Higher Order Expectations in Economics and Finance: An Overview

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"...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgement, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees." Keynes (1936), page 156.

Late Rational Expectations Era

Phelps (1983), Townsend (1983), Lucas (1982, 1983).

- Rational Expectations with Heterogeneous Expectations
- Monetary Policy depends on higher order expectations of monetary policy
- Individual i sets a price a_i, money supply is θ
- $a_i = (1-r)\,E_i(\theta) + rE_i(\overline{a})$ where $\overline{a} = \frac{1}{N}\sum_{i=1}^{N} a_i$

- Now
$$\overline{a} = (1 - r)\,\overline{E}\,(\theta) + r\overline{E}\,(\overline{a})$$
where $\overline{E}$ represents average expectations.

- Progressive substitution of $a_i = (1 - r)\,E_i\,(\theta) + rE_i\,(\overline{a})$ implies

$$\begin{aligned}
\overline{a} &= (1 - r)\,\overline{E}\,(\theta) + r\overline{E}\,(\overline{a}) \\
&= (1 - r)\,\overline{E}\,(\theta) + r\overline{E}\,\big((1 - r)\,\overline{E}\,(\theta) + r\overline{E}\,(\overline{a})\big) \\
&= ... \\
&= (1 - r)\left(\overline{E}\,(\theta) + r\overline{E}^{2}\,(\theta) + r\overline{E}^{3}\,(\theta) + ...\right)
\end{aligned}$$

Distinctive Role of Public Information

"If the leaders of a country's economic policy are determined to embark on a program of disinflation, then they ought... to devise a way to convey the idea that expectation of disinflation should be presumed, that it should be taken to be a fact. The need is for a "signal" that everyone can and will assume to be interpreted by others in a like-minded way. If the mere announcement of the new disinflationary policy by the central bank constitutes a *public* signal, then well and good! Unfortunately, historical research has increasingly suggested that more visible and palpable signals are needed to do the trick. The German hyperinflation was ended in 1923 when the government fired tens of thousands of postal workers - a useful symbol....In any case, there seems to be a need for a symbol, a catalyst, that will precipitate the expectation on the part of all persons." Phelps (1983)

History?

- MACRO: The "Higher Order Expectations" work of late 70s / early 80s had limited impact...

- FINANCE: Keynes' beauty contest ideas played limited role in finance despite new asset pricing models with asymmetric information (e.g., Grossman (1976), Hellwig (1980), Diamond and Verrecchia (1981)). Asset pricing models that do focus on higher order expectations do not fit into the mainstream (e.g., Allen, Morris and Postlwaite (1993), Biais and Boessaerts (1998)).

- Hypothesis: Applied economists are adept at finding tools to tame higher order beliefs (in the name of tractability).

- MACRO. Woodford (2003) on Lucas (1983): "(the) rejection of the Phelpsian insight that information imperfections play a crucial role in the monetary transmission mechanism may have been premature. For the unfortunate predictions (of low persistence) relate to the specific model presented by Lucas (1972), but not necessarily to alternative versions of the imperfect-information theory."

- FINANCE. Asset pricing theorists find tricks to combine asymmetric information with versions of martingale asset pricing, e.g., a most informed trader (Wang (1993))), a risk neutral market maker (Kyle (1995))).

Outline

- Static Models
  - Welfare Analysis of Transparency
- Dynamic Models
  - Macro
  - Finance

"Transparency": Good Public Information

- Bernanke (2005) (see also (Blinder (1998))

  "The Federal Open Market Committee controls very short-term interest rates fairly directly. However... the Committee's control over longer-term yields and over the prices of long-lived financial assets depend crucially on its ability to influence market expectations about the likely course of policy. In the past decade or so, the Federal Reserve has become substantially more transparent and open in its communication with the public. Growing appreciation of the fact that greater openness makes monetary policy more effective is, I believe, an important reason for this welcome trend."

Sunspots and Bubbles: Bad Public Information

- Not very informative signals can also coordinate market expectations....
  - Enron's profit projections
  - "Finally, there is the CNBC effect... now that CNBC can no longer run breathless tales about the economic boom, it has started to feature equally breathless (and potentially self-fulfilling) speculations about the economic crisis." Paul Krugman in New York Times.

Sunspots and Bubbles: Bad Public Information

- The way information is communicated may matter......
  - "The history of speculative bubbles begins roughly with the advent of newspapers. One can assume that, although the record of these early newpapers is mostly lost, they regularly reported on the first bubble of any consequence, the Dutch tulipmania of the 1630s. Although the news media - newspapers, magazines, and broadcast media, along with their new outlets on the Internet - present themselves as detached observers of market events, they are themselves an integral part of these event. Significant market events generally occur only if there is similar thinking among large groups of people, and the news media are essential vehicles for the spread of ideas." Shiller "Irrational Exuberance".

Sunspots and Bubbles: Bad Public Information

- High strategic complementarities in economic decisions leads to potential overreaction to public information.
  - "Firms making investment decisions are starting to emulate the hair-trigger beahvior of financial investors. That means that a growing part of the economy may be starting to act like a financial market, with all that implies - like the potential for bubbles and panics. One could argue that far from making the economy more stable, the rapid responses of today's corporations make their investment in equipment and software vulnerable to the kind of self-fulfilling pessimism that used to be possible only for investment in paper assets." Paul Krugman in New York Times.

Sunspots and Bubbles: Bad Public Information

- Central bankers always have and still are worried about markets' overeaction to what they say.....
  - "Undue importance may be attributed to (announcments of the central bank), more importance perhaps than they merit... It has been our practise to leave our actions to explain our policy... It is a dangerous thing to start to give reasons." Deputy Governor of the Bank of England Harvey at the Macmillan Committee hearings (1931).
  - "I'd also like to comment that as a consequence of our general rule... of not speaking to the press, not returning their phone calls, and essentially avoiding contact with the press, I can now say, having started the next meeting, that we succeeded. [Laughter]". Alan Greenspan, minutes of the Open Market Committee, August 1992.

Good and Bad Public Information in the Same Model

- A very bad dichotomy in the economics literature: fundamental-based models/equilibria vs. non-fundamentals/sunspots/animal spirits models/equilibria.
- This dichotomy does not make sense where higher order beliefs are taken seriously
- Tendency to associate ”good public information” with fundamental models/equilibria, ”bad public information” with non-fundamental models/equilibria

- Would like a model where both interact

- Return to Phelps' beauty contest game....

$$a_i = (1 - r) E_i(\theta) + r E_i(\overline{a})$$

Higher Order Expectations in simple information structure

- Morris and Shin (2002)
- Let $\theta \sim N\left(y, \frac{1}{\alpha}\right)$: y is public signal about θ
- Let $x_i \sim N\left(\theta, \frac{1}{\beta}\right)$ in continuum population
- Now

$$E_i(\theta) = \frac{\alpha y + \beta x_i}{\alpha + \beta}$$

- So

$$\overline{E}(\theta) = \int_0^1 E_i(\theta)\, di = \frac{\alpha y + \beta\theta}{\alpha + \beta}$$

- Now i's expectation of the average expectation of θ is

$$\begin{aligned} E_i\left(\overline{E}(\theta)\right) &= E_i\left(\frac{\alpha y + \beta\theta}{\alpha + \beta}\right) \\ &= \frac{\alpha y + \beta\left(\frac{\alpha y + \beta x_i}{\alpha + \beta}\right)}{\alpha + \beta} \\ &= \frac{\left((\alpha + \beta)^2 - \beta^2\right) y + \beta^2 x_i}{(\alpha + \beta)^2} \end{aligned}$$

- ...and av. exp. of av. exp. of θ is

$$\begin{aligned}\overline{E}^2(\theta) &= \overline{E}\left(\overline{E}(\theta)\right) \\ &= \frac{\left((\alpha+\beta)^2-\beta^2\right)y+\beta^2\theta}{(\alpha+\beta)^2}\end{aligned}$$

- ...more generally

$$\begin{aligned}\overline{E}^k(\theta) &= \left(1-\left(\frac{\beta}{\alpha+\beta}\right)^k\right)y+\left(\frac{\beta}{\alpha+\beta}\right)^k\theta \\ E_i\left(\overline{E}^k(\theta)\right) &= \left(1-\left(\frac{\beta}{\alpha+\beta}\right)^k\right)y+\left(\frac{\beta}{\alpha+\beta}\right)^k x_i\end{aligned}$$

- General Result: As $k \to \infty$, $\overline{E}^k(\theta)$ tends to the expectation of θ based on public information alone. Samet (1998).

- recall the first order condition from the beauty contest game

$$a_i = (1-r)\left(E_i(\theta) + rE_i\left(\overline{E}(\theta)\right) + rE_i\left(\overline{E}^2(\theta)\right) + ...\right)$$

$$
\begin{aligned}
a_i &= (1-r)\sum_{k=0}^{\infty} r^k \left[\left(1-\left(\frac{\beta}{\alpha+\beta}\right)^{k+1}\right) y + \left(\frac{\beta}{\alpha+\beta}\right)^{k+1} x_i\right] \\
&= \left(1-\frac{\mu\,(1-r)}{1-r\mu}\right) y + \left(\frac{\mu\,(1-r)}{1-r\mu}\right) x_i \\
&= \frac{\alpha y + \beta\,(1-r)\,x_i}{\alpha + \beta\,(1-r)}
\end{aligned}
$$

Welfare

Case 1: High actions are good

$$\begin{aligned} W(\mathbf{a},\theta) &= \begin{cases} 1, \text{ if } \overline{a} \geq a^* \\ 0, \text{ if } \overline{a} \leq a^* \end{cases} \\ &= 1-\Phi\left(\sqrt{\alpha}\left(1+\frac{\beta(1-r)}{\alpha}\right)(a^*-\theta)\right) \end{aligned}$$

- For small β, welfare is decreasing in the precision of public information if $a^* > \theta$

Welfare

Case 2: Reducing heterogeneity

$$
\begin{aligned}
W(\mathbf{a},\theta) &= -\int\limits_{i,j} (a_i - a_j)^2 \, di dj \\
&= -\frac{\beta (1-r)^2}{(\alpha + \beta (1-r))^2}
\end{aligned}
$$

- Welfare is decreasing in precision of private information β.

- Welfare is increasing in precision of public information α.

Welfare

Case 3: Getting the actions right

$$W\left(\mathbf{a},\theta\right)=-\int\limits_{i}\left(a_{i}-\theta\right)^{2}di$$

Then

$$\begin{aligned} W(\mathbf{a},\theta) &= -\frac{\alpha^2 E\left(\eta^2\right) + \beta^2 (1-r)^2 \left[E\left(\varepsilon_i^2\right)\right]}{(\alpha + \beta(1-r))^2} \\ &= -\frac{\alpha + \beta(1-r)^2}{(\alpha + \beta(1-r))^2} \end{aligned}$$

- Welfare is increasing in precision of private information β.
- Welfare is increasing in precision of public information α and only if $r \leq \frac{1}{2}$ or $r > \frac{1}{2}$ and $\alpha \geq \beta(2r-1)(1-r)$
- Svensson (2005): this is good news for transparency

Reduced Form Payoffs giving best response and welfare functions

Payoff Function

$$u_i(\mathbf{a}, \theta) \equiv -(1-r)(a_i - \theta)^2 - r\left(L_i - \bar{L}\right)$$

where

$$0 < r < 1$$

and

$$\begin{aligned} L_i &\equiv \int_0^1 (a_j - a_i)^2 \, dj \\ \bar{L} &\equiv \int_0^1 L_j dj. \end{aligned}$$

Foundational Analysis of Payoffs

- Currency crises: Metz (2003)
- Pricing: Ui (2003), Stasavage (2004), Hellwig (2004). In the latter model of price formation with monopolistic competition, social value to getting relative prices right, public information is always valuable.
- Coordination: Angeletos and Pavan (2004a, 2004b): also social value to actions reflecting level, but outweighted by social value of coordination
- Real Investment: Foundations for concern about actions reflecting information? Third parties must make real investment decisions based on information revealed by actions. Billions of dollars of fiber optic cable arising from technology bubble?

Optimal Communication Policy

- Morris and Shin, in progress.
- $\theta \sim U(\mathbb{R})$.
- continuum of individuals observe private signals $x_i \sim N\left(\theta, \frac{1}{\beta}\right)$.
- there are n "semi-public" announcements; each $z_k \sim N\left(\theta, \frac{1}{\gamma}\right)$
- each individual i observes random $m \leq n$ of the announcements: $(z_k)_{k \in I_i}$, where $I_i \subseteq \{1, ..., n\}$ and $\# I_i = m$

- write $\overline{z}_i = \frac{1}{m} \sum\limits_{k \in I_i} z_k$ and $\overline{z} = \frac{1}{n} \sum\limits_{k=1}^{n} z_k$.

- $E_i \left(\overline{z} \right) = \frac{m}{n} \overline{z}_i + \frac{m-n}{n} E_i \left(\theta \right)$

- Now

$$\begin{aligned}
E_i(\theta) &= \left(\frac{\beta}{\beta+m\gamma}\right)x_i + \left(\frac{m\gamma}{\beta+m\gamma}\right)\overline{z}_i \\
\overline{E}(\theta) &= \left(\frac{\beta}{\beta+m\gamma}\right)\theta + \left(\frac{m\gamma}{\beta+m\gamma}\right)\overline{z} \\
E_i\left(\overline{E}(\theta)\right) &= E_i\left(\left(\frac{\beta}{\beta+m\gamma}\right)\theta + \left(\frac{m\gamma}{\beta+m\gamma}\right)\overline{z}\right) \\
&= \left(\frac{\beta}{\beta+m\gamma}\right)E_i(\theta) + \left(\frac{m\gamma}{\beta+m\gamma}\right)E_i(\overline{z}) \\
&= \left(\frac{\beta+m\gamma\left(\frac{n-m}{n}\right)}{\beta+m\gamma}\right)E_i(\theta) + \left(\frac{m\gamma\frac{m}{n}}{\beta+m\gamma}\right)\overline{z}_i \\
&= \lambda_2 x_i + (1-\lambda_2)\,\overline{z}_i
\end{aligned}$$

where

$$\lambda_2 = \left(\frac{\beta}{\beta + m\gamma}\right)\left(\frac{\beta + m\gamma\left(\frac{n-m}{n}\right)}{\beta + m\gamma}\right).$$

Now

$$\overline{E}^k(\theta) = \lambda_k x_i + (1 - \lambda_k)\,\overline{z}$$

where

$$\lambda_{k+1} = \left(\frac{\beta}{\beta + m\gamma}\right)\left(\lambda_k + (1 - \lambda_k)\left(\frac{n-m}{n}\right)\right)$$

$$\lambda_k = \frac{1}{1 - \frac{m}{n}\left(\frac{\beta}{\beta+m\gamma}\right)}\left[\left(\frac{n-m}{n}\right)\left(\frac{\beta}{\beta + m\gamma}\right) + \left(\frac{m\gamma}{\beta + m\gamma}\right)\left(\frac{m}{n}\left(\frac{\beta}{\beta + m\gamma}\right)\right)^k\right]$$

$$\overline{E}^\infty(\theta) = \lambda_\infty \theta + (1 - \lambda_\infty)\overline{z}$$

$$\lambda_\infty = \frac{\left(\frac{n-m}{n}\right)\left(\frac{\beta}{\beta+m\gamma}\right)}{1-\frac{m}{n}\left(\frac{\beta}{\beta+m\gamma}\right)}$$

- Thus in beauty contest game with $r \to 1$, $\overline{a} \to \lambda_\infty \theta + (1-\lambda_\infty)\overline{z}$

- case 2: Reducing Heterogeneity

$$a_i - a_j = \lambda_\infty (\varepsilon_i - \varepsilon_j) + (1-\lambda_\infty)(\overline{z}_i - \overline{z}_j)$$

  – welfare....

$$-\left(\lambda_\infty^2\left(\frac{1}{\beta}\right) + (1-\lambda_\infty)^2 \sum_{k=2m-n}^{m} \left(\begin{array}{c} m \\ k \end{array}\right) \frac{m!\,(n-m)!\,(n-m)!}{(m-k)!\,(n-2m+k)!n!}\right)$$

– increasing in m for fix n; decreasing in $\frac{m}{n}$ for fixed m....

- case 3: getting the actions right

$$a_i - \theta = \lambda_\infty \varepsilon_i + (1 - \lambda_\infty)\overline{z}_i$$

- – welfare

$$-\left(\lambda_\infty^2\left(\frac{1}{\beta}\right) + (1 - \lambda_\infty)^2\left(\frac{1}{m\gamma}\right)\right)$$

Martingales and Average Expectations

- If agents learn nothing, then

$$\begin{aligned}\overline{E}_t\left(\overline{E}_{t+1}(\theta)\right) &= \left(1-\left(\frac{\beta}{\alpha+\beta}\right)^2\right)y+\left(\frac{\beta}{\alpha+\beta}\right)^2\theta \\ &\neq \frac{\alpha y+\beta\theta}{\alpha+\beta}=\overline{E}_t(\theta).\end{aligned}$$

Average expectations do NOT satisfy a martingale property.

Expectation of expectation is biased to y.

$$\overline{E}_t\left(....(\overline{E}_{t+k}\left(\theta\right))\right)=\left(1-\left(\frac{\beta}{\alpha+\beta}\right)^{k+1}\right)y+\left(\frac{\beta}{\alpha+\beta}\right)^{k+1}\theta$$

As $k\to\infty$,

$$\overline{E}_t\left(....(\overline{E}_{t+k}\left(\theta\right))\right)\to y$$

Inertia

- Amato and Shin (2003, 2004), Hellwig (2002), Lorenzoni (2005), Woodford (2003).

- $\overline{a}_{T+1} = \theta$

- Arrive at time 0 with static information structure.

- for $t = 1, ..., T$

$$\overline{a}_t = \overline{E}_t \left(\overline{a}_{t+1} \right).$$

- With no learning:

$$\overline{a}_t = \left(1 - \left(\frac{\beta}{\alpha+\beta}\right)^{T-t+1}\right) y + \left(\frac{\beta}{\alpha+\beta}\right)^{T-t+1} \theta$$

- News arrives at time 0, is reflected in actions with a lag.

- Looks like adaptive expectations and rational inattention (Mankiw and Reis).

Keynesian Asset Pricing Equation

- $\overline{p}_{T+1} = \theta$

- for $t = 1, ..., T$

$$\overline{p}_t = \overline{E}_t \left(\overline{p}_{t+1} \right).$$

Noisy REE

- Singleton (1987), Grundy and McNichols (1989), Brown and Jennings (1989), He and Wang (1995).
- random supply of asset in each trading period, $s_t \sim N\left(0, \frac{1}{\gamma}\right)$
- before first period of trade, a public signal y is observed; $y \sim N\left(\theta, \frac{1}{\alpha}\right)$
- continuum of traders observe private signals x_i; $x_i \sim N\left(0, \frac{1}{\beta}\right)$
- overlapping generations of short-lived traders; period t traders have utility

$$u(w) = e^{-\tau w}$$

for period $t + 1$ consumption

- each generation inherits the information of the previous generation

- Keynesian asset pricing equation holds for average prices when noise is sufficiently large in a finite horizon model (Allen, Morris and Shin (2003))

- Rich asset price dynamics in infinite horizon models (Bacchetta and van Wincoop (2003, 2005))

- With "reasonable" noise, not large difference between short-lived and long-lived traders (Brown-Jennings, He-Wang, Bacchetta-van Wincoop)

Maintaining a Lack of Common Knowledge?

- In order for interesting dynamics with inertia to arise from lack of common knowledge, lack of common knowledge must be maintained

- Macro and finance models endogenously generate public information that push the model back to common knowledge

- Closely related issue arises in global games models

- Atkeson (2001), Angeletos, Hellwig and Pavan (2003, 2004), Tarashev (2003), Hellwig, Mukerji and Tsyvinski (2005), Angeletos and Werning (2005)....

Multidimensional Higher Order Beliefs

- Kondor (2005)
- Morris and Shin in progress
- $\theta \sim N\left(\mu, \frac{1}{\gamma}\right)$
- Continuum of agents, i observes private signals (x_{i1}, x_{i2}), with $x_{ik} = \theta + \eta_k + \varepsilon_{ik}$, each $\eta_k \sim N\left(0, \frac{1}{\alpha_k}\right)$ and $\varepsilon_{ik} \sim N\left(0, \frac{1}{\beta_k}\right)$

- Since each $x_{ik} \sim N\left(\theta, \frac{\alpha_k \beta_k}{\alpha_k + \beta_k}\right)$,

$$E_i(\theta) = \frac{\gamma\mu + \frac{\alpha_1+\beta_1}{\alpha_1\beta_1} x_{i1} + \frac{\alpha_2+\beta_2}{\alpha_2\beta_2} x_{i2}}{\gamma + \frac{\alpha_1+\beta_1}{\alpha_1\beta_1} + \frac{\alpha_2+\beta_2}{\alpha_2\beta_2}}.$$

- The average expectation of θ in the population is then

$$\overline{E}(\theta) = \frac{\gamma\mu + \frac{\alpha_1+\beta_1}{\alpha_1\beta_1} \eta_1 + \frac{\alpha_2+\beta_2}{\alpha_2\beta_2} \eta_2}{\gamma + \frac{\alpha_1+\beta_1}{\alpha_1\beta_1} + \frac{\alpha_2+\beta_2}{\alpha_2\beta_2}}.$$

- Higher order average expectations $\overline{E}^k(\theta)$ will then differ from $\overline{E}(\theta)$ and will converge to μ as $k \to \infty$.

- Now suppose that a public opinion poll is announced, and $\psi = \overline{E}(\theta)$ becomes common knowledge. Now the agents will have observed two public signals (μ and ψ) and two private signals (x_{i1} and x_{i2}) concerning θ. But despite the new information, their views do not become common knowledge.

- Also observe that an individual who observed all the agents' private information could aggregate it into $E^*(\theta) = \frac{\gamma\mu + \alpha_1\eta_1 + \alpha_2\eta_2}{\gamma + \alpha_1 + \alpha_2}$.

- Although this is a one dimensional variable, it is a sufficient statistic for (η_1, η_2), and a public announcement of this single statistic would create common knowledge beliefs about θ.