Seminar 4
Early Space Age
Launch Vehicle Dynamics
FRS 104, Princeton University
Robert Stengel

• The Cold War and Sputnik
• America Before Sputnik
• Technology, the State, and the Birth of Deterrence
• The ICBM and the 1st American Space Program
• Building Blocks of NASA
• Launch Vehicle Flight Dynamics

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http://www.princeton.edu/~stengel/FRS.html

Pre-World War II Rocket Societies
Political Rains and First Fruit: The Cold War and Sputnik

Development of Rocketry in the Soviet Union up to the Launch of Sputnik

German Vengeance (Aggregate) Weapons

V-1 “Buzz Bomb

A-3

A-4 (V-2)

A-4b

http://en.wikipedia.org/wiki/Aggregate_(rocket_family)
German Vengeance Weapons and Space Launch Vehicles

V-2 (A-4) Rocket

- Gyro Control
- Thrust and aero steering
- 6,084 built during WWII
- 1000+ test flights
- 3,225 launched in combat
Sänger “Antipodal” Bomber Concept
(1935-1941)

Dr. Eugen Sänger (1905-1964), Austrian designer

http://www.luft46.com/misc/sanger.html

Peenemünde, Wernher von Braun, and the German Rocket Team
Project Bumper (V-2/WAC Corporal)
1948-1950

- White Sands, NM
- Cape Canaveral, FL
- 2 stages
- 8 flights, 4 failures
- Engineering development
- High-altitude photography
- Atmospheric temperature profile
- Cosmic radiation

Nuclear Weapons and the Balance of Power

Trinity Test

Hiroshima Explosion

“Fat Man” Atomic Bomb
Balance of Power in Europe
Stalin’s 5-Yr Plan

Post-WWII Soviet Rocket Development
Dawn of the Missile Age, 1956

<table>
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<th>Hydrogen Bomb</th>
<th>First Test</th>
<th>First Deliverable Device Tested</th>
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<tr>
<td>USSR</td>
<td>Aug 1953</td>
<td>Nov 1955</td>
</tr>
<tr>
<td>US</td>
<td>Nov 1952</td>
<td>May 1956</td>
</tr>
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</table>

Sputnik 1 and the R-7  
(October 4, 1957)

R-7 (Semyorka) launch vehicle (ICBM)
The Space Race had begun

Evolution of the Soyuz Booster from the R-7
Early Russian Manned Spacecraft

<table>
<thead>
<tr>
<th>Vostok</th>
<th>Vostok</th>
<th>Soyuz</th>
</tr>
</thead>
</table>

Eight Characteristics of Soviet R&D

1) Tension between the need to borrow foreign technology and the desire to foster national creativity
2) Lack of a competitive stimulus and a planning system that inhibited innovation by production managers
3) Inhibitions due to terror (risk of failure)
4) Tensions between professionalization of R&D and adherence to the party line
5) Scarcity of skilled workers, need to keep unskilled workers occupied
6) Strong tradition of pure research, priority given to science not technology
7) Organizational distance between R&D and production
8) Less concern for economics than technical performance among R&D workers
Modern Arms and Free Men
America Before Sputnik

Bashful Behemoth
Buildup of Federal Support for R&D

- Civil War spawned National Academy of Sciences, 1863
- American pattern: research the business of diffuse entities. Army and navy arsenals the exception. American assembly lines, but lack of "command technology" of Europe.
- NACA established in 1915 [by President Woodrow Wilson] at Langley Research Center.
- 1920s: Veblen lobbies for federal support of R&D. Hoover lobbies for federal support of R&D.
- Great Depression: 1929 to 1939: longest and most severe depression ever experienced by the industrialized Western world.

US R&D in the 1930s

- 1935-1936: Universities spent about $50M for research; $6M came from the government.
- Total federal R&D: about $70M, plus $50M for social sciences and statistics.
- Industry spent $100M. Total: about $264M.
- Aloofness toward public finance of R&D, but US was not anti-tech
- Hoover and Roosevelt were big boosters of research, engineering, and federal intervention.
- Goddard, LOX, pressurized feed, integration with gasoline fuel (like kerosene, or RP-1). 83 patents
Robert Goddard (1882-1945)

- Physics Professor, Clark University, Worcester MA
- “A Method of Reaching Extreme Altitudes”, 1919
- *New York Times’ s editorial* comment (1920):

  "after the rocket quits our air … it will neither be accelerated nor maintained by the explosion of the charges it then might have left.”

  “To claim that it would be is to deny a fundamental law of dynamics”

  (The NYT Editors) expressed disbelief that Professor Goddard actually "does not know of the relation of action to reaction, and the need to have something better than a vacuum against which to react.”

- *New York Times* regretted its error on July 17, 1969, the day after Apollo 11 was launched to the Moon

Robert Goddard’ s Rockets, 1926-1941
US Rocket R&D in the 1930s

- American Rocket Society
- Guggenheim Aeronautical Laboratory, CIT
- Reaction Motors, Inc.

US R&D Through 1946

- R&D turnaround in 1941, effect of military needs, establishment of numerous federally funded labs, "Arsenal of democracy"
- Vannevar Bush, wartime science leader, favored extension of Office of Scientific Research and Development to civilian needs
- R&D without politics -- is this a good idea?
- Military R&D accounted for 62% of all federal R&D, 1948
- Henry Smyth (Princeton) report on peaceful use of atomic energy, 1944
- Atomic Energy Commission, 1945
- Canadians crack Russian atomic spy ring, 1946.
Post-World War II US Rocketry

• 1945:
  - Von Braun team to US Army
  - Navy: satellite studies

• 1948:
  - Air Force: will study satellites “at the proper time”
  - Army: begins satellite studies

• 1949:
  - 1st flight to space (Project Bumper, 244 miles)

• 1950:
  - Army: Redstone ballistic missile; launched in 1953
  - NACA: Becomes interested in rockets

• 1954:
  - Navy: Viking sub-orbital rocket
  - Army: plans for Project Orbiter based on Redstone
  - US science community proposes to orbit satellite for International Geophysical Year (1957-58)

• 1955:
  - Eisenhower: 1st US satellite should not be launched by a military rocket
  - Army: Project Orbiter cancelled
  - Navy: Project Vanguard begins, with Viking as 1st stage

Technology, the State, and the Birth of Deterrence

American attitude about science & technology up to the start of The Cold War
Military-Industrial Complex

- UN debates.
  - Baruch Plan (international control)
  - Gromyko's response (unilateral US disarmament)
- How would ban on nuclear weapons have affected the military power balance?
- Roles and missions contested within American military -- mentally inseparable from the Cold War (Daniel Yergin). Navy, ground power, air power, ...
- Tradition, size, power, sick aviation industry, reconversion of factories.
- Importance of aviation industry to US security.
- Truman's recognition (1946), continued growth of R&D in that area, including development of rockets and missiles.

Post-War Developments

- Cruise missiles (e.g., Northrop Snark) vs ballistic missiles.
- USAF got responsibility for strategic ballistic missiles
- Army got responsibility for tactical ballistic missiles.
- Marshall, 1943: Case for deterrence, long-range bomber atomic strike capability.
- 1947, George Kennan's Foreign Affairs article, "The Sources of Soviet Conduct," suggested containment.
- Truman's Board of Budget director: James Webb. Who is he?
- 1945-49: US sought a counterweight to Soviet's conventional military might. The answer was Strategic Air Command
Sub-Orbital (Sounding) Rockets
1945 - Present

Project Vanguard (1957-1959)

- Vanguard TV3 failure, December 7, 1957
- Vanguard 1 launched March 17, 1958
Juno 1 Launched Explorer 1 to Orbit

(January 31, 1958)

- Resurrection of Project Orbiter
- Juno 1 derived from
  - Redstone missile / V-2 missile

Van Allen Belts
Commonalities Between Post-War US and USSR

- Continental superstates
- Both born of revolutions inspired by ideologies of progress
- Faith in the works of man
- Patriotism rooted in
  - common ideas, values, and experience rather than
  - tradition, religion, or ethnicity ...
  - or not?
- Both came of age internationally during the world wars and prevailed because of
  - geographical expanse
  - remoteness
  - unprecedented mobilization of technical resources
- Both emerged from isolationism to find themselves superpowers

The ICBM and the First American Space Program

Development of the ICBM in America
Precursors to US ICBMs, 1946

- Convair Project MX-774.
  - Swiveling engines
  - Separable nose cone
  - Outer skin was propellant-tank wall
- RAND study of satellite launch

**Post-World War II Ballistic Missiles**

- Redstone
- Thor
- Atlas
- Titan
Resistance to ICBMs and Satellites

- Invasion of South Korea by North Korea, June 1950
- Bureaucratic resistance to ICBMs
- Guidance systems, MIT Instrumentation Lab
- Blunt body re-entry vehicle proposed, NACA Langley
- Reports of Soviet rocketry supported need for a crash ICBM program
- Atlas given highest priority, June 1954
- RAND studied utility of satellites for strategic and meteorological reconnaissance, 1950
- Political problems with over-flight, even in orbit.
  - How would the Soviets respond?
  - This worried Eisenhower
  - Policy must be charted in advance
- WS-117L

Inside NASA

NASA’s Organizational Culture: Building Blocks
1958: NACA Becomes NASA

National Advisory Committee for Aeronautics

National Aeronautics and Space Administration

1920

1939

1940

1946

1936/58

1959

1960

1962

1963

1961

NACA/NASA Research Laboratories

Ames Research Center
Moffett Field, CA

Dryden Flight Research Center
Edwards, CA

Jet Propulsion Laboratory
Pasadena, CA

NASA Shared Services Center
SSC, MS

Stennis Space Center
SSC, MS

Marshall Space Flight Center
MSFC, AL

Kennedy Space Center
KSC, FL

Glenn Research Center
Cleveland, OH

Goddard Space Flight Center
Greenbelt, MD

NASA Headquarters
Washington, DC

Langley Research Center
Hampton, VA
The Rocket Engineers

- Clash of cultures
  - ABMA
  - USAF
  - USN
  - NASA
- Sense of ownership
- Different approaches to R&D

Human Space Flight

- Began with the NACA Langley Pilotless Aircraft Research Division!
- Space Task Group

*Wallops Island, VA, 1961*
Early US Manned Spacecraft

Mercury (1959-63)

Gemini (1965-6)

“The NASA Science Centers

- Goddard Space Center
  - Near-Earth Space Craft
- Jet Propulsion Laboratory
  - Deep Space Probes

A Confederation of Cultures

- Resilience of cultures among individuals
- Norms are persistent
- Complexity and diversity of cultures within NASA

Launch Vehicle
Flight Dynamics
Position and Velocity Following Launch Determine Orbital Elements

Equations of Motion for a Point Mass

\[ \frac{d\mathbf{r}}{dt} = \mathbf{v} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \]

\[ \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \frac{1}{m} \mathbf{F} = \begin{bmatrix} 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} \]
Equations of Motion for a Point Mass

Velocity and position dynamics expressed in a single equation

\[
\dot{x}(t) = \frac{d x(t)}{dt} = f[x(t), F]
\]

Combined Equations of Motion for a Point Mass

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z} \\
\dot{v}_x \\
\dot{v}_y \\
\dot{v}_z \\
\end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
f_x/m & f_y/m & f_z/m & 0 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix} x \\
y \\
z \\
v_x \\
v_y \\
v_z \\
\end{bmatrix} + \begin{bmatrix} f_x \\
f_y \\
f_z/m \\
\end{bmatrix}
\]

With

\[
F_l = \begin{bmatrix} f_x \\
f_y \\
f_z/m \\
\end{bmatrix} = \left[ F_{\text{gravity}} + F_{\text{aerodynamics}} + F_{\text{thrust}} \right]_l
\]
Approximate Inertial Frames of Reference

- **Flat-earth approximation**
  - \( g \) is gravitational acceleration
  - \( mg \) is gravitational force
  - Independent of position

- **Round, rotating earth**
  - Inverse-square gravitation
  - “Centrifugal acceleration”
  - Non-linear function of position
  - \( \mu = 3.986 \times 10^5 \text{ km}^3/\text{s}^2 \)
  - \( \Omega = 7.29 \times 10^{-5} \text{ rad/s} \)

\[ mg_f = m \begin{bmatrix} 0 \\ 0 \\ g_o \end{bmatrix} \; ; \; g_o = 9.807 \text{ m/s}^2 \]

\[
g_r = g_{\text{gravity}} + g_{\text{rotation}} \; \text{ [rotating frame]} \]
\[
= \frac{\mu}{r^3} \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \Omega^2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} \; ; \; r = \left[ x^2 + y^2 + z^2 \right]^{1/2}
\]
Equations of Motion with Round-Earth Gravity Model
(Inertial, Non-Rotating Frame)

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z} \\
\dot{v}_x \\
\dot{v}_y \\
\dot{v}_{z-E}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\mu/r^3 & 0 & 0 & 0 & 0 & 0 \\
0 & -\mu/r^3 & 0 & 0 & 0 & 0 \\
0 & 0 & -\mu/r^3 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
v_x \\
v_y \\
v_{z-E}
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/m & 0 & 0 \\
0 & 0 & 0 & 0 & 1/m & 0
\end{bmatrix}
\begin{bmatrix}
f_x \\
f_y \\
f_z_{aero} + f_z_{thrust}
\end{bmatrix}
\]

Position of the vehicle (in spherical coordinates)

\[
\begin{align*}
r &= \begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
\cos L \cos \lambda \\
\cos L \sin \lambda \\
\sin L
\end{bmatrix} (R + h)
\end{align*}
\]

Effect of Launch Site on Launch Velocity

Launch site and azimuth

Earth’s rotation adds up to 465 m/s to final inertial velocity

Function of launch latitude and azimuth angles

\[
\Delta V_{\text{launch}} \approx \Omega R \cos L \cos \beta
\]

\( \beta \): Launch azimuth angle (rotating frame, from East)
Properties of the Lower Atmosphere

- Air density and pressure decay exponentially with altitude.
- Air temperature and speed of sound are linear functions of altitude.

US Standard Atmosphere, 1976

Lower Atmosphere Rotates With The Earth

- Zero wind at Earth’s surface = Inertially rotating air mass.
- Wind measured with respect to Earth’s rotating surface.
- Jet stream magnitude typically peaks at 10-15-km altitude.
Aerodynamic Forces

Drag measured opposite to the air-relative velocity vector
Lift and side force are perpendicular to the velocity vector

\[
\begin{bmatrix}
\text{Drag} \\
\text{Side Force} \\
\text{Lift}
\end{bmatrix} = 
\begin{bmatrix}
C_D \\
C_Y \\
C_L
\end{bmatrix} \frac{1}{2} \rho V^2 S
\]

- \( V = \text{air-relative velocity} = \text{velocity w.r.t. air mass} \)
- \( \text{Drag} \) measured opposite to the air-relative velocity vector
- \( \text{Lift} \) and \( \text{side force} \) are perpendicular to the velocity vector

Aerodynamic Force Parameters

\[ \rho = \text{air density}, \text{ function of height}, h \]
\[ = \rho_{\text{sealevel}} e^{-\beta h} \]
\[ \rho_{\text{sealevel}} = 1.225 \text{ kg/m}^3; \ \beta = 1 / 9,042 \text{ m} \]
\[ V = \left[ v_x^2 + v_y^2 + v_z^2 \right]^{1/2} = \left[ \mathbf{v}^T \mathbf{v} \right]^{1/2}, \text{ m/s} \]

Dynamic pressure \( \bar{q} = \frac{1}{2} \rho V^2, \text{ N/m}^2 \)

\( S = \text{reference area}, \text{ m}^2 \)

\[
\begin{bmatrix}
C_D \\
C_Y \\
C_L
\end{bmatrix} = \text{non-dimensional aerodynamic coefficients}
\]
Aerodynamic Drag

\[ Drag = C_D \frac{1}{2} \rho V^2 S \]

- Drag components sum to produce total drag
  - Skin friction
  - Base pressure differential
  - Forebody pressure differential (M > 1)

Aerodynamic Lift Force

\[ Lift = C_L \frac{1}{2} \rho V^2 S \approx \frac{\partial C_L}{\partial \alpha} \frac{1}{2} \rho V^2 S \]

- Angle between x axis and airstream = angle of attack, \( \alpha \)
- Lift components integrate over length to produce net lift
  - Increase in cross-sectional area
  - Tail fins
- For symmetric vehicle, lift = 0 if \( \alpha = 0 \)
2-D Equations of Motion for a Point Mass

Restrict motions to a vertical plane (i.e., motions in y direction = 0)

\[
\begin{bmatrix}
\dot{x} \\
\dot{z} \\
\dot{v}_x \\
\dot{v}_z
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
z \\
v_x \\
v_z
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & \frac{f_x}{m} \\
0 & 0 & 0 & 1 & 0 & 0 & \frac{f_z}{m}
\end{bmatrix}
\]

2-D Equations of Motion for a Point Mass

Transform velocity from Cartesian to polar coordinates

\[
\begin{bmatrix}
\dot{r} \\
\dot{h}
\end{bmatrix} =
\begin{bmatrix}
V \cos \gamma \\
V \sin \gamma
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
V \\
\gamma
\end{bmatrix} = \begin{bmatrix}
\sqrt{\dot{x}^2 + \dot{z}^2} \\
\sin^{-1} \left( \frac{h}{V} \right)
\end{bmatrix}
= \begin{bmatrix}
\text{Velocity} \\
\text{Flight path angle}
\end{bmatrix}
\]
Flat-Earth Model

- Ignore round, rotating Earth effects (!)
- i.e., assume that flat-Earth-relative frame is inertial

\[
\begin{bmatrix}
\dot{x} \\
\dot{z}
\end{bmatrix} = \begin{bmatrix}
v_x \\
v_z
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x \\
z
\end{bmatrix} + \begin{bmatrix}
f_x \\
f_z
\end{bmatrix}
\]

- Adequate model for investigating early phase of launch

Simplified Launch Trajectory Equations of Motion

- Gravity-turn, flat earth, vertical plane
  - Thrust aligned with velocity vector \((\alpha = 0)\)
  - Lift = 0
  - Round, rotating earth effects neglected

\[
\dot{V}(t) = \frac{Thrust - \left(Drag + m(t)g \sin \gamma(t)\right)}{m(t)} \\
= \left(Thrust - C_D \frac{1}{2} \rho(h)V^2(t)\right) / m(t) - g \sin \gamma(t)
\]

\[
\dot{\gamma}(t) = -g \cos \gamma(t) / V(t)
\]

\[
\dot{h}(t) = -\dot{z}(t) = V(t) \sin \gamma(t)
\]

\[
\dot{r}(t) = \dot{x}(t) = V(t) \cos \gamma(t)
\]

\(V = velocity\)  
\(\gamma = flight\ path\ angle\)  
\(h = height\ (altitude)\)  
\(r = range\)
Gravity-Turn Flight Path

- For vertical launch,
  - trajectory is vertical unless
  - vehicle is pitched over via thrust-vector control

- Following pitch-over,
  - if thrust is aligned with the velocity vector,
  - the result is called a gravity turn, with $\alpha = 0$

- Gravity-turn flight path is a function of 3 variables
  - Initial pitch-over angle (from vertical launch)
  - Velocity at pitch-over
  - Acceleration profile determined by thrust-to-weight ratio, $T/W$

Typical Velocity Loss due to Drag During Launch

- Aerodynamic effects on launch vehicle are most important below ~50-km altitude
- Maintain angle of attack and sideslip angle near zero to minimize side force and lift
- Typical velocity loss due to drag for vertical launch
  - Constant thrust-to-weight ratio
  - $C_D S/m = 0.0002 \ m^2/kg$
  - Final altitude above 80 km
Effects of Gravity and Drag on the Velocity Vector

- Significant reduction in velocity magnitude
- Strong curvature of the flight path

Thrust/Weight = $T/W = 2$

Thrust = 1960

$C_D = 0.2$

$S = 0.1$

Mass = 100

$\dot{V}(t) = \frac{\text{Thrust} - \frac{1}{2} \rho \pi \frac{1}{2} V^2(t) + mg \sin \gamma(t)}{m}$

$\dot{\gamma}(t) = -g \cos \gamma(t)/V(t)$

Typical Properties of Launch Trajectories

- Maximum dynamic pressure, Mach = 1, maximum drag, and maximum jet stream magnitude tend to occur at similar altitudes
- Aerodynamic effects on launch vehicle become negligible above ~50-km altitude
Typical Properties of Launch Trajectories

- Thrust/Weight = \( T/W = 4 \)

- Maximum dynamic pressure, Mach = 1, maximum drag, and maximum jet stream magnitude tend to occur at similar altitudes
- Aerodynamic effects on launch vehicle become negligible above ~50-km altitude

Gravity and Drag Effects during Single-Stage Orbital Launch

- Launch trajectory using flat-earth model
- Red line signifies velocity due to rocket alone
- Several km/s lost to gravity and drag

- With higher \( T/W \)
  - Shorter time to orbit
  - Increased loss due to drag
  - Decreased loss due to gravity
Typical Ariane 4 Launch Profile

Mass-Ratio Effect on Final Load Factor

• Thrust-to-weight ratio = load factor

\[ \text{Thrust} = n \text{ (load factor)} = \frac{\text{Thrust}}{m g_0} \]

\[ n_{\text{initial}} = \frac{\text{Thrust}}{m_{\text{initial}} g_0}; \quad n_{\text{final}} = \frac{\text{Thrust}}{m_{\text{final}} g_0} \]

• If thrust is constant

\[ \frac{n_{\text{final}}}{n_{\text{initial}}} = \frac{m_{\text{initial}}}{m_{\text{final}}} = \mu \]

• If thrust is reduced, limit load factor can be enforced

<table>
<thead>
<tr>
<th>Initial Load Factor</th>
<th>Final Load Factor</th>
<th>Mass Ratio = 2</th>
<th>Mass Ratio = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>2.6</td>
<td>6.5</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>
Jet Stream Profiles

• Launch vehicle must be able to fly through strong wind profiles
• Design profiles assume 95th-99th-percentile worst winds and wind shear

Aerodynamic Normal Force ~ Lift

\[ \text{Normal Force} = C_N \frac{1}{2} \rho V^2 S \approx \frac{\partial C_N}{\partial \alpha} \alpha \frac{1}{2} \rho V^2 S \]

• For small angle of attack, normal force is approximately the same as lift
Aerodynamic Pitching Moment

\[
Pitching \, Moment = C_m \frac{1}{2} \rho V^2 Sr \approx \frac{\partial C_m}{\partial \alpha} \frac{1}{2} \rho V^2 Sr
\]

\( r = \text{Reference Length} \)

- Pitching moment components integrate over length to produce net pitching moment
  - Increase in cross-sectional area
  - Tail fins

- ... plus pitching moment due to thrust vectoring for control

Angular Attitude Perturbations

- Pitch-angle perturbation, \( \Delta \theta \), is about the same as angle-of-attack perturbation, \( \Delta \alpha \)

\[
\Delta \ddot{\theta} \approx \Delta \ddot{\alpha} = \frac{\text{Net Pitching Moment}}{\text{Pitching Moment of Inertia}}
\]

- Then

\[
\Delta \ddot{\alpha} = \frac{M_{\text{net}}}{I_{yy}} \approx \frac{M_{\text{net}}}{I_{yy}} \approx \frac{1}{I_{yy}} \frac{\partial M_{\text{net}}}{\partial \alpha} \Delta \alpha + \frac{\partial M_{\text{net}}}{\partial \alpha} \Delta \alpha
\]
Attitude Stability

\[
\Delta \dot{\alpha} = \frac{M_{\text{servo}} + M_{\text{thrust}}}{I_{yy}} = \frac{M_{\text{ext}}}{I_{yy}} \approx \frac{1}{I_{yy}} \frac{\partial M_{\text{ext}}}{\partial \alpha} \Delta \dot{\alpha} + \frac{\partial M_{\text{ext}}}{\partial \alpha} \Delta \alpha
\]

- Attitude perturbations are stable if
  \[
  \frac{\partial M_{\text{ext}}}{\partial \alpha} < 0, \quad \frac{\partial M_{\text{ext}}}{\partial \alpha} < 0
  \]

- Oscillatory divergence if
  \[
  \frac{\partial M_{\text{ext}}}{\partial \alpha} > 0 \quad \text{Dynamic Instability}
  \]

- Non-oscillatory divergence if
  \[
  \frac{\partial M_{\text{ext}}}{\partial \alpha} > 0 \quad \text{Static Instability}
  \]

*Thrust-vector feedback control normally required to provide static and dynamic stability*

Typical Thrust-Vector Angle Requirements

Example: Concept study for solid-fueled Saturn-class vehicles (NASA TN D-4662, 1968)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variation</th>
<th>Apollo</th>
<th>Voyager</th>
<th>SSOPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Steady stage winds</td>
<td>99 percent</td>
<td>1.35</td>
<td>2.30</td>
<td>1.17</td>
</tr>
<tr>
<td>2. Wind gusts</td>
<td>3σ</td>
<td>.15</td>
<td>.26</td>
<td>.13</td>
</tr>
<tr>
<td>3. Thrust misalignment</td>
<td>3σ</td>
<td>.25</td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td>4. Thrust and weights</td>
<td>3σ</td>
<td>.15</td>
<td>.15</td>
<td>.15</td>
</tr>
<tr>
<td>5. Pitch program</td>
<td>maximum</td>
<td>.50</td>
<td>.50</td>
<td>.50</td>
</tr>
</tbody>
</table>

\[a\] Total consists of item 1 plus root sum square of items 2, 3, and 4.
Pitching Moment Distribution Causes Large Bending Effects

Aerodynamic and thrust-vectoring effects bend the vehicle
Trajectory shaped to reduce structural loads

Trajectory shaped to reduce structural loads
Launch Phases and Loading Issues-1

- Liftoff
  - Reverberation from the ground
  - Random vibrations
  - Thrust transients
- Winds and Transonic Aerodynamics
  - High-altitude jet stream
  - Buffeting
- Staging
  - High sustained acceleration
  - Thrust transients

Launch Phases and Loading Issues-2

- Heat shield separation
  - Mechanical and pyrotechnic transients
- Spin stabilization
  - Tangential and centripetal acceleration
  - Steady-state rotation
- Separation
  - Pyrotechnic transients
Next Time:

*NASA Enchanted Rendezvous Houbolt.pdf*

**Rocket Propulsion and Staging:** [Understanding Space] Sec 14.2

**Antecedents to the Apollo Program:** [... the Heavens and the Earth] Ch 5 to 7; [Inside NASA] Ch 2; [Enchanted Rendezvous]

Supplemental Material
Drag Coefficients of Cones and Cone Frustums

\[ M = \frac{\text{Air - RelativeVelocity}}{\text{Speed of Sound}} \]