Political Rains and First Fruit:
The Cold War and Sputnik
German Vengeance (Aggregate) Weapons

http://en.wikipedia.org/wiki/Aggregate_(rocket_family)

V-1 “Buzz Bomb

A-3

A-4 (V-2)

A-4b

V-2 (A-4) Rocket

- Gyro Control
- Thrust and aero steering
- 6,084 built during WWII
- 1000+ test flights
- 3,225 launched in combat
Peenemünde, Wernher von Braun, and the German Rocket Team

German Vengeance Weapons and Space Launch Vehicles
Sänger “Antipodal” Bomber Concept
(1935-1941)

Dr. Eugen Sänger (1905-1964), Austrian designer

Sub-Orbital “Skip” Trajectory

http://www.luft46.com/misc/sanger.html

Nuclear Weapons and the
Balance of Power

Trinity Test

Hiroshima Explosion

“Fat Man” Atomic Bomb

“Little Boy” Atomic Bomb

“Enola Gay” B-29

Hiroshima Explosion

Trinity Test
Balance of Power in Europe
Stalin’s 5-Yr Plan

Post-WWII Soviet Rocket Development
Sputnik 1 and the R-7
(October 4, 1957)
R-7 (Semyorka) launch vehicle (ICBM)

Eight Characteristics of Soviet R&D

1) **Need** to borrow foreign technology and **desire** to foster national creativity
2) Lack of competitive stimulus and planning system that **inhibited innovation** by production managers
3) Inhibitions due to **terror** (risk of failure)
4) **Tensions** between professionalization of R&D and adherence to the party line
5) **Scarcity** of skilled workers, need to keep unskilled workers occupied
6) **Tradition** of pure research, priority to science not technology
7) Organizational **distance** between R&D and production
8) Less concern for economics than **technical performance**
America Before Sputnik; Technology, the State, and the Birth of Deterrence

Buildup of Federal Support for R&D

- **Civil War spawned** National Academy of Sciences, 1863
- **American pattern**
  - Diffuse entities for research
  - Army and navy arsenals the exception, American assembly lines
  - Lack of European "command technology"
- **NACA established in 1915 [by President Woodrow Wilson] at Langley Research Center**
- **1920s**
  - Veblen lobbies for federal support of R&D
  - Hoover lobbies for federal support of R&D
- **Great Depression, 1929 to 1939**
Robert Goddard (1882-1945)

- Physics Professor, Clark University, Worcester MA
- “A Method of Reaching Extreme Altitudes”, 1919
- *New York Times*’s editorial comment (1920):

> "after the rocket quits our air … it will neither be accelerated nor maintained by the explosion of the charges it then might have left.”

> “To claim that it would be is to deny a fundamental law of dynamics”

(The NYT Editors) expressed disbelief that Professor Goddard actually "does not know of the relation of action to reaction, and the need to have something better than a vacuum against which to react.”

*New York Times* regretted its error on July 17, 1969, the day after Apollo 11 was launched to the Moon

Robert Goddard’s Rockets, 1926-1941
US Rocket R&D in the 1930s

American Rocket Society
Guggenheim Aeronautical Laboratory, CIT
Reaction Motors, Inc.

US R&D Through 1948

- **1941**: R&D turnaround
  - Military needs
  - Federally funded labs
  - "Arsenal of democracy"
- Vannevar Bush: Civilian extension of Office of Scientific Research and Development
- Henry Stimson and James Forrestal (Princeton, Forrestal campus), Research Board for National Security
- R&D without politics – good idea?
- **1944**: Henry Smyth (Princeton) report on peaceful use of atomic energy
- **1945**: Atomic Energy Commission
- **1946**: Canadians crack Russian atomic spy ring
- **1948**: Military R&D: 62% of federal total
Post-War Developments

- **1943**: Marshall: Deterrence, long-range bomber atomic strike capability
- **1947**: George Kennan's *Foreign Affairs* article, "The Sources of Soviet Conduct": containment
- Truman's Board of Budget director: James Webb
- **1945-49**: US sought a counterweight to Soviet's conventional military might: Strategic Air Command
- Cruise missiles (e.g., Northrop *Snark*) vs ballistic missiles
- USAF: responsibility for strategic ballistic missiles
- Army: responsibility for tactical ballistic missiles

---

Project Bumper (V-2/WAC Corporal)

*1948-1950*

- White Sands, NM
- Cape Canaveral, FL
- 2 stages
- 8 flights, 4 failures
- Engineering development
- High-altitude photography
- Atmospheric temperature profile
- Cosmic radiation
Dawn of the Missile Age, 1956

<table>
<thead>
<tr>
<th>Hydrogen Bomb</th>
<th>First Test</th>
<th>First Deliverable Device Tested</th>
</tr>
</thead>
<tbody>
<tr>
<td>USSR</td>
<td>Aug 1953</td>
<td>Nov 1955</td>
</tr>
<tr>
<td>US</td>
<td>Nov 1952</td>
<td>May 1956</td>
</tr>
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</table>

Contest for influence in the “3rd World”

The ICBM and the First American Space Program
Precursors to US ICBMs, 1946-1948

- Convair Project MX-774
  - Swiveling engines
  - Separable nose cone
  - Outer skin was propellant-tank wall
- RAND study of satellite launch

Post-World War II Science Fact and Fiction

Catalyzed human imagination
Hydrogen Bombs, 1953-1954

Military-Industrial Complex

- UN debates
  - Baruch Plan (international control)
  - Gromyko's response (unilateral US disarmament)
- How would ban on nuclear weapons have affected military power balance?
- Roles and missions contested within American military -- inseparable from the Cold War (Daniel Yergin)
  - Navy, ground power, air power, ...
- Tradition, size, power, sick aviation industry, conversion of factories
- Importance of aviation industry to US security
- Truman's recognition (1946)
  - Continued growth of aviation R&D, including rockets and missiles
Resistance to ICBMs and Satellites

- Invasion of South Korea by North Korea, June 1950
- Bureaucratic resistance to ICBMs
- Guidance systems, MIT Instrumentation Lab
- Blunt body re-entry vehicle proposed, NACA Langley
- Reports of Soviet rocketry supported need for a crash ICBM program
- Atlas given highest priority, June 1954
- RAND studied utility of satellites for strategic and meteorological reconnaissance, 1950
- Political problems with over-flight, even in orbit.
  - How would the Soviets respond?
  - This worried Eisenhower
  - Policy must be charted in advance
- WS-117L

Post-World War II Intermediate-Range and Inter-Continental Ballistic Missiles
USAF Project A119, 1958
A proposal to “nuke” the Moon

http://en.wikipedia.org/wiki/Project_A119#cite_note-FOIA-19

Commonalities Between Post-War US and USSR

- Continental superstates
- Born of revolutions inspired by ideology of progress
- Faith in the works of man
- Patriotism rooted in
  - common ideas, values, and experience rather than
  - tradition, religion, or ethnicity ...
  - or not?
- Came of age internationally during the world wars and prevailed because of
  - geographical expanse
  - remoteness
  - unprecedented mobilization of technical resources
- Emerged from isolationism to become superpowers
President Dwight D. Eisenhower, 1953-61

“The Chance for Peace” Address
American Society of Newspaper Editors
April 16, 1953

http://www.eisenhower.archives.gov/all_about_ike/speeches/chance_for_peace.pdf

“IN THIS SPRING of 1953 the free world weighs one question above all others: the chance for a just peace for all peoples….

…another recent moment of great decision…. that yet more hopeful spring of 1945, bright with the promise of victory and of freedom…. a just and lasting peace….

This common purpose lasted an instant and perished. The nations of the world divided to follow two distinct roads. The United States and our valued friends, the other free nations, chose one road. The leaders of the Soviet Union chose another….

We are ready, in short, to dedicate our strength to serving the needs, rather than the fears, of the world…..”
Post-World War II US Rocketry

• **1945:**
  - Army: Von Braun team
  - Navy: satellite studies

• **1948:**
  - Air Force: will study satellites “at the proper time”
  - Army: begins satellite studies

• **1949:**
  - 1st flight to space (Project Bumper, 244 miles)

• **1950:**
  - Army: Redstone ballistic missile; launched in 1953
  - NACA: Becomes interested in rockets

• **1954:**
  - Navy: Viking sub-orbital rocket
  - Army: plans for Project Orbiter based on Redstone
  - Proposes to orbit scientific satellite for International Geophysical Year (1957-58)

• **1955:**
  - Eisenhower: 1st US satellite should not be launched by military rocket
  - Army: Project Orbiter cancelled
  - Navy: Project Vanguard begins, with Viking as 1st stage

---

**Project Vanguard (1957-1959)**

- Vanguard TV3 failure, December 7, 1957
- Vanguard 1 launched March 17, 1958
Juno 1 Launched Explorer 1 to Orbit  
(January 31, 1958)

- Resurrection of Project Orbiter
- Juno 1 derived from  
  - Redstone missile / V-2 missile

Van Allen Belts
Launch Phases and Loading Issues-1

- **Liftoff**
  - Reverberation from the ground
  - Random vibrations
  - Thrust transients
- **Winds and Transonic Aerodynamics**
  - High-altitude jet stream
  - Buffeting
- **Staging**
  - High sustained acceleration
  - Thrust transients
Launch Phases and Loading Issues-2

- Heat shield separation
  - Mechanical and pyrotechnic transients
- Spin stabilization
  - Tangential and centripetal acceleration
  - Steady-state rotation
- Separation
  - Pyrotechnic transients

Equations of Motion for a Point Mass

\[
\frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \mathbf{v} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}
\]

\[
\frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \frac{1}{m} \mathbf{F} = \begin{bmatrix} 1/m & 0 & 0 & f_x \\ 0 & 1/m & 0 & f_y \\ 0 & 0 & 1/m & f_z \end{bmatrix}
\]
Equations of Motion for a Point Mass

Velocity and position dynamics expressed in a single equation

\[ \dot{x}(t) = \frac{d\mathbf{x}(t)}{dt} = \mathbf{f}[\mathbf{x}(t), \mathbf{F}] \]

Combined Equations of Motion for a Point Mass

\[ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ f_x/m & f_y/m & f_z/m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} \]

With

\[ \mathbf{F}_I = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{\text{gravity}} + \mathbf{F}_{\text{aerodynamics}} + \mathbf{F}_{\text{thrust}} \end{bmatrix}_I \]
Force due to Gravity

- Flat-earth approximation
  - \( g \) is gravitational acceleration
  - \( mg \) is gravitational force
  - Independent of position
- Round, rotating earth
  - Inverse-square gravitation
  - “Centripetal acceleration”
  - Non-linear function of position
  - \( \mu = 3.986 \times 10^5 \text{ km}^3/\text{s}^2 \)
  - \( \Omega = 7.29 \times 10^{-5} \text{ rad/s} \)

\[
mg_f = m \begin{bmatrix} 0 \\ 0 \\ g_o \end{bmatrix} ; \quad g_o = 9.807 \text{ m/s}^2
\]

\[
g_r = \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = g_{\text{gravity}} \quad \text{[non-rotating frame]}
\]

\[
g_r = g_{\text{gravity}} + g_{\text{rotation}} \quad \text{[rotating frame]}
\]

\[
\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \Omega^2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; \quad r = \left[ x^2 + y^2 + z^2 \right]^{1/2}
\]

Equations of Motion with Round-Earth Gravity Model
(Inertial, Non-Rotating Frame)

\[
\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\mu/r^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\mu/r^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu/r^3 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} + \begin{bmatrix} f_{\text{aero}} \\ f_{\text{thrust}} \end{bmatrix}
\]

Position of the vehicle (in spherical coordinates)

\[
r = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos L \cos \lambda \\ \cos L \sin \lambda \\ \sin L \end{bmatrix} (R + h)
\]

\( R \): Earth’s radius
\( h \): Altitude (height)
\( L \): Latitude
\( \lambda \): Longitude
Effect of Launch Site on Launch Velocity

Launch site and azimuth
Earth’s rotation adds up to 465 m/s to final inertial velocity
Function of launch latitude and azimuth angles

\[ \Delta V_{\text{launch}} \approx \Omega R \cos \theta \cos \beta \]

\( \beta \): Launch azimuth angle (rotating frame, from East)

Properties of the Lower Atmosphere

- Air density and pressure decay exponentially with altitude
- Air temperature and speed of sound are linear functions of altitude

US Standard Atmosphere, 1976
Lower Atmosphere Rotates With The Earth

- Zero wind at Earth’s surface = Inertially rotating air mass
- Wind measured with respect to Earth’s rotating surface
- Jet stream magnitude typically peaks at 10-15-km altitude

Jet Stream (typical)

Aerodynamic Forces

\[
\begin{bmatrix}
\text{Drag} \\
\text{Side Force} \\
\text{Lift}
\end{bmatrix} = \begin{bmatrix}
C_D \\
C_Y \\
C_L
\end{bmatrix} \frac{1}{2} \rho V^2 S
\]

- \( V \) = air-relative velocity = velocity w.r.t. air mass
- Drag measured opposite to the air-relative velocity vector
- Lift and side force are perpendicular to the velocity vector
Aerodynamic Force Parameters

\[ \rho = \text{air density}, \text{ function of height, } h \]
\[ = \rho_{\text{sealevel}} e^{-\beta h} \]

\[ \rho_{\text{sealevel}} = 1.225 \text{ kg/ m}^3; \quad \beta = 1/9,042 \text{ m} \]

\[ V = \left[ v_x^2 + v_y^2 + v_z^2 \right]^{1/2} = \left[ \mathbf{v}^T \mathbf{v} \right]^{1/2}, \text{ m/s} \]

Dynamic pressure \( \bar{q} = \frac{1}{2} \rho V^2, \text{ N/ m}^2 \)

\[ S = \text{reference area, } \text{ m}^2 \]

\[ \begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} = \text{non-dimensional aerodynamic coefficients} \]

Aerodynamic Drag

\[ \text{Drag} = C_D \frac{1}{2} \rho V^2 S \]

- Drag components sum to produce total drag
  - Skin friction
  - Base pressure differential
  - Forebody pressure differential (\( M > 1 \))
Flat-Earth 2-D Equations of Motion for a Point Mass

Restrict motions to a vertical plane (i.e., motions in y direction = 0)

\[
\begin{bmatrix}
\dot{x} \\
\dot{z} \\
\dot{v}_x \\
\dot{v}_z \\
\end{bmatrix}
= 
\begin{bmatrix}
v_x \\
v_z \\
f_x/m \\
f_z/m \\
\end{bmatrix}
= 
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x \\
z \\
v_x \\
v_z \\
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0 \\
1/m \\
0 \\
\end{bmatrix}
\begin{bmatrix}
f_x \\
f_z \\
\end{bmatrix}
\]

Flat-Earth 2-D Equations of Motion for a Point Mass

Transform velocity from Cartesian to polar coordinates

\[
\begin{bmatrix}
\dot{x} \\
\dot{z} \\
\dot{\hat{h}} \\
\end{bmatrix}
\triangleq 
\begin{bmatrix}
\dot{r} \\
\dot{\gamma} \\
\end{bmatrix}
= 
\begin{bmatrix}
V \cos \gamma \\
V \sin \gamma \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
V \\
\gamma \\
\end{bmatrix}
= 
\begin{bmatrix}
\sqrt{\dot{x}^2 + \dot{z}^2} \\
\sin^{-1}\left(\frac{\dot{\hat{h}}}{V}\right) \\
\end{bmatrix}
\begin{bmatrix}
Velocity \\
Flight path angle \\
\end{bmatrix}
\]
Flat-Earth Model

- Ignore round, rotating Earth effects (!)
- i.e., assume that flat-Earth-relative frame is inertial

\[
\begin{bmatrix}
\dot{x} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
z
\end{bmatrix} +
\begin{bmatrix}
f_x \\
f_z
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{x} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
V \cos \gamma \\
-V \sin \gamma
\end{bmatrix}
\]

- Adequate model for investigating early phase of launch

Simplified Launch Trajectory Equations of Motion

- Gravity-turn, flat earth, vertical plane
  - Thrust aligned with velocity vector \((\alpha = 0)\)
  - Lift = 0
  - Round, rotating earth effects neglected

\[
\dot{V}(t) = \frac{\text{Thrust} - \left[\text{Drag} + m(t)g \sin \gamma(t)\right]}{m(t)}
= \left[\left(\text{Thrust} - C_D V^2(t) \frac{1}{2} \rho h V^2(t)\right) / m(t) - g \sin \gamma(t)\right]
\]

\[
\dot{\gamma}(t) = -\frac{g \cos \gamma(t)}{V(t)}
\]

\[
\dot{h}(t) = \dot{z}(t) = V(t) \sin \gamma(t)
\]

\[
\dot{r}(t) = \dot{x}(t) = V(t) \cos \gamma(t)
\]

\(V = \text{velocity}\)

\(\gamma = \text{flight path angle}\)

\(h = \text{height (altitude)}\)

\(r = \text{range}\)
Gravity-Turn Flight Path

• For vertical launch,
  – trajectory is vertical unless
  – vehicle is pitched over via thrust-vector control

• Following pitch-over,
  – if thrust is aligned with the velocity vector,
  – the result is called a gravity turn, with $a = 0$

• Gravity-turn flight path is a function of 3 variables
  – Initial pitch-over angle (from vertical launch)
  – Velocity at pitch-over
  – Acceleration profile determined by thrust-to-weight ratio, $T/W$

Typical Velocity Loss due to Drag During Launch

• Aerodynamic effects on launch vehicle are most important below ~50-km altitude

• Maintain angle of attack and sideslip angle near zero to minimize side force and lift

• Typical velocity loss due to drag for vertical launch
  – Constant thrust-to-weight ratio
  – $C_D S/m = 0.0002 \ m^2/kg$
  – Final altitude above 80 km
Effects of Gravity and Drag on the Velocity Vector

- Significant reduction in velocity magnitude
- Strong curvature of the flight path

\[
\dot{V}(t) = \frac{\text{Thrust} - C_p S V^2(t) + mg \sin \gamma(t)}{m} \]

\[
\dot{\gamma}(t) = -g \cos \gamma(t) / V(t)
\]

Thrust/Weight = T/W = 2

Thrust = 1960

\( C_p = 0.2 \)

\( S = 0.1 \)

Mass = 100

Typical Properties of Launch Trajectories

- Maximum dynamic pressure, Mach = 1, maximum drag, and maximum jet stream magnitude tend to occur at similar altitudes
- Aerodynamic effects on launch vehicle become negligible above ~50-km altitude

Thrusted/Weight = 2
Typical Properties of Launch Trajectories

- Thrust/Weight = 4

- Maximum dynamic pressure, Mach = 1, maximum drag, and maximum jet stream magnitude tend to occur at similar altitudes

- Aerodynamic effects on launch vehicle become negligible above ~50-km altitude

Gravity and Drag Effects during Single-Stage Orbital Launch

- Launch trajectory using flat-earth model
- Red line signifies velocity due to rocket alone
- Several km/s lost to gravity and drag

- With higher T/W
  - Shorter time to orbit
  - Increased loss due to drag
  - Decreased loss due to gravity
Typical Ariane 4 Launch Profile

Mass-Ratio Effect on Final Load Factor

- Thrust-to-weight ratio = load factor

\[
\frac{\text{Thrust}}{\text{Weight}} = n = \left( \frac{\text{load factor}}{mg_o} \right) = \frac{\text{Thrust}}{m_{\text{initial}}g_o}
\]

- If thrust is constant

\[
\frac{n_{\text{final}}}{n_{\text{initial}}} = \frac{m_{\text{initial}}}{m_{\text{final}}} = \mu
\]

- If thrust is reduced, limit load factor can be enforced

<table>
<thead>
<tr>
<th>Initial Load Factor</th>
<th>Mass Ratio = 2</th>
<th>Mass Ratio = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>2.6</td>
<td>6.5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>15</td>
</tr>
</tbody>
</table>
Expendable vs. Reusable Launch Vehicles

- **Expendable Vehicle**
  - Low cost per vehicle
  - New vehicle for each launch
  - Low structural ratio
  - Continued production
  - Launch preparation
  - Upgrade in production

- **Reusable Vehicle**
  - High initial cost
  - High structural ratio
  - Maintenance and repair
  - Non-reusable parts and supplies
  - Launch preparation
  - Return to launch site
  - Upgrade
  - Replacement cost

Evolution of the Soyuz Booster from the R-7
“Big Dumb Boosters”, c. 1963

**Objective: 450,000 kg to low earth orbit**

**Douglas Single-Stage-to-Orbit**
- Plug nozzle
- Nozzle = Reentry Heat Shield
- Fully recoverable

**General Dynamics, Martin, and Douglas Concepts**
- 1-1/2 stage, fully recoverable
- Recovery at sea
- Ducted rocket

---

**SATURN-NOVA COMPARISON**

- C-1: 21.9' dia., 21' 9.5' long
- C-5: 26' long
- NOVA: 22' dia., 40' long
Specific Energy Contributed in Boost Phase

- **Total Energy** = Kinetic plus Potential Energy (relative to flat earth)

\[ E = \frac{mV^2}{2} + mgh \]

- **Specific Total Energy** = Energy per unit weight = Energy Height (km)

\[ E' = \frac{V^2}{2g} + h \]
Specific Energy Contributed in Boost Phase

- Specific Energy contributed by first stage of launch vehicle
  - Less remaining drag loss (typical)
  - Plus Earth’s rotation speed (typical)

<table>
<thead>
<tr>
<th>Altitude, km</th>
<th>Mach Number</th>
<th>Earth-Relative Velocity, km/s</th>
<th>Remaining Drag Loss, km/s</th>
<th>Earth Rotation Speed, km/s</th>
<th>Specific Kinetic Energy, km</th>
<th>Total Specific Energy, km</th>
<th>Percent of Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scout 1st-Stage Burnout</td>
<td>22</td>
<td>4</td>
<td>1.2</td>
<td>0.05</td>
<td>0.4</td>
<td>123.42</td>
<td>145.42</td>
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<tr>
<td>Subsonic Horizontal Launch</td>
<td>12</td>
<td>0.8</td>
<td>0.235</td>
<td>0.15</td>
<td>0.4</td>
<td>12.05</td>
<td>24.05</td>
</tr>
<tr>
<td>Supersonic Horizontal Launch</td>
<td>25</td>
<td>3</td>
<td>0.93</td>
<td>0.04</td>
<td>0.4</td>
<td>85.57</td>
<td>110.57</td>
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<tr>
<td>Scramjet Horizontal Launch</td>
<td>50</td>
<td>12</td>
<td>3.6</td>
<td>0</td>
<td>0.4</td>
<td>829.19</td>
<td>879.19</td>
</tr>
<tr>
<td>Target Orbit</td>
<td>300</td>
<td>25</td>
<td>7.4</td>
<td>0.4</td>
<td>3403.34</td>
<td>3703.34</td>
<td></td>
</tr>
</tbody>
</table>

Aerodynamic Lift Force

\[ \text{Lift} = C_L \frac{1}{2} \rho V^2 S \approx \frac{\partial C_L}{\partial \alpha} \alpha \frac{1}{2} \rho V^2 S \]

- Angle between x axis and airstream = angle of attack, \( \alpha \)
- Lift components integrate over length to produce net lift
  - Increase in cross-sectional area
  - Tail fins
- For symmetric vehicle, lift = 0 if \( \alpha = 0 \)
Aerodynamic Normal Force ~ Lift

\[ Normal Force = C_N \frac{1}{2} \rho V^2 S = \frac{\partial C_N}{\partial \alpha} \frac{1}{2} \rho V^2 S \]

- For small angle of attack, normal force is approximately the same as lift

Aerodynamic Pitching Moment

\[ Pitching Moment = C_m \frac{1}{2} \rho V^2 Sr = \frac{\partial C_m}{\partial \alpha} \frac{1}{2} \rho V^2 Sr \]

\[ r = Reference \text{ Length} \]

- Pitching moment components integrate over length to produce net pitching moment
  - Increase in cross-sectional area
  - Tail fins

- ... plus pitching moment due to thrust vectoring for control
Pitching Moment Distribution Causes Large Bending Effects

Aerodynamic and thrust-vectoring effects bend the vehicle
Trajectory shaped to reduce structural loads

Angular Attitude Perturbations

- Pitch-angle perturbation, $\Delta \theta$, is about the same as angle-of-attack perturbation, $\Delta \alpha$

$$\Delta \theta \approx \Delta \alpha = \frac{\text{Net Pitching Moment}}{\text{Pitching Moment of Inertia}}$$

- Then

$$\Delta \alpha = \frac{M_{\text{aero}} + M_{\text{thrust}}}{I_{yy}} \equiv \frac{M_{\text{net}}}{I_{yy}} \approx \frac{1}{I_{yy}} \left[ \frac{\partial M_{\text{net}}}{\partial \alpha} \Delta \alpha + \frac{\partial M_{\text{net}}}{\partial \alpha} \Delta \alpha \right]$$
Attitude Stability

\[ \Delta \dot{\alpha} = \frac{M_{\text{atroc}} + M_{\text{thrust}}}{I_{yy}} \approx \frac{M_{\text{atroc}}}{I_{yy}} \approx \frac{1}{I_{yy}} \left( \frac{\partial M_{\text{atroc}}}{\partial \alpha} \Delta \dot{\alpha} + \frac{\partial M_{\text{atroc}}}{\partial \alpha} \Delta \alpha \right) \]

- Attitude perturbations are stable if
  \[ \frac{\partial M_{\text{atroc}}}{\partial \alpha} < 0, \quad \frac{\partial M_{\text{atroc}}}{\partial \alpha} < 0 \]
- Oscillatory divergence if
  \[ \frac{\partial M_{\text{atroc}}}{\partial \alpha} > 0 \quad \text{Dynamic Instability} \]
- Non-oscillatory divergence if
  \[ \frac{\partial M_{\text{atroc}}}{\partial \alpha} > 0 \quad \text{Static Instability} \]

Thrust-vector feedback control normally required to provide static and dynamic stability

Jet Stream Profiles

- Launch vehicle must be able to fly through strong wind profiles
- Design profiles assume 95th-99th-percentile worst winds and wind shear

![Jet Stream Profiles Diagram](image)
Typical Thrust-Vector Angle Requirements

Example: Concept study for solid-fueled Saturn-class vehicles (NASA TN D-4662, 1968)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variation</th>
<th>Apollo</th>
<th>Voyager</th>
<th>SSOPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflection angle, deg</td>
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<td></td>
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<tr>
<td>1. Steady stage winds</td>
<td>99 percent</td>
<td>1.35</td>
<td>2.30</td>
<td>1.17</td>
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<td>2. Wind gusts</td>
<td>3σ</td>
<td>.15</td>
<td>.26</td>
<td>.13</td>
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<tr>
<td>3. Thrust misalignment</td>
<td>3σ</td>
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<td>.25</td>
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<tr>
<td>4. Thrust and weights</td>
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<td>.15</td>
<td>.15</td>
<td>.15</td>
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<tr>
<td>5. Pitch program</td>
<td>maximum</td>
<td>.50</td>
<td>.50</td>
<td>.50</td>
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<tr>
<td>Total</td>
<td></td>
<td>1.68</td>
<td>2.69</td>
<td>1.99</td>
</tr>
</tbody>
</table>

*Total consists of item 1 plus root sum square of items 2, 3, and 4.

Trajectory shaped to reduce structural loads
Next Time:

Antecedents to the Apollo Program:
[... the Heavens and the Earth] Ch 6 to 9

Rocket Propulsion and Staging:
[Understanding Space] Sec 14.2

Supplemental Material
US R&D in the 1930s

- 1935-1936: Universities spent about $50M for research; $6M came from the government.
- Total federal R&D: about $70M, plus $50M for social sciences and statistics.
- Industry spent $100M. Total: about $264M.
- Aloofness toward public finance of R&D, but US was not anti-tech
- Hoover and Roosevelt were big boosters of research, engineering, and federal intervention.
- Goddard, LOX, pressurized feed, integration with gasoline fuel (like kerosene, or RP-1). 83 patents