Discussion of Gizatulina and Hellwig "Payoffs Can be Inferred From Beliefs, Generically, When Beliefs are Conditioned on Information."

Stephen Morris
"Information and Dynamic Mechanism Design" workshop, Hausdorff Institute, Bonn

June 2009
Example

- Two individuals, single object, correlated valuations

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>H</th>
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<tbody>
<tr>
<td>L</td>
<td>$\frac{6}{14}$</td>
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</tr>
<tr>
<td>H</td>
<td>$\frac{3}{14}$</td>
<td>$\frac{2}{14}$</td>
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- Myerson, Cremer-McLean: a simple mechanism allocates the object efficiently and extracts full surplus: direct mechanism where agents bet on others’ types
Efficient Allocation

- allocate object to agent with highest value (flip a coin if equal) and make following payments to agent 1, with payments

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<tbody>
<tr>
<td>L</td>
<td>M</td>
<td>$-2M$</td>
</tr>
<tr>
<td>H</td>
<td>$-2M$</td>
<td>$3M$</td>
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- agent 1’s posterior beliefs are

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<tbody>
<tr>
<td>L</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>H</td>
<td>$\frac{3}{5}$</td>
<td>$\frac{2}{5}$</td>
</tr>
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</table>

- for sufficiently large $M$, truth-telling is an equilibrium....
Richer Information Structure

<table>
<thead>
<tr>
<th>t_1</th>
<th>( t_2 )</th>
<th>( t'_2 )</th>
<th>( t''_2 )</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t'_1 )</td>
<td>( \frac{1}{14} )</td>
<td>( \frac{1}{14} )</td>
<td>( \frac{1}{14} )</td>
<td>L</td>
</tr>
<tr>
<td>( t''_1 )</td>
<td>( \frac{1}{14} )</td>
<td>( \frac{2}{14} )</td>
<td>( \frac{2}{14} )</td>
<td>H</td>
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<tr>
<td>L</td>
<td>L</td>
<td>H</td>
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Gizatulina Hellwig discussion
Conditional beliefs of agent 1

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<tr>
<th></th>
<th>$t_2$</th>
<th>$t'_2$</th>
<th>$t''_2$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
<td>$L$</td>
</tr>
<tr>
<td>$t'_1$</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{2}{5}$</td>
<td>$\frac{2}{5}$</td>
<td>$L$</td>
</tr>
<tr>
<td>$t''_1$</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{2}{5}$</td>
<td>$\frac{2}{5}$</td>
<td>$H$</td>
</tr>
<tr>
<td>$L$</td>
<td>$L$</td>
<td>$H$</td>
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Full surplus extraction impossible. Type $t''_1$ must earn informational rent.
Type Space

- "Payoff types" of agent $i$: $\Theta_i$
- "Types" of agent $i$: $T_i$
  - $\tilde{\theta}_i : T_i \rightarrow \Theta_i$
  - $\tilde{\pi}_i : T_i \rightarrow \Delta(T_{-i})$
- Common prior:
  - there exists $\pi^* \in \Delta(T)$ with

$$
\tilde{\pi}_i(t_{-i}|t_i) = \frac{\pi^*(t_i, t_{-i})}{\sum_{t'_{-i}} \pi^*(t_i, t'_{-i})}
$$
Without Loss of Generality.....

- There exists a universal type spaces $T_i^*$ such that

$$T_i^* \approx \Theta_i \times \Delta(T_{-i}^*)$$

and thus all type spaces are embedded within $T^*$

1. Belief Determine Preferences (BDP): for each $\pi_i \in \text{range}(\tilde{\pi}_i)$, there exists $\theta_i \in \Theta_i$ such that

$$\tilde{\pi}_i (t_i) = \pi_i \Rightarrow \tilde{\theta}_i (t_i) = \theta_i.$$
"Beliefs Determine Preferences" and Genericity

**BDP underlies Cremer-McLean (1988)**

Linear Independence of Beliefs (LIB): there do not exist \( \lambda_i \) such that \( \lambda_i \neq 0 \) and

\[
\sum_{\pi_i \in \Pi_i} \lambda_i \pi_i = 0
\]

beliefs in range(\( \pi_i \))

- BDP is necessary condition for full surplus extraction
- BDP and LIB are sufficient for full surplus extraction
"Beliefs Determine Preferences" and Genericity

Is BDP "generic" for common prior types spaces?

- Cremer-McLean: yes, BDP holds for almost all $\pi^*$ on a fixed finite type space
- Heifetz-Neeman (2006): no, under geometric and (infinite dimensional) measure theoretic notions of genericity
- Barelli (2008): yes, under topological notion of genericity
Gizatulina and Hellwig (2009)

- CM pick beliefs "exogenously"
- Derive beliefs endogenously from signal structure
- Rich set of examples with and without BDP
- Finite and infinite type space results formalizing the idea that rich enough space of uncertainty
Important Question

- Zero consensus in literature or among researchers on this issue
- Central to mechanism design
Comments

1. Motivation and examples are well understood. But what extra structure do they add to genericity arguments? Doesn’t seem to be much in finite case. What about the infinite case?

2. Implicit common knowledge assumptions. GH are assuming common knowledge of the distribution of signals

3. The common prior assumption is key.

4. Other natural signal stories lead to failure of BDP and/or full surplus extraction
2. Implicit Common Knowledge Assumptions

Generic choice of common prior on fixed finite space ⇒ the modeller is certain that

1. knowing agent 2’s beliefs allows agent 1 to deduce - for sure - agent 2’s preferences
2. every agent knows that (1) is true
3. every agent knows that (2) is true
4. ......

i.e. BDP
GH push back the common knowledge assumption

GH assume that the modeller is certain that

1. every agent is certain of the distribution of the signals other agents’ are observing as a function of agents’ preferences...
2. every agent knows that (1) is true
3. every agent knows that (2) is true
4. ..... 

and perturbations reflect this.
The Common Prior Assumption

- With the common prior assumption, BDP fails on the universal type space.
The Common Prior Assumption

- With the common prior assumption, BDP fails on the universal type space.
- Recall that there exists a universal type spaces $T_i^*$ such that

$$T_i^* \approx \Theta_i \times \Delta(T_{-i}^*)$$

and thus all type spaces are embedded within $T^*$.
- Thus for every $\theta_i$ and $\pi_i \in \Delta(T_{-i}^*)$, there exists $t_i \in T_i^*$ such that $\tilde{\pi}_i(t_i) = \pi_i \Rightarrow \tilde{\theta}_i(t_i) = \theta_i$
The Common Prior Assumption

- With the common prior assumption, BDP fails on the universal type space.
- Recall that there exists a universal type space $T^*_i$ such that
  \[ T^*_i \approx \Theta_i \times \Delta (T^*_{-i}) \]
  and thus all type spaces are embedded within $T^*$.
- Thus for every $\theta_i$ and $\pi_i \in \Delta (T^*_{-i})$, there exists $t_i \in T^*_i$ such that $\tilde{\pi}_i (t_i) = \pi_i \Rightarrow \tilde{\theta}_i (t_i) = \theta_i$
Common Value payoff states $\Phi$.
Each agent observes signal in $S_i$ about $\Phi$ and idiosyncratic payoff component in $X_i$.
Value depends on $\phi$ and $x_i$.
Common prior $h \in \Delta (\Phi \times S_1 \times X_1 \times ... S_n \times X_n)$

$$h (\phi, s_1, x_1, ..., s_n, x_n) = \pi^* (\phi) \prod_{i=1}^{n} g_i (s_i | \phi) f_i (x_i)$$
McLean and Postlewaite (RES 05)

- Common prior \( h \in \Delta (\Phi \times S_1 \times X_1 \times ... S_n \times X_n) \)

\[
h (\phi, s_1, x_1, ..., s_n, x_n) = \pi^* (\phi) \prod_{i=1}^{n} g_i (s_i | \phi) f_i (x_i)
\]

- Not independent, fails BDP (satisfies LIP for generic distributions...)

- (McLean-Postlewaite show efficiency can be achieved combined CM surplus extraction with VCG extraction of private value component)
Parreiras (JET 05)

- Agents are uncertain how informed other agents are...
- Uncertainty about others’ information creates linear dependence
- Example: 1’s valuation is $H$ or $L$, each with prob. $\frac{1}{2}$; 1 is informed of his type with prob. $\frac{1}{2}$.

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<thead>
<tr>
<th></th>
<th>$H$</th>
<th>$L$</th>
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<tbody>
<tr>
<td>Uninformed</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Informed $H$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Informed $L$</td>
<td>0</td>
<td>1</td>
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Parreiras (JET 05) converse

When rents are left to an agent, there is an interpretation of the information structure in terms of uncertainty about others being more informed in Blackwell’s sense