

Virtual Robust Implementation and Strategic Revealed Preference

Dirk Bergemann and Stephen Morris

Workshop of Mathematical Economics
Celebrating the 60th birthday of Aloisio Araujo
IMPA Rio de Janeiro

December 2006

Definitions

- "implementation": requires ALL equilibria deliver the right outcome, a.k.a. full implementation
- "robust": same mechanism works independent of agents' beliefs and higher order beliefs about the environment
- "virtual": enough if correct outcome arises with probability $1 - \varepsilon$

Single Good, Private Values I

- I agents
- agent i has valuation $\theta_i \in [0, 1]$
- Second Price Sealed Bid Auction
 - object goes to highest bidder
 - winner pays second highest bid

Single Good, Private Values I

- I agents
- agent i has valuation $\theta_i \in [0, 1]$
- Second Price Sealed Bid Auction
 - object goes to highest bidder
 - winner pays second highest bid
- Truth-Telling is a dominant strategy, but there are inefficient equilibria

Single Good, Private Values II

Modified Second Price Sealed Bid Auction

- With probability $1 - \varepsilon$,
 - allocate object to highest bidder
 - winner pays second highest bid
- For each i , with probability $\frac{\varepsilon b_i}{I}$
 - i gets object
 - pays $\frac{1}{2}b_i$

Single Good, Private Values II

Modified Second Price Sealed Bid Auction

- With probability $1 - \varepsilon$,
 - allocate object to highest bidder
 - winner pays second highest bid
- For each i , with probability $\frac{\varepsilon b_i}{I}$
 - i gets object
 - pays $\frac{1}{2}b_i$

Truth-Telling is a strictly dominant strategy, so Robust Virtual Implementation

Single Good, Interdependent Values I

- I bidders
- Bidder i has type $\theta_i \in [0, 1]$
- Bidder i 's valuation is $v_i = \theta_i + \gamma \sum_{j \neq i} \theta_j$
- Modified Second Price Sealed Bid Auction
 - object goes to highest bidder
 - winner pays $\max_{j \neq i} b_j + \gamma \sum_{j \neq i} b_j$

Single Good, Interdependent Values I

- I bidders
- Bidder i has type $\theta_i \in [0, 1]$
- Bidder i 's valuation is $v_i = \theta_i + \gamma \sum_{j \neq i} \theta_j$
- Modified Second Price Sealed Bid Auction
 - object goes to highest bidder
 - winner pays $\max_{j \neq i} b_j + \gamma \sum_{j \neq i} b_j$
- If $\gamma \leq 1$, truth-telling is a "ex post" equilibrium, but there are inefficient equilibria

Single Good, Interdependent Values II

Modified Second Price Sealed Bid Auction

- With probability $1 - \varepsilon$,
 - allocate object to highest bidder i
 - winner pays $\max_{j \neq i} b_j + \gamma \sum_{j \neq i} b_j$
- For each i with probability $\frac{\varepsilon b_i}{I}$,
 - i gets object
 - pays $\frac{1}{2} b_i + \gamma \sum_{j \neq i} b_j$

Single Good, Interdependent Values II

Modified Second Price Sealed Bid Auction

- With probability $1 - \varepsilon$,
 - allocate object to highest bidder i
 - winner pays $\max_{j \neq i} b_j + \gamma \sum_{j \neq i} b_j$

- For each i with probability $\frac{\varepsilon b_i}{I}$,
 - i gets object
 - pays $\frac{1}{2}b_i + \gamma \sum_{j \neq i} b_j$

Truth telling is a strict ex post equilibrium

Single Good, Interdependent Values III

Two cases

- if $\gamma < \frac{1}{I-1}$, Robust Virtual Implementation (in the modified direct mechanism)
- if $\gamma \geq \frac{1}{I-1}$, inefficient multiple equilibria in this AND ALL OTHER mechanisms

This Paper

Necessary and Sufficient Conditions for Virtual Robust Implementation in General Environment:

1. Ex Post Incentive Compatibility

- in example, $\gamma \leq 1$

2. "Robust Measurability" or Not Too Much Interdependence

- in example, $\gamma < \frac{1}{I-1}$

Abreu-Matsushima (1992) Incomplete Information: Setup

- Standard "Bayesian" incomplete information setting, i.e., common knowledge of common prior on type space
- More demanding in two ways:
 - Iterated deletion of strictly dominated strategies
 - Compact mechanisms
- Less demanding in one way:
 - "Virtual" implementation

Abreu-Matsushima (1992) Incomplete Information: Results

Necessary conditions for virtual implementation

- Bayesian incentive compatibility
- Abreu-Matsushima measurability: types are iteratively distinguishable
 - reduces to "value distinction" in private values case

"Slicing" mechanism shows sufficiency of Bayesian incentive compatibility and Abreu-Matsushima measurability

Adding Robustness I

With robustness, full implementation equivalent to belief free version of iterated deletion of strictly dominated strategies

Adding Robustness II

Generalizing Abreu-Matsushima, necessary conditions become:

1. Ex post incentive compatibility (instead of Bayesian IC)
 - Bergemann-Morris "Robust Mechanism Design"

Adding Robustness II

Generalizing Abreu-Matsushima, necessary conditions become:

1. Ex post incentive compatibility (instead of Bayesian IC)
 - Bergemann-Morris "Robust Mechanism Design"
2. A belief free version of iterative distinguishability (instead of AM measurability)

Strategic Revealed Preference I

Forget implementation question....

- Two types are "distinguishable" if there exists a mechanism (game) such that those types choose different messages (action) in every equilibrium on every type space
- We characterize distinguishability for general environments
 - in single good environment
 - * every pair of types are indistinguishable if $\gamma \geq \frac{1}{I-1}$
 - * every pair of distinct types are distinguishable if $\gamma < \frac{1}{I-1}$

Strategic Revealed Preference II

Gul and Pesendorfer (2005): "The Canonical Type Space for Interdependent Types." What is meant by interdependent utility representations of preferences?

1. Neuroeconomics procedural interpretation
2. Traditional "mindless" revealed preference interpretation

Our exercise differs from Gul-Pesendorfer...

1. explicitly strategic
2. static mechanism \Rightarrow no counterfactual information used

3. GP require that distinguishable types MIGHT behave differently; we require that they MUST behave differently

Back to Virtual Implementation

- Definition of "Robust Measurability": social choice function treats indistinguishable types the same
- Ex post incentive compatibility and robust measurability are necessary for virtual robust implementation
- Adaption of Abreu Matsushima slicing mechanism will show them to be sufficient

Exact Implementation I

Following Maskin methods, necessary and sufficient conditions for exact robust implementation - using ANY mechanism:

(Bergemann-Morris "Robust Implementation: The Role of Large Type Spaces")

1. ex post incentive compatibility
2. "robust monotonicity": not too much interdependence

Exact Implementation II

in general,

- robust monotonicity $\not\Rightarrow$ robust measurability
- robust measurability $\not\Rightarrow$ robust monotonicity
- gap from
 - virtual vs. exact
 - compact vs. unbounded mechanism
- restricting to compact mechanism, strict ex post incentive compatibility and robust measurability necessary for full implementation

Exact Implementation III

in large class of economically interesting "monotonic aggregator" environments:

(Bergemann-Morris "Robust Implementation: The Role of Direct Mechanisms")

1. robust monotonicity = robust measurability
2. natural generalization of $\gamma < \frac{1}{I-1}$ condition
3. when robust virtual implementation is possible, it arises in modified direct mechanism

Outline

1. Environment and Solution Concepts
2. Strategic Revealed Preference: A Characterization Result
3. Virtual Robust Implementation

PART 1:
ENVIRONMENT AND SOLUTION CONCEPTS

Environment

- I agents
- Lottery outcome space $Y = \Delta(X)$, X finite
- Finite types Θ_i
- vNM utilities: $u_i : Y \times \Theta \rightarrow \mathbb{R}$
- Economic Assumption (can be relaxed)
 - \bar{y} is uniform lottery over X
 - for each i , there exists $b_i \in Y$ s.t. $u_i(\bar{y}, \theta) > u_i(b_i, \theta)$ for all θ

Mechanism

A mechanism \mathcal{M} is a collection $((M_i)_{i=1}^I, g)$

- each M_i is a finite message set
- outcome function $g : M \rightarrow Y$

Rationalizable Messages

$$S_i^{\mathcal{M},0}(\theta_i) = M_i,$$

$$S_i^{\mathcal{M},k+1}(\theta_i) = \left. \begin{array}{l} \exists \mu_i \in \Delta(\Theta_{-i} \times M_{-i}) \text{ s.t.:} \\ (1) \mu_i(\theta_{-i}, m_{-i}) > 0 \Rightarrow m_{-i} \in S_{-i}^{\mathcal{M},k}(\theta_{-i}) \\ (2) m_i \in \arg \max_{m'_i} \sum_{\theta_{-i}, m_{-i}} \mu_i(\theta_{-i}, m_{-i}) u_i(g(m'_i, m_{-i}), (\theta_i, \theta_{-i})) \end{array} \right\}$$

$$S_i^{\mathcal{M}}(\theta_i) = \bigcap_{k \geq 1} S_i^{\mathcal{M},k}(\theta_i).$$

$S_i^{\mathcal{M}}(\theta_i)$ are *rationalizable actions* of type θ_i in mechanism \mathcal{M}

Epistemic Foundations I

- Type Space $\mathcal{T} = \left(T_i, \hat{\pi}_i, \hat{\theta}_i \right)_{i=1}^I$
 1. T_i countable types of agent i
 2. $\hat{\pi}_i : T_i \rightarrow \Delta(T_{-i})$
 3. $\hat{\theta}_i : T_i \rightarrow \Theta_i$
- Incomplete information game $(\mathcal{T}, \mathcal{M})$
 - i 's strategy: $\sigma_i : T_i \rightarrow \Delta(M_i)$
 - strategy profile σ is an equilibrium if $\sigma_i(m_i|t_i) > 0$ implies m_i is in

$$\arg \max_{m'_i} \sum_{t_{-i}, m_{-i}} \hat{\pi}_i[t_i](t_{-i}) \left(\prod_{j \neq i} \sigma_j(m_j|t_j) \right) u_i \left(g(m'_i, m_{-i}), \hat{\theta}(t) \right)$$

Epistemic Foundations II

PROPOSITION. $m_i \in S_i^{\mathcal{M}}(\theta_i)$ if and only if there exist

1. a type space \mathcal{T} ,
2. an equilibrium σ of $(\mathcal{T}, \mathcal{M})$ and
3. a type $t_i \in T_i$, such that
 - (a) $\sigma_i(m_i|t_i) > 0$ and
 - (b) $\hat{\theta}_i(t_i) = \theta_i$.

Brandenburger and Dekel (1987), Battigalli (1996), Bergemann and Morris (2001), Battigalli and Siniscalchi (2003).

PART 2:

STRATEGIC REVEALED PREFERENCE

Defining Distinguishability I

DEFINITION. R_{θ_i, λ_i} is preference relation of agent i with payoff type θ_i and conjecture $\lambda_i \in \Delta(\Theta_{-i})$ about types of others:

$$y R_{\theta_i, \lambda_i} y' \Leftrightarrow \sum_{\theta_{-i} \in \Theta_{-i}} \lambda_i(\theta_{-i}) u_i(y, (\theta_i, \theta_{-i})) \geq \sum_{\theta_{-i} \in \Theta_{-i}} \lambda_i(\theta_{-i}) u_i(y', (\theta_i, \theta_{-i}))$$

- DEFINITION. Type set Ψ_i is *distinguishable* given type set profile Ψ_{-i} if for all R_i there exists $\theta_i \in \Psi_i$ such that $R_{\theta_i, \lambda_i} \neq R_i$ for all $\lambda_i \in \Delta(\Psi_{-i})$.

Defining Distinguishability II

$$\begin{aligned}\Xi_i^0 &= 2^{\Theta_i} \\ \Xi_i^{k+1} &= \{ \Psi_i \in 2^{\Theta_i} \mid \exists \Psi_{-i} \in \Xi_{-i}^k \text{ s.t. } \Psi_i \text{ is indistinguishable given } \Psi_{-i} \} \\ \Xi_i^* &= \bigcap_{k \geq 1} \Xi_i^k\end{aligned}$$

DEFINITION. Type θ_i is indistinguishable from θ'_i if $\{\theta_i, \theta'_i\} \in \Xi_i^*$.

Single Good Example I

- $R_{\theta_i, \lambda_i} = R_{\theta'_i, \lambda'_i}$ if and only if

$$\theta_i + \gamma E_{\lambda_i} \left(\sum_{j \neq i} \theta_j \right) = \theta'_i + \gamma E_{\lambda'_i} \left(\sum_{j \neq i} \theta_j \right)$$

- Suppose $\theta_i < \theta'_i$; there exist $\lambda_i, \lambda'_i \in \Delta(\Psi_{-i})$ such that $R_{\theta_i, \lambda_i} = R_{\theta'_i, \lambda'_i}$ if and only if

$$\theta_i + \gamma \sum_{j \neq i} \max \Psi_j \geq \theta'_i + \gamma \sum_{j \neq i} \min \Psi_j$$

Single Good Example II

- Ψ_i is distinguishable given Ψ_{-i} if and only if

$$\max \Psi_i - \min \Psi_i > \gamma \left(\sum_{j \neq i} \max \Psi_j - \min \Psi_j \right)$$

- Thus

$$\Xi_i^k = \left\{ \Psi_i \mid \max \Psi_i - \min \Psi_i \leq [\gamma (I - 1)]^k \right\}$$

- If $\gamma \geq \frac{1}{I-1}$, all θ_i, θ'_i are indistinguishable
- If $\gamma < \frac{1}{I-1}$, $\theta_i \neq \theta'_i \Rightarrow \theta_i$ and θ'_i are distinguishable

Fixed Point Distinguishability

Consider a collection of sets $\Xi = (\Xi_i)_{i=1}^I$, each $\Xi_i \subseteq 2^{\Theta_i}$.

DEFINITION. The collection Ξ is mutually indistinguishable if, for each i and $\Psi_i \in \Xi_i$, there exists $\Psi_{-i} \in \Xi_{-i}$ such that Ψ_i is indistinguishable given Ψ_{-i} .

LEMMA. Type θ_i is indistinguishable from θ'_i if and only if there does not exist mutually indistinguishable Ξ such that $\{\theta_i, \theta'_i\} \subseteq \Psi_i$ for some $\Psi_i \in \Xi_i$.

Strategic Revealed Preference Characterization

PROPOSITION 1: If θ_i and θ'_i are indistinguishable, then $S_i^{\mathcal{M}}(\theta_i) \cap S_i^{\mathcal{M}}(\theta'_i) \neq \emptyset$ in any mechanism \mathcal{M} .

PROPOSITION 2: There exists a mechanism \mathcal{M} such that if θ_i and θ'_i are distinguishable, then $S_i^{\mathcal{M}}(\theta_i) \cap S_i^{\mathcal{M}}(\theta'_i) = \emptyset$.

Proof of Proposition 1

PROPOSITION 1: If θ_i and θ'_i are indistinguishable, then $S_i^{\mathcal{M}}(\theta_i) \cap S_i^{\mathcal{M}}(\theta'_i) \neq \emptyset$ in any mechanism \mathcal{M} .

Suppose Ξ is mutually indistinguishable

Fix any finite mechanism

Proof of Proposition 1

By induction on k , for each k , i and $\Psi_i \in \Xi_i$, there exists $m_i^k(\Psi_i)$ such that $m_i^k(\Psi_i) \in S_i^k(\theta_i)$ for each $\theta_i \in \Psi_i$

1. True by definition for $k = 0$.

2. Suppose true for $k - 1$

- fix any i and $\Psi_i \in \Xi_i$
- since Ξ is mutually indistinguishable, $\exists \Psi_{-i} \in \Xi_{-i}$, R_i and, for each $\theta_i \in \Psi_i$, $\lambda_i^{\theta_i} \in \Delta(\Psi_{-i})$ such that $R_{\theta_i, \lambda_i^{\theta_i}} = R_i$
- $m_i^k(\Psi_i)$ be any element of the argmax under R_i of $g(m_i, m_{-i}^{k-1}(\Psi_{-i}))$
- by construction, $m_i^k(\Psi_i) \in S_i^{\mathcal{M}, k}(\theta_i)$ for all $\theta_i \in \Psi_i$.

Strategic Revealed Preference

PROPOSITION 2: There exists a mechanism \mathcal{M} such that if θ_i and θ'_i are distinguishable, then $S_i^{\mathcal{M}}(\theta_i) \cap S_i^{\mathcal{M}}(\theta'_i) = \emptyset$.

PROOF: By construction of "maximally revealing mechanism".

Proof of Proposition 2

KEY LEMMA: Set Ψ_i is distinguishable given Ψ_{-i} if and only if there exists $\tilde{y} : \Psi_i \rightarrow Y$ such that

$$\sum_{\theta_i \in \Psi_i} (\tilde{y}(\theta_i) - \bar{y}) = 0$$

and, for each $\theta_i \in \Psi_i$ and $\lambda_i \in \Delta(\Psi_{-i})$,

$$\tilde{y}(\theta_i) P_{\theta_i, \lambda_i} \bar{y}.$$

Proof of Proposition 2

LEMMA (Morris 1994, Samet 1998): Let V_1, \dots, V_L be closed, convex, subsets of the N -dimensional simplex Δ^N . These sets have an empty intersection if and only if there exist $z_1, \dots, z_L \in \mathbb{R}^N$ such that

$$\sum_{l=1}^L z_l = 0$$

and $v \cdot z_l > 0$ for each $l = 1, \dots, L$ and $v \in V_l$.

Key lemma follows from this lemma, letting $\Theta_i = \{1, \dots, L\}$ and V_l be the set of possible utility weights of type $\theta_i = l$ with any $\lambda_i \in \Delta(\Psi_{-i})$.

Proof of Proposition 2

TEST SET LEMMA. There exists a finite set $Y^* \subseteq Y$ such that

1. for each i , θ_i and $\lambda_i \in \Delta(\Theta_{-i})$, $B_i(\theta_i, \lambda_i) \neq Y^*$
2. for each i , Ψ_i and Ψ_{-i} , if Ψ_i is distinguishable given Ψ_{-i} , then for each $\theta_i \in \Psi_i$ and $\lambda_i \in \Delta(\Psi_{-i})$, there exists $\theta'_i \in \Psi_i$ such that

$$B_i(\theta_i, \lambda_i) \cap B_i(\theta'_i, \Psi_{-i}) = \emptyset$$

where

$$\begin{aligned} B_i(\theta_i, \lambda_i) &= \{y \in Y^* \mid y R_{\theta_i, \lambda_i} y' \text{ for all } y' \in Y^*\} \\ B_i(\theta_i, \Psi_{-i}) &= \bigcup_{\lambda_i \in \Delta(\Psi_{-i})} B_i(\theta_i, \lambda_i) \end{aligned}$$

Maximally Revealing Mechanism: In Words

- Each player makes K simultaneous announcements:
 1. an element of test set Y^*
 2. a profile of first round announcements of other players he thinks possible, plus an element of Y^*
 3. a profile of second round announcements of other players he thinks possible, plus an element of Y^*
 4.

- All chosen outcomes selected with positive probability, with much higher weight on "earlier" announcements

Maximally Revealing Mechanism: In Algebra

Mechanism $\mathcal{M}^{K,\varepsilon} = \left((M_i^K)_{i=1}^I, g^{K,\varepsilon} \right)$ parameterized by

1. $\varepsilon > 0$
2. integer K

Maximally Revealing Mechanism: In Algebra

Message Sets:

- i 's message set is M_i^K where
 - $M_i^0 = \{\bar{m}_i^0\}$
 - $M_i^{k+1} = M_{-i}^k \times Y^*$
- typical element $m_i^k = \{\bar{m}_i^0, r_i^1, y_i^1, \dots, r_i^K, y_i^K\}$

Allocation Rule:

$$g^{K,\varepsilon}(m) = \bar{y} + \frac{1 - \varepsilon^K}{(1 - \varepsilon)I} \sum_{k=1}^K \varepsilon^{k-1} \sum_{i=1}^I \mathbb{I}(r_i^k, m_{-i}^{k-1}) (y_i^k - \bar{y})$$

Conclusion of Proof of Proposition 2

1. Let

$$\bar{\Theta}_i^k(m_i^k) = \bar{\Theta}_i^k((m_i^{k-1}, r_i^k, y_i^k)) = \left\{ \theta_i \left| \begin{array}{l} \theta_i \in \bar{\Theta}_i^{k-1}(m_i^{k-1}) \\ \bar{\Theta}_{-i}^{k-1}(r_i^k) \neq \emptyset \\ y_i^k \in B_i(\theta_i, \bar{\Theta}_{-i}^{k-1}(r_i^k)) \end{array} \right. \right\}$$

2. There exists $\bar{\varepsilon} > 0$ such that

$$\left\{ \theta_i \in \Theta_i \mid m_i^k \in S_i^{\mathcal{M}^{k,\varepsilon}}(\theta_i) \right\} \subseteq \bar{\Theta}_i^k(m_i^k)$$

for all $\varepsilon \leq \bar{\varepsilon}$ and $m_i^k \in M_i^k$.

3. There exists $\bar{\varepsilon} > 0$ and K such that

$$\left\{ \theta_i \in \Theta_i \mid m_i^K \in S_i^{\mathcal{M}^{K,\varepsilon}}(\theta_i) \right\} \in \Xi_i^*$$

for all $\varepsilon \leq \bar{\varepsilon}$ and $m_i^K \in M_i^K$.

PART 3:
VIRTUAL ROBUST IMPLEMENTATION

Robust Virtual Implementation Definitions

DEFINITION: A social choice function $f : \Theta \rightarrow Y$.

Write $\|y - y'\|$ for the Euclidean distance between a pair of lotteries y and y' , i.e.,

$$\|y - y'\| = \sqrt{\sum_{x \in X} (y(x) - y'(x))^2}.$$

DEFINITION: Social choice function f is robustly ε -implementable if there exists a mechanism \mathcal{M} such that

$$m \in S^{\mathcal{M}}(\theta) \Rightarrow \|g(m) - f(\theta)\| \leq \varepsilon.$$

DEFINITION: Social choice function f is robustly virtually implementable if, for every $\varepsilon > 0$, f is robustly ε -implementable.

Robust Virtual Implementation Result

DEFINITION: Social choice function f satisfies ex post incentive compatibility if, for all i , θ_i , θ_{-i} and θ'_i :

$$u_i (f (\theta_i, \theta_{-i}), (\theta_i, \theta_{-i})) \geq u_i (f (\theta'_i, \theta_{-i}), (\theta_i, \theta_{-i})) .$$

DEFINITION: Social choice function f satisfies robust measurability if $f (\theta_i, \theta_{-i}) \neq f (\theta'_i, \theta_{-i}) \Rightarrow \theta_i$ is distinguishable from θ'_i .

THEOREM. Social choice function f is robustly virtually implementable if and only if f satisfies ex post incentive compatibility and robust measurability.

Canonical Mechanism: In Words

- Fix small $\delta > 0$ and large integer L .
- Announce message from maximally revealing mechanism plus L copies of set of indistinguishable types
- With prob. δ , follow allocation rule of maximally revealing mechanism
- For each $l = 1, \dots, L$, with prob. $\frac{1-\delta-\delta^2}{L}$, apply f to l th announcements of players
- For each $i = 1, \dots, I$, with prob. $\frac{\delta^2}{I}$, give player i a small fine if he was one of the "first" players to report a type inconsistent with his message in maximally revealing mechanism

Canonical Mechanism: In Algebra

- Write $\mathcal{M}^* = \left((M_i^*)_{i=1}^I, g^* \right)$ for the maximally revealing mechanism
- Let \mathcal{P}_i be partition of i 's types into indistinguishable sets; let $\phi_i(\theta_i)$ be the element of \mathcal{P}_i containing θ_i
- Let $f(\theta) = f^*(\phi(\theta))$ where $f^* : \mathcal{P} \rightarrow Y$
- Let $M_i = M_i^* \times \mathcal{P}_i^L$, with typical element $m_i = (m_i^0, m_i^1, \dots, m_i^L)$

Canonical Mechanism: In Algebra

Let

$$g(m) = \frac{1 - \delta - \delta^2}{L} \sum_{l=1}^L f^*(m^l) + \delta g^*(m^0) + \frac{\delta^2}{I} \sum_{i=1}^I r_i(m)$$

where

$$r_i(m) = \begin{cases} b_i, & \text{if } \exists k \text{ s.t. } m_i^k \neq \phi_i(m_i^0) \text{ and } m^j = \phi(m^0) \text{ for } j = 1, \dots, k-1 \\ \bar{y}, & \text{otherwise} \end{cases}$$

Monotonic Aggregator Environment

There exist $h_i : \Theta \rightarrow \mathbb{R}$ and $v_i : Y \times \mathbb{R} \rightarrow \mathbb{R}$ such that

1. h_i is continuous and strictly increasing in θ_i
2. v_i is continuous and, for any y, y' , there exists at most one x with $v_i(y, x) = v_i(y', x)$
3. $u_i(y, \theta) = v_i(y, h_i(\theta))$

Monotonic Aggregator Environment Results

1. Robust Monotonicity = Robust Measurability = "Contraction Property"
2. If $h_i(\theta) = \theta_i + \sum_{j \neq i} \gamma_{ij} \theta_j$, contraction property holds if and only if largest eigenvalue of matrix

$$\begin{pmatrix} 0 & |\gamma_{12}| & |\gamma_{13}| & \cdot & |\gamma_{1I}| \\ |\gamma_{21}| & 0 & |\gamma_{23}| & \cdot & |\gamma_{2I}| \\ |\gamma_{31}| & |\gamma_{32}| & 0 & \cdot & |\gamma_{3I}| \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ |\gamma_{I1}| & |\gamma_{I2}| & |\gamma_{I3}| & \cdot & 0 \end{pmatrix}$$

is greater than one.....