E}^k(\theta) &= \left(1 - \left(\frac{\beta}{\alpha + \beta}\right)^k\right) y + \left(\frac{\beta}{\alpha + \beta}\right)^k \theta \\ E_i\left(\overline{E}^k(\theta)\right) &= \left(1 - \left(\frac{\beta}{\alpha + \beta}\right)^k\right) y + \left(\frac{\beta}{\alpha + \beta}\right)^k x_i\end{aligned}$$

- General Result: As  $k \rightarrow \infty$ ,  $\overline{E}^k(\theta)$  tends to the expectation of  $\theta$  based on public information alone. Samet (1998).

- recall the first order condition from the beauty contest game

$$a_i = (1 - r) \left( E_i(\theta) + r E_i(\overline{E}(\theta)) + r E_i(\overline{E}^2(\theta)) + \dots \right)$$

$$\begin{aligned}
a_i &= (1-r) \sum_{k=0}^{\infty} r^k \left[ \left( 1 - \left( \frac{\beta}{\alpha + \beta} \right)^{k+1} \right) y + \left( \frac{\beta}{\alpha + \beta} \right)^{k+1} x_i \right] \\
&= \left( 1 - \frac{\mu(1-r)}{1-r\mu} \right) y + \left( \frac{\mu(1-r)}{1-r\mu} \right) x_i \\
&= \frac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)}
\end{aligned}$$

- strategic complementarities lead to overweighting of the public signal

$$\bar{a} = \frac{\alpha y + \beta(1-r)\theta}{\alpha + \beta(1-r)}$$

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## ”Transparency”: Good Public Information

- Bernanke (2005) (see also (Blinder (1998)))

”The Federal Open Market Committee controls very short-term interest rates fairly directly. However... the Committee’s control over longer-term yields and over the prices of long-lived financial assets depend crucially on its ability to influence market expectations about the likely course of policy. In the past decade or so, the Federal Reserve has become substantially more transparent and open in its communication with the public. Growing appreciation of the fact that greater openness makes monetary policy more effective is, I believe, an important reason for this welcome trend.”

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## Sunspots and Bubbles: Bad Public Information

- Not very informative signals can also coordinate market expectations....
  - Enron's profit projections
  - "Finally, there is the CNBC effect... now that CNBC can no longer run breathless tales about the economic boom, it has started to feature equally breathless (and potentially self-fulfilling) speculations about the economic crisis." Paul Krugman in New York Times.

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# Good and Bad Public Information in the Same Model

- A very bad dichotomy in the economics literature: fundamental-based models/equilibria vs. non-fundamentals/sunspots/animal spirits models/equilibria.
- Both effects in simple model

## Welfare Analysis

- If social welfare

$$- \int_i (a_i - \theta)^2 di,$$

welfare is

$$-\frac{\beta (1 - r)^2}{(\alpha + \beta (1 - r))^2};$$

increasing in public precision  $\alpha$  only if  $r$  is small or  $\beta$  is small.

- Morris and Shin (2002), Svensson (2005), Morris, Shin and Tong (2005)
- Microfoundations: Angeletos and Pavan (2004, 2005), Hellwig (2004).

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# Optimal Communication Policy

- Morris and Shin, in progress
- private signals and "semi-public" signals
- more announcements lead to less common knowledge
- optimal design of accountancy standards

## Dynamics without Learning

- If agents learn nothing, then

$$\begin{aligned}\bar{E}_t(\bar{E}_{t+1}(\theta)) &= \left(1 - \left(\frac{\beta}{\alpha + \beta}\right)^2\right)y + \left(\frac{\beta}{\alpha + \beta}\right)^2\theta \\ &\neq \frac{\alpha y + \beta\theta}{\alpha + \beta} = \bar{E}_t(\theta).\end{aligned}$$

- Average expectations do NOT satisfy a martingale property.

## Transmission Mechanisms and Inertia

- Townsend (1983), Woodford (2003), Amato and Shin (2003, 2004), Hellwig (2002), Lorenzoni (2005).

- A shock  $\theta$  arrives at time 0. Static info structure.

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$$a_{it} = E_i(\bar{a}_t)$$

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$$\bar{a}_{T+1} = \theta$$

- for  $t = 1, \dots, T$

$$\bar{a}_t = \bar{E}_t(\bar{a}_{t+1}).$$

- With no learning:

$$\bar{a}_t = \left( 1 - \left( \frac{\beta}{\alpha + \beta} \right)^{T-t+1} \right) y + \left( \frac{\beta}{\alpha + \beta} \right)^{T-t+1} \theta$$

- Looks like adaptive expectations and rational inattention (Mankiw and Reis).
- Adding learning: observe  $\bar{a}_t + \varepsilon_t$
- If  $T$  is large, learning is very slow
- If  $T$  is small, learning is fast

## Keynesian Asset Pricing Equation

- $\bar{p}_{T+1} = \theta$

- for  $t = 1, \dots, T$

$$\bar{p}_t = \bar{E}_t (\bar{p}_{t+1}) .$$

- response to information about future dividend: large and fast reaction to public information, smaller delayed reaction to private assessments....

## Noisy REE

- Singleton (1987), Grundy and McNichols (1989), Brown and Jennings (1989), He and Wang (1995).
- random supply of asset in each trading period,  $s_t \sim N\left(0, \frac{1}{\gamma}\right)$
- before first period of trade, a public signal  $y$  is observed;  $y \sim N\left(\theta, \frac{1}{\alpha}\right)$
- continuum of traders observe private signals  $x_i$ ;  $x_i \sim N\left(0, \frac{1}{\beta}\right)$
- overlapping generations of short-lived traders; period  $t$  traders have utility

$$u(w) = e^{-\tau w}$$

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for period  $t + 1$  consumption

- each generation inherits the information of the previous generation
- Keynesian asset pricing equation holds for average prices when noise is sufficiently large in a finite horizon model (Allen, Morris and Shin (2003))
- Rich asset price dynamics in infinite horizon models (Bacchetta and van Wincoop (2003, 2005))
- With "reasonable" noise, not large difference between short-lived and long-lived traders (Brown-Jennings, He-Wang, Bacchetta-van Wincoop)

## Maintaining a Lack of Common Knowledge?

- In order for interesting dynamics with inertia to arise from lack of common knowledge, lack of common knowledge must be maintained
- Macro and finance models endogenously generate public information that push the model back to common knowledge: Lucas (1982, 1983), Woodford (2003), Lorenzoni (2005)
- Closely related issue arises in global games models: Atkeson (2001), Angeletos, Hellwig and Pavan (2003, 2004), Tarashev (2003), Hellwig, Mukerji and Tsyvinski (2005), Angeletos and Werning (2005)..
- Richer modelling of higher order beliefs will limit endogenous common knowledge?

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## Conclusion

- Psychologists and Game theory experimentors agree: reasoning to only 2 or 3 levels; even Keynes asserts at most 4 to 6....
- But simple heuristic: distinguish those things that are "commonly known" /public and private understanding
- "High confidence" in the economy: common understanding that the conditions are good.