Seminar 5

Antecedents to the Apollo Program
Rocket Propulsion and Staging

FRS 148, Princeton University
Robert Stengel

- *The Satellite Decision*
- *"A New Era of History" and a Media Riot*
- *The Birth of NASA*

Chemical Rocket Fundamentals
Multi-Stage Rockets

… the Heavens and the Earth, Ch 5 to 7
Understanding Space, Sec 14.2

1

The Satellite Decision

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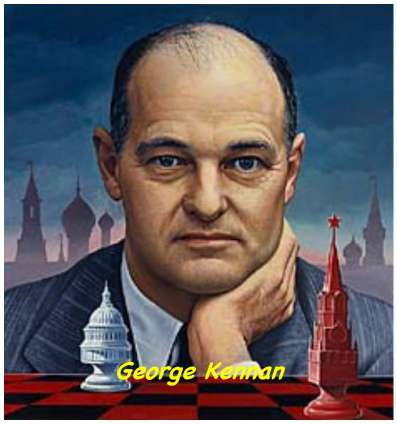
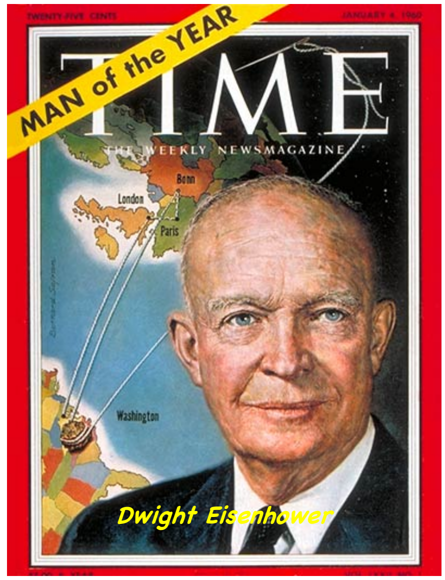
Eisenhower and the Cynical Critic





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President Dwight D. Eisenhower, 1953-61

"The Chance for Peace" Address
American Society of Newspaper Editors
April 16, 1953

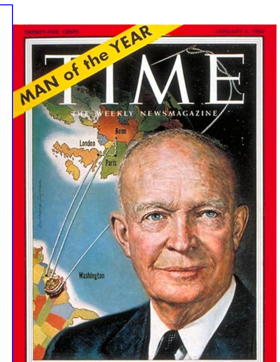
http://www.eisenhower.archives.gov/all_about_ike/speeches/chance_for_peace.pdf

"IN THIS SPRING of 1953 the free world weighs one question above all others: the chance for a just peace for all peoples….

…another recent moment of great decision…. that yet more hopeful spring of 1945, bright with the promise of victory and of freedom…. a just and lasting peace….

This common purpose lasted an instant and perished. The nations of the world divided to follow two distinct roads. The United States and our valued friends, the other free nations, chose one road. The leaders of the Soviet Union chose another….

We are ready, in short, to dedicate our strength to serving the needs, rather than the fears, of the world….."



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The Killian Report (1954)

- Period I (54-55)
  - Vulnerability to attack
  - Lack of reliable warning
  - Inadequate air defense
  - Growing Soviet bomber force
- Period II (56-60)
  - Great offensive advantage of US
  - Opportunity for diplomatic initiatives
- Period III (60-?)
  - Rapid increase in Soviet bombers
- Period IV (--)
  - Mutually assured retaliation and destruction

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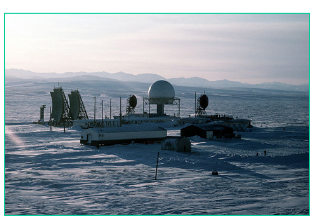
DEW Line and the CIA's U-2

- Need for warning and surveillance
- Secret overflights of the Soviet Union



http://en.wikipedia.org/wiki/Space_policy_of_the_United_States

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Post-World War II US Rocketry

- **1945-54: Von Braun team to US Army**
  - **(V-2 + JPL 2nd stage) flies to space**
  - **Satellite launch studies**
  - **Redstone missile based on V-2**
  - **Viking scientific sounding rocket**
- **1954: Recognition of need for military surveillance satellites**

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Post-World War II US Rocketry

- **1955: Decision to launch "civilian" satellite during International Geophysical Year**
  - **Political implications of overflight**
  - **No concern to be 1st to orbit**
  - **Project Orbiter based on Redstone**
  - **Project Vanguard based on Viking**

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IGY, Politics, and Defense

- USSR and US stated intent to orbit a satellite
- Proposed US scientific satellite launchers
  - Redstone
  - Vanguard
  - Atlas
- What led to the choice of Vanguard for IGY?
- Which were more important to US: military or scientific satellites?

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Ike's Policies and Style

- Budget surplus
  - Sound economy
  - adequate defense
  - unmortgaged future
- Dominant boss, but seen as slothful and senile
- Fiscally conservative, but challenged by need for progress

- "… get the Federal Government out of every unnecessary activity."
- Sputnik demanded a response in kind
- Eisenhower space policy: sufficiency not superiority

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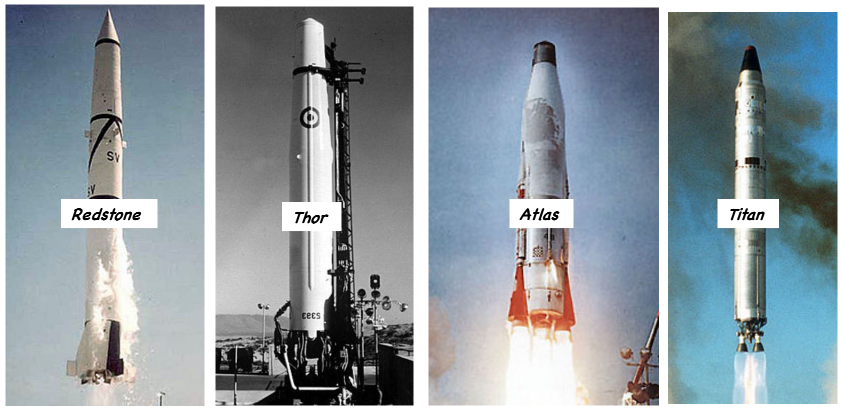
Post-World War II Intermediate-Range and Inter-Continental Ballistic Missiles



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Post-WW II Science Fact and Fiction

Catalyzed human imagination

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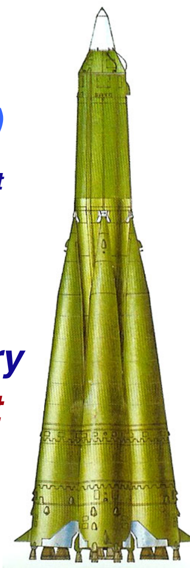
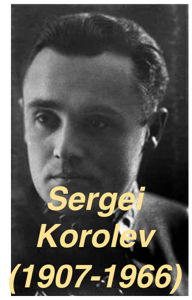
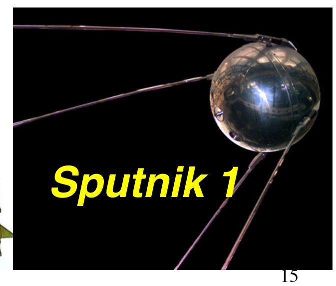
Sputnik 1 and the R-7
(October 4, 1957)

- USSR launches 1st satellite for IGY with ICBM
- Solved the US overflight quandary
- ... but that was not the way the American public saw it







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"A New Era of History" and a Media Riot

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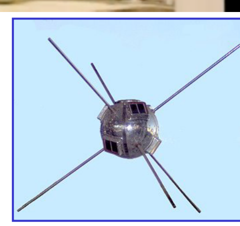
Project Vanguard (1957-1959)

Vanguard 1 success, *March 17, 1958*

[still in orbit]



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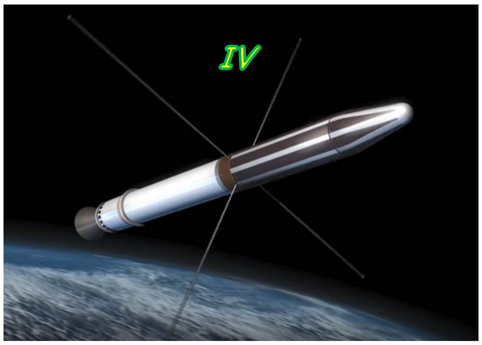
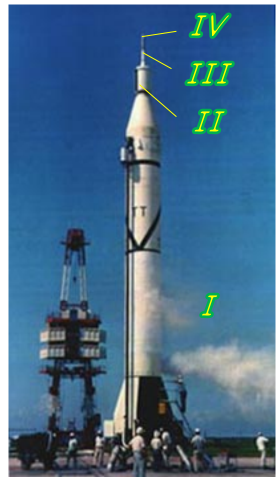
Project Orbiter Resurrected

(January 31, 1958)





Explorer 1 (Army/JPL)

Launched 84 days after approval

V-2 -> Redstone -> 4-stage Juno 1

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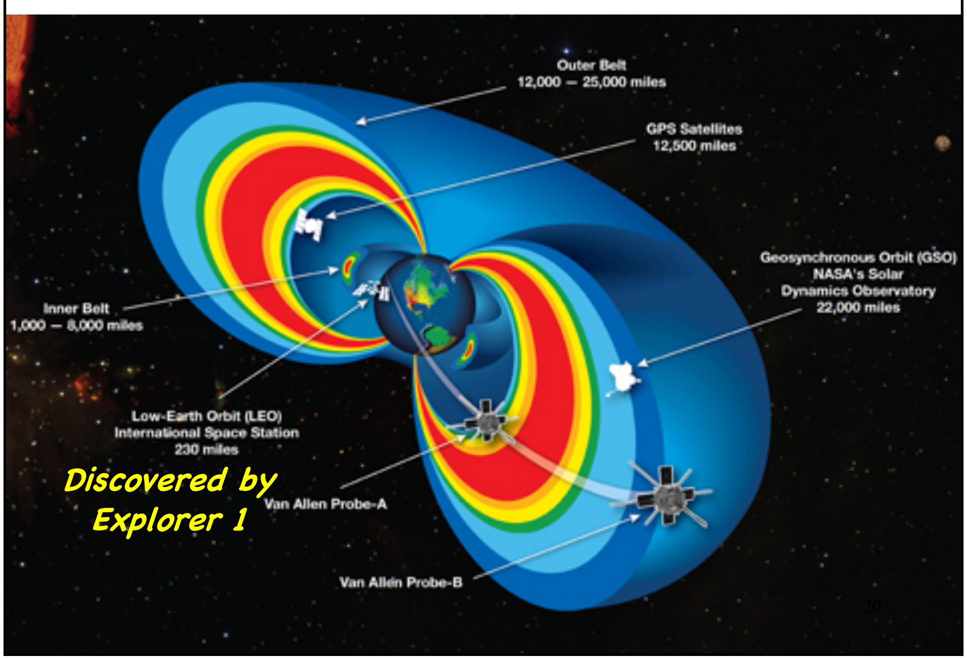
Van Allen Radiation Belts



Frenzy and Realpolitik

- Eisenhower: affable but simple?
- Rise of LBJ as Senate Majority Leader
  - What did he know about space?
- Public response to Sputniks
- LBJ "scoops people like peanuts"
- The space race was "on"

- Russian space feats got better press than American program Why?
- Bureaucratic response,
- Frenzy, journalistic excess
- Academics supported increased support for education!
- Rockefeller admonition
- Politics and publicity
- Sputnik as a "durable permacrisis" Eisenhower on the defensive

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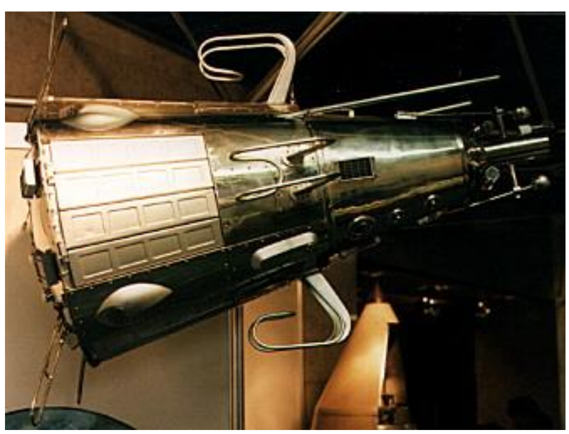
Sputnik 3, May 15, 1958



IGY

Geophysical payload, R-7 launcher

1,327 kg

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The Birth of NASA

National Defense Education Act (1958), National Aeronautics and Space Act of 1958

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Research, Development, and Education

- *NSF supposed to support basic science, diminishing the role of DOD and AEC*
  - *but NSF was not given an adequate budget*
- *1957: Scientists advise Ike that R&D should be funded by the military*
  - *WHICH scientists?*
- *Presidential Assistant for Science and Technology*
- *NDEA of 1958 designed as a stopgap, following GI Bill*
- *Attacks on Dewey's "Progressive Education"*
- *Some Southerners advocated increased educational support but feared forced integration*

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Technology vs. Politics?

- Former Harvard president Conant
  - "What will be needed is not more engineers and scientists but … political leaders of wisdom, courage, and devotion"
  - Eisenhower agreed
  - Then-current Harvard president Pusey disagreed
- Concern for ability to deliver nuclear weapons
- NACA slumping by mid-50s
- USAF took the lead in "X-Plane" programs
- Little NACA funding for space as late as 1955
  - "Skeptical, conservative, and reticent," von Karman
- NACA budget shrank steadily after WWII
  - no powerful allies
  - channeled only 2% of its budget to contractors
- Military pleased to take up the slack
  - Army and Navy were racing to launch the first satellite
  - where was the Air Force?
- President's Science Advisory Committee (PSAC): 2 goals in space
  - exploration
  - control (of what?)
- 1957: NACA space budget increased by 20%, a timid US response to Sputnik

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Struggle Among Government Agencies

- Sentiment for AEC to take the lead
- 15-year plan, booster development by ABMA (von Braun),
  - cognizance for JPL
- USAF: X-15, Atlas, Thor, Agena for WS-117L
- Navy: Vanguard and Polaris
- Killian (MIT president), President's Science Advisory Committee (PSAC)
- Congress's influence on space policy
- ARPA
- Major goals of spaceflight seen as scientific and political, favoring civilian agency for non-military goals
  - But NACA was too small
- National Aeronautics and Space Act of 1958 split responsibilities between the new NASA and DOD
  - Did not commit US to a space race
- USAF was not happy about the split, even rallied existing NACA to its side regarding R&D responsibilities
  - NACA reluctant to take quasi-military responsibilities
- Different bills in House and Senate committees, have to be resolved

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1958 State of the Union Address

- Communist imperialism still the threat
- No "call to arms" but waging total cold war

- 7 points:
  - defense reorganization
  - accelerated R&D
  - mutual aid to allies
  - increased trade with allies
  - scientific cooperation with allies
  - increased investment in education and research
  - supplemental appropriations for defense

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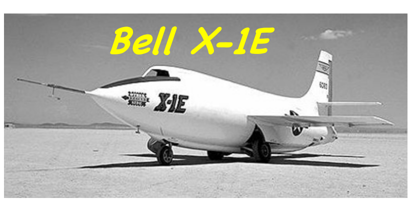
NACA Rocket X-Planes



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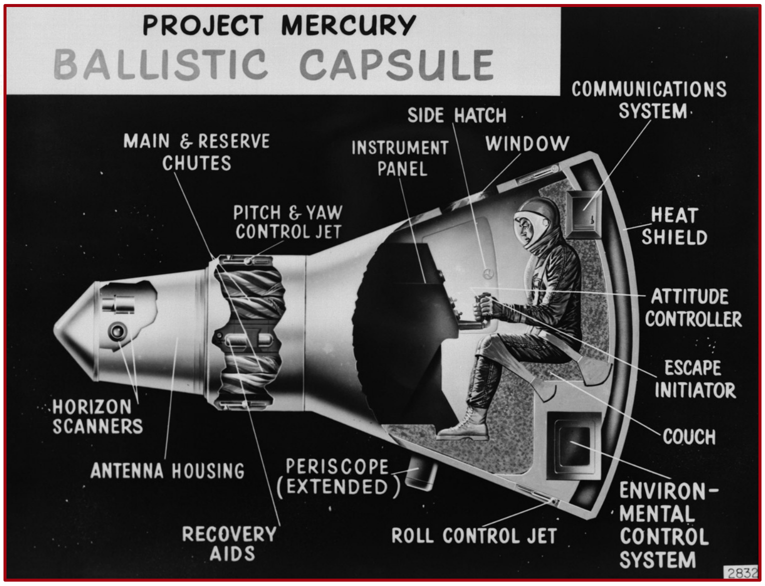
1958: NACA Becomes NASA



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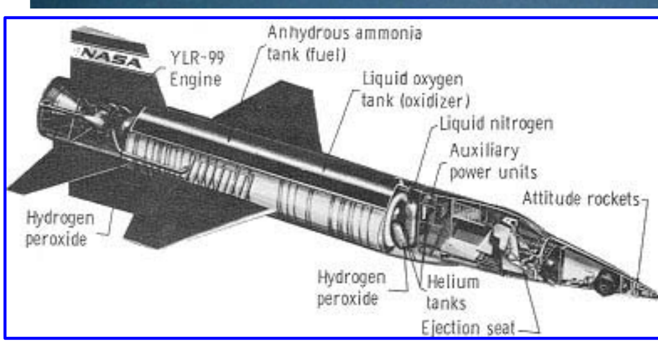
NASA/USAF X-15



Forrest Petersen, Princeton GS

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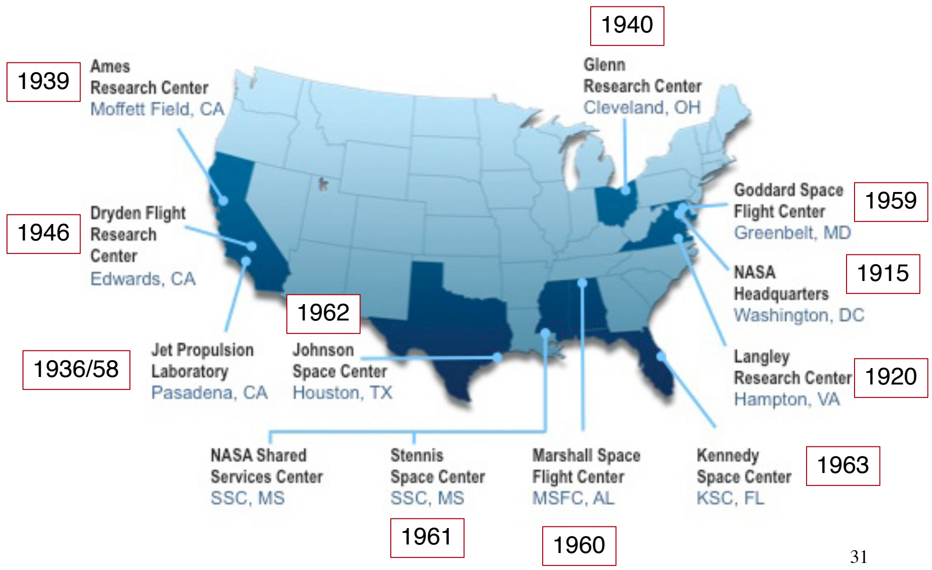
NACA/NASA Research Laboratories



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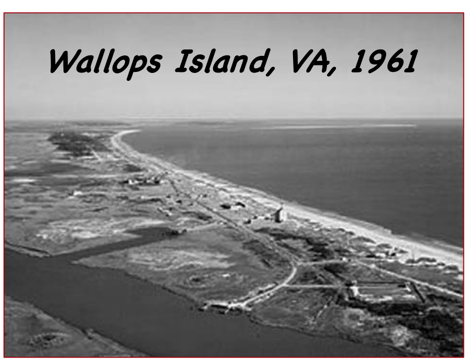
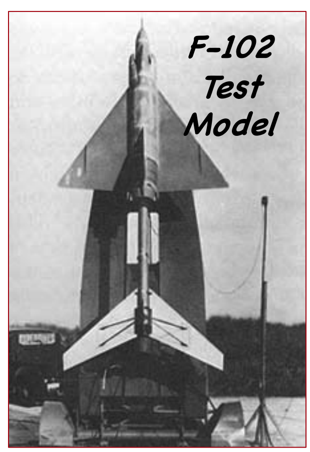
Human Space Flight

- *NACA Langley Pilotless Aircraft Research Division evolved to the …*
- *NASA Space Task Group*





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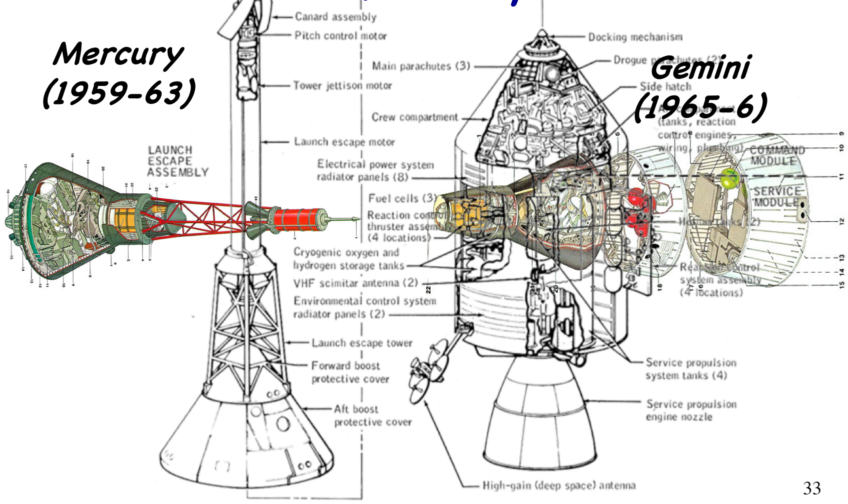
Space Task Group
Created the designs for Mercury, Gemini, and Apollo



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Space Task Group became the NASA Manned Spacecraft (Johnson Space) Center, Houston

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Rocket Propulsion and Staging

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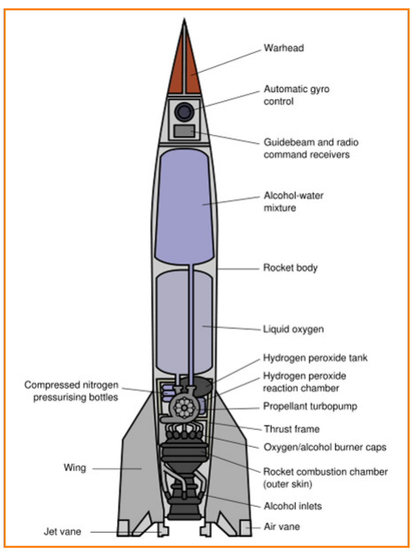
Single-Stage Launch Vehicle Systems



- *Propulsion and Power*
  - *Main engines*
  - *Turbo-pumps*
  - *Batteries, fuel cells*
  - *Pressurizing bottles*
- *Structure*
  - *Skin, frames,*
  - *Propellant tanks*
  - *Fins, control surfaces*
  - *Heat shield, insulation*
- *Electronics*
  - *Computers*
  - *Sensors and actuators*
  - *Transmitters and receivers*
  - *Transponders*
- *Payload*

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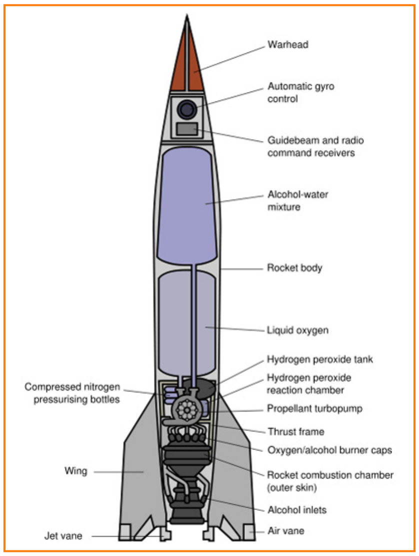
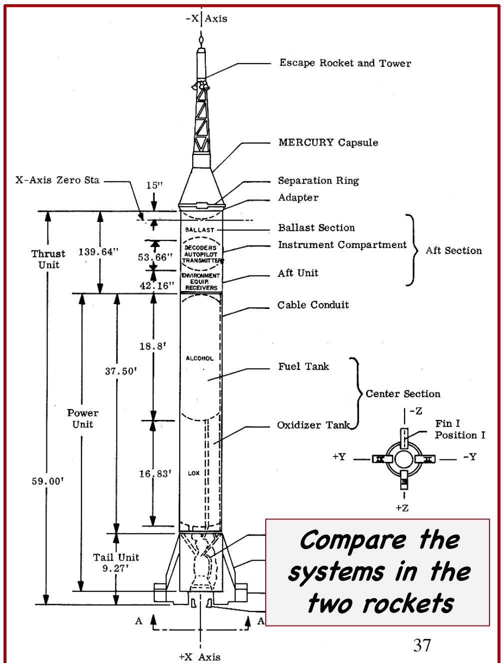
V-2 and Mercury Redstone





Compare the systems in the two rockets

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Chemical (Thermal) Rockets

- **Liquid/Gas Propellant**
  - Monopropellant
    - Catalytic ignition
    - Chemical decomposition
  - Bipropellant
    - Separate oxidizer and fuel
    - Hypergolic (spontaneous) ignition
    - External ignition
    - Storage
      - Ambient temperature and pressure
      - Cryogenic
      - Pressurized tank
  - Throttlable
  - Start/stop cycling
- **Solid Propellant**
  - Mixed oxidizer and fuel
  - External ignition
  - Burn to completion
- **Hybrid Propellant**
  - Liquid oxidizer, solid fuel
  - Throttlable
  - Start/stop cycling

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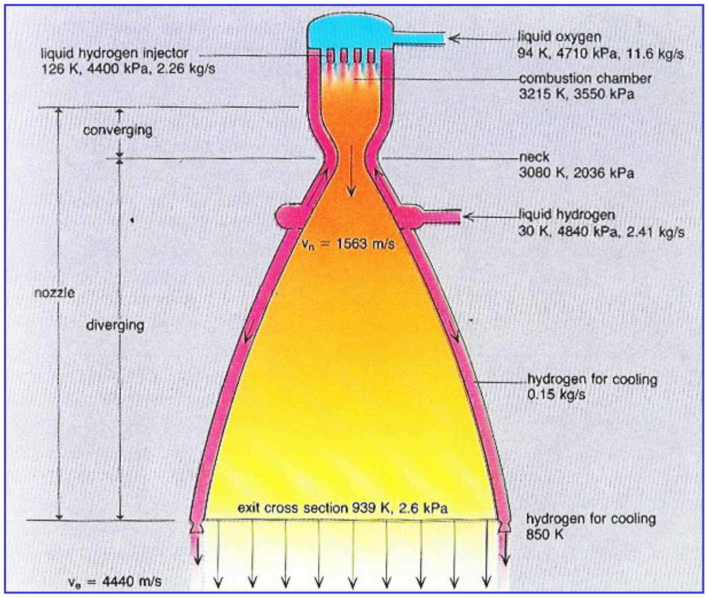
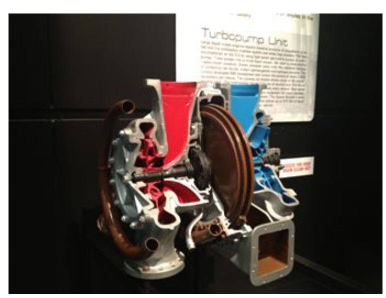
Liquid-Propellant Rocket Motor



German V-2 Rocket Motor, Fuel Injectors, and Turbopump



USSR RD-107/8 Rocket Motors

RD-107
4 combustion chambers, 2 verniers

RD-108
4 combustion chambers, 4 verniers

R-7 Base
4-RD-107, 1-RD-108

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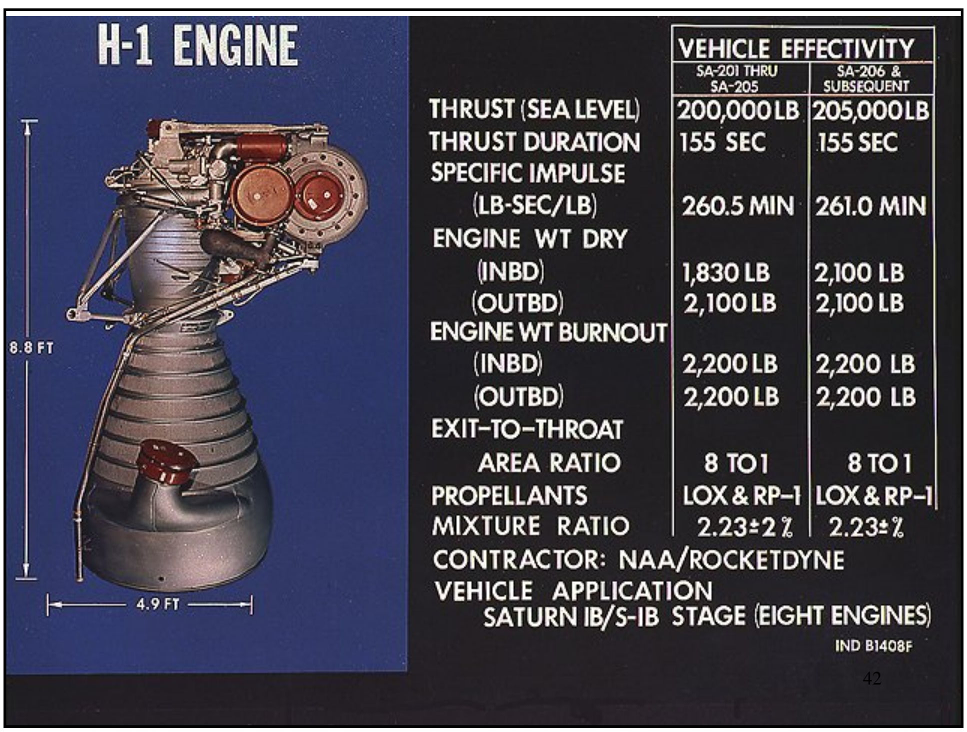

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US Saturn F-1 Engine

	VEHICLE EFFECTIVITY	
	SA-501 THRU SA-503	SA-504 & SUBSEQUENT
THRUST (SEA LEVEL)	1,500,000 LB	1,522,000 LB
THRUST DURATION	150 SEC	165 SEC
SPECIFIC IMPULSE (LB-SEC/LB)	260 SEC MIN	263 MIN
ENGINE WEIGHT DRY	18,416 LB	18,500 LB
ENGINE WEIGHT BURNOUT	20,096 LB	20,180 LB
EXIT-TO-THROAT AREA RATIO	16 TO 1	16 TO 1
PROPELLANTS	LOX & RP 1	LOX & RP 1
MIXTURE RATIO	2.27±2%	2.27±2%

CONTRACTOR: NAA/ROCKETDYNE
VEHICLE APPLICATION:
SATURN V/S-IC STAGE (FIVE ENGINES)

IND B1413D

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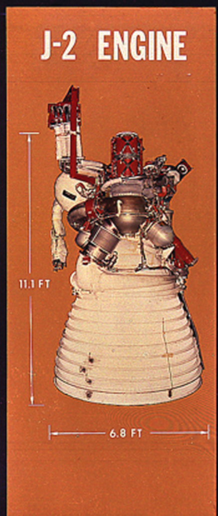
US Saturn J-2 Engine



	VEHICLE EFFECTIVITY		
	SA-201 THRU SA-203	SA-204 THRU SA-207 & SA-501 THRU SA-503	SA-208 & SUBSEQUENT; AND SA-504 & SUBSEQUENT
THRUST (ALTITUDE)	200,000LB	225,000LB	230,000LB
THRUST DURATION	500 SEC	500 SEC	500 SEC
SPECIFIC IMPULSE (LB-SEC/LB)	418 MIN	419 MIN	421 MIN
ENGINE WEIGHT DRY	3,480LB	3,480LB	3,492LB
ENGINE WEIGHT BURNOUT	3,609LB	3,609LB	3,621LB
EXIT TO THROAT AREA RATIO	27.5 TO 1	27.5 TO 1	27.5 TO 1
PROPELLANTS	LOX&LH_2	LOX&LH_2	LOX&LH_2
MIXTURE RATIO	5.00±2%	5.50±2%	5.50±2%

CONTRACTOR: NAA/ROCKETDYNE
VEHICLE APPLICATION:
SAT IB/S-IVB STAGE (ONE ENGINE)
SAT V/S-II STAGE (FIVE ENGINES)
SAT V/S-IVB STAGE (ONE ENGINE)

IND B1411E

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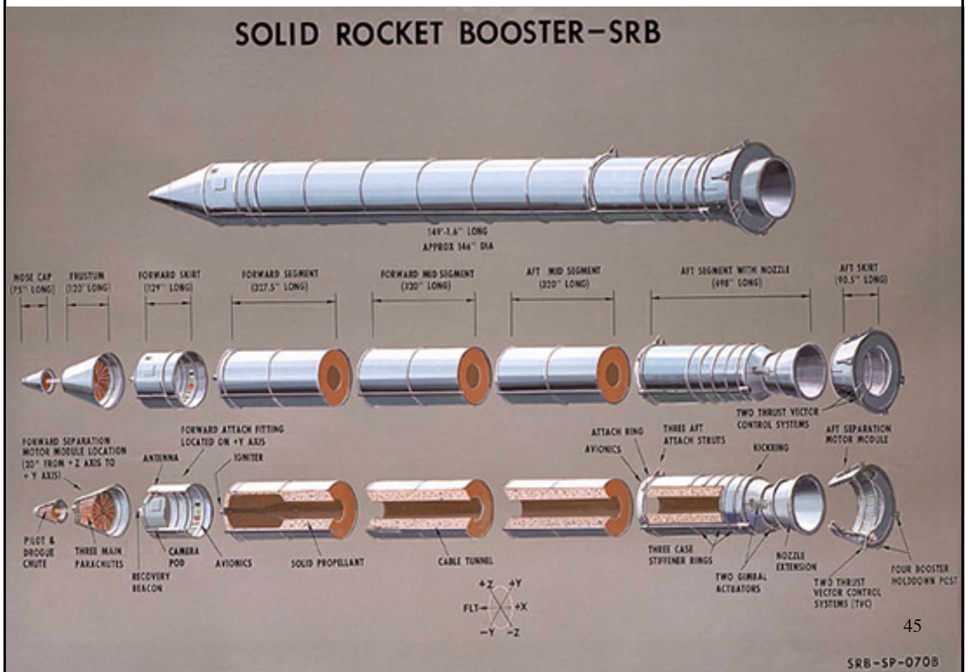

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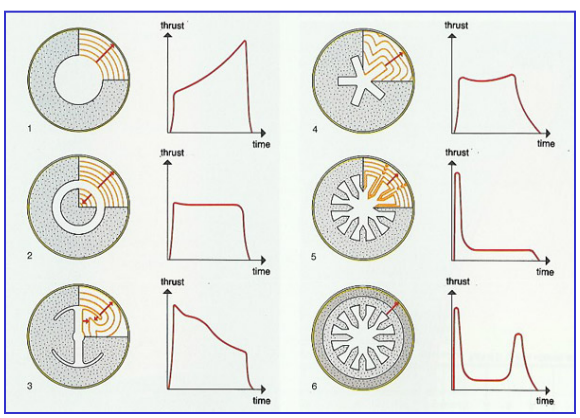
Solid-Propellant Rocket Grain and Thrust Profile



Thrust is proportional to burning area
Rocket grain patterns affect thrust profile
Propellant chamber must sustain high pressure and temperature
Environmentally unfriendly exhaust gas

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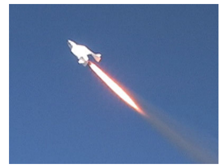
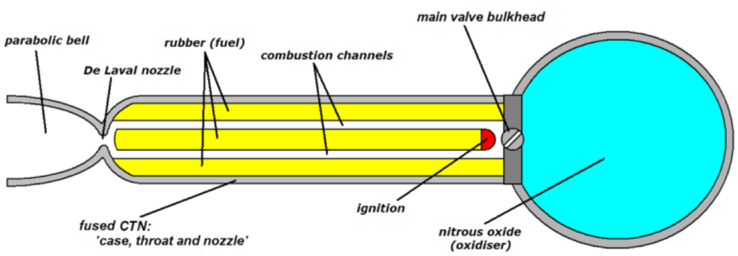
Hybrid-Fuel Rocket Motor



- SpaceShipOne motor
  - Nitrous oxide
  - Hydroxy-terminated polybutadiene (HTPB)
- Issues
  - Hard start
  - Blow back
  - Complete mixing of oxidizer and fuel to completion



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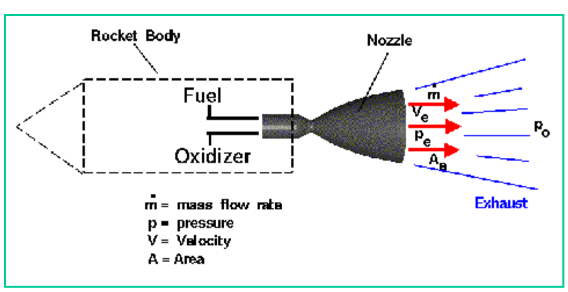
Rocket Thrust

$$\textbf{Thrust} = \dot{m}_{propellant} V_{exhaust} + A_{exit}\left(p_{exit} - p_{ambient}\right)$$

$$\triangleq \dot{m}\, c_{eff}$$

$$c_{eff} = \frac{Thrust}{\dot{m}} = \textbf{Effective exhaust velocity}$$

$$\dot{m} \equiv \textbf{Mass flow rate of on-board propellant}$$



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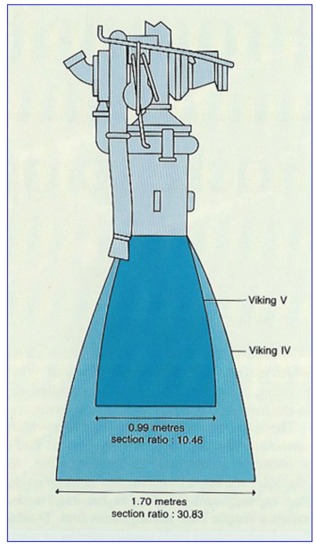
Rocket Nozzles

- Expansion ratio, A_e/A_t , chosen to match exhaust pressure to average ambient pressure
  - Ariane rockets: Viking V for sea level, Viking IV for high altitude
- Rocket nozzle types
  - DeLaval nozzle
  - Isentropic expansion nozzle
  - Spike/plug nozzles
  - Expansion-deflection nozzle



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Specific Impulse

$$I_{sp} = \frac{Thrust}{\dot{m}\, g_o} = \frac{c_{eff}}{g_o}, \quad Units = \frac{m/s}{m/s^2} = s$$

$$g_o \equiv Gravitational\ acceleration\ at\ earth's\ surface$$

g_o is a normalizing factor for the definition

Chemical rocket specific impulse (vacuum)

Solid propellants: < 295 s

Liquid propellants: < 510 s

- Space Shuttle Specific Impulses
  - Solid boosters: 242-269 s
  - Main engines: 455 s
  - OMS: 313 s
  - RCS: 260-280 s

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Specific Impulse

- Specific impulse is a product of characteristic velocity, c*, and rocket thrust coefficient, C_F

$$I_{sp} = \frac{Thrust}{\dot{m}\, g_o} = \frac{c_{eff}}{g_o} = C_F\, c^*/g_o = \frac{V_{exhaust}}{g_o}$$

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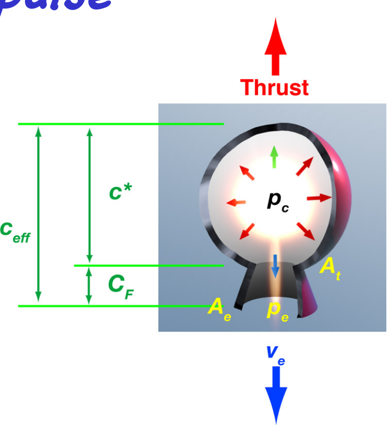
Specific Impulse

- Characteristic velocity is related to
  - combustion chamber performance
  - propellant characteristics
- Thrust coefficient is related to
  - nozzle shape
  - exit/ambient pressure differential



$$when\ C_F = 1, \quad p_e = p_{ambient}$$

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Rockets 101

- Initial mass:
  - Payload
  - Structure, Systems
  - Rocket motors
  - Propellant
- Final mass:
  - Initial mass less propellant
- Final velocity depends on mass ratio, initial to final mass, μ

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The Rocket Equation

Ideal velocity increment of a rocket stage, ΔV_I
(gravity and aerodynamic effects neglected)

$$\text{Acceleration} = \frac{dV}{dt} = \frac{Thrust}{m} = \frac{\dot{m}\, c_{eff}}{m} = -\frac{dm/dt\, I_{sp} g_o}{m}$$

$$\int_{V_i}^{V_f} dV = -I_{sp} g_o \int_{m_i}^{m_f} dm/m = -I_{sp} g_o \ln m\Big|_{m_i}^{m_f}$$

$$\left(V_f - V_i\right) \equiv \Delta V_I = I_{sp} g_o \ln\left(\frac{m_i}{m_f}\right) \equiv I_{sp} g_o \ln \mu$$

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Ratios Characterizing a Rocket Stage

Mass ratio

$$\mu = {m_{initial}}/{m_{final}}$$

Payload ratio (as large as possible)

$$\lambda = {m_{payload}}/{m_{initial}}$$

Structural ratio (typically 0.1 - 0.2)

$$\eta = {m_{structure/engine}}/{m_{initial}}$$

Propellant ratio

$$\varepsilon = {m_{propellant}}/{m_{initial}} = \frac{\mu - 1}{\mu}$$

For a single stage

$$\lambda + \eta + \varepsilon = 1$$

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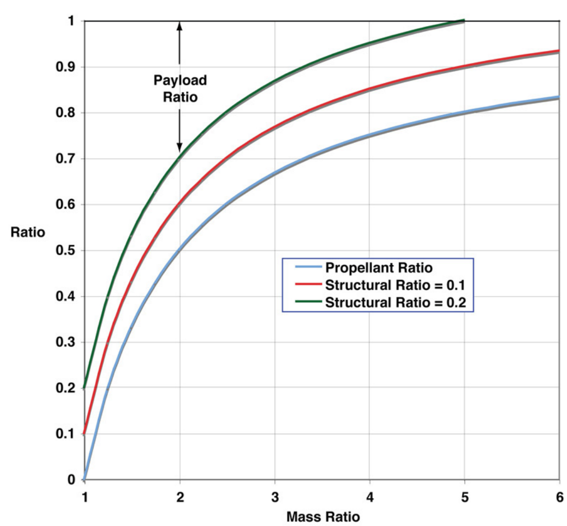
Ratios Characterizing a Rocket Stage



Payload is what's left after propellant and structure are subtracted

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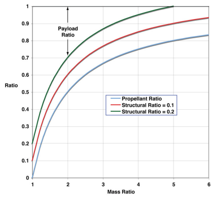

Ideal Velocity Increment for Single Stage With Various Specific Impulses

	Ideal Velocity Increment, km/s			
Mass Ratio	Isp = 220 s	= 275 s	= 400 s	= 500 s
2	1.5	1.9	2.7	3.4
3	2.4	3	4.3	5.3
4	3	3.8	5.4	6.8
5	3.5	4.3	6.3	7.9

$$\mu_{required} = e^{\Delta V_I / I_{sp} g_0}$$

Single stage to orbit with payload (ΔV_I ~ 7.3 km/s)? Not easy.

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Required Mass Ratio for Various Velocity Increments

$$\mu_{required} = e^{\Delta V_I / I_{sp} g_0}$$

	Required Mass Ratio	
Ideal Velocity Increment, km/s	Isp = 240 s	= 400 s
7	19.6	6.0
8	29.9	7.7
9	45.7	9.9
10	69.9	12.8
11	106.9	16.5
12	163.5	21.3

... and there are velocity losses due to gravity and aerodynamic drag

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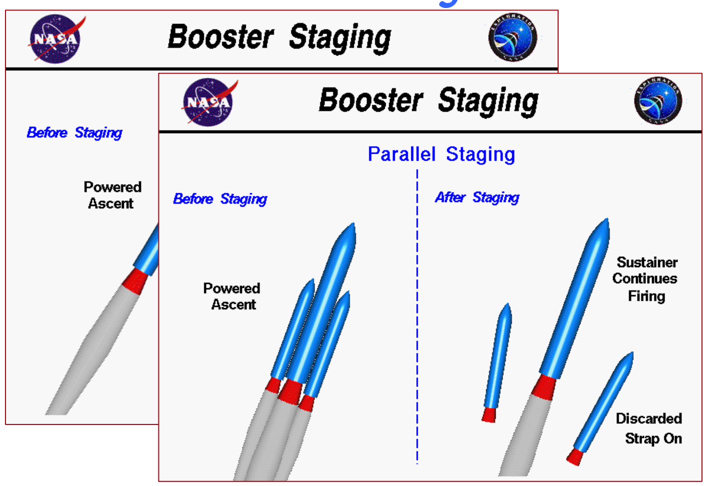
Mass Ratio for Space Missions Difficult to Obtain without Staging

Final mass can be reduced by getting rid of structure when no longer needed



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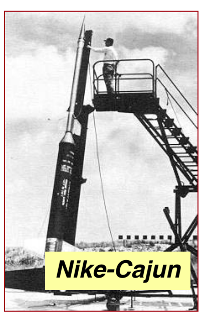

Ideal Velocity Increment of a 2-Stage Rocket

$$\Delta V_I = \left(I_{sp}\right)_1 g_o \ln\left(\frac{m_i}{m_f}\right)_1 + \left(I_{sp}\right)_2 g_o \ln\left(\frac{m_i}{m_f}\right)_2$$

$$\triangleq g_o\left[\left(I_{sp}\right)_1 \ln \mu_1 + \left(I_{sp}\right)_2 \ln \mu_2\right]$$

μ_j is the mass ratio of the j^{th} stage

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Ideal Velocity Increment for a Multiple-Stage Rocket

- *Ideal velocity increment of an n-stage rocket*

$$\Delta V_I = g_o \sum_{j=1}^{n} \left(I_{sp}\right)_j \ln \mu_j$$

- *Optimal ideal velocity increment with equal specific impulses*

$$\Delta V_I = I_{sp} g_o \ln\left(\mu_1 \bullet \mu_2 \bullet \ldots \mu_n\right) \equiv I_{sp} g_o \ln\left(\mu_{overall}\right)$$
$$= I_{sp} g_o \ln \mu^n$$

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Required Mass Ratios for Multiple-Stage Rockets

- *Staging reduces mass ratios to achievable values*
- *With equal specific impulses for each stage*

	Required Mass Ratio					
	Single Stage		Two Stages		Three Stages	
Ideal Velocity Increment, km/s	Isp = 240 s	= 400 s	Isp = 240 s	= 400 s	Isp = 240 s	= 400 s
7	19.6	6.0	4.4	2.4	2.7	1.8
8	29.9	7.7	5.5	2.8	3.1	2.0
9	45.7	9.9	6.8	3.1	3.6	2.1
10	69.9	12.8	8.4	3.6	4.1	2.3
11	106.9	16.5	10.3	4.1	4.7	2.5
12	163.5	21.3	12.8	4.6	5.5	2.8

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Overall Payload Ratio of a Multiple-Stage Rocket

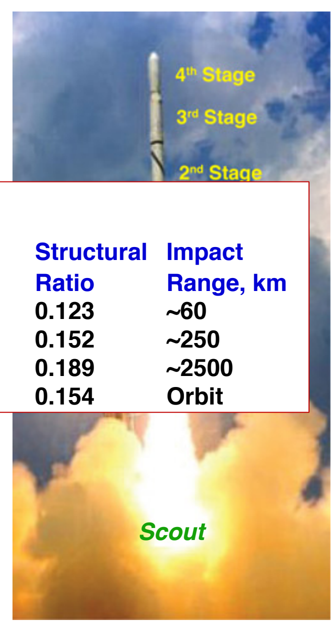
$$\lambda_{overall} = \frac{(m_{payload})_n}{(m_{initial})_1} = \frac{(m_{payload})_n}{(m_{initial})_n} \bullet \frac{(m_{payload})_{n-1}}{(m_{initial})_{n-1}} \bullet \dots \frac{(m_{payload})_1}{(m_{initial})_1}$$

$$= \lambda_1 \bullet \lambda_2 \bullet \dots \lambda_n$$

Feasible design goal: *Choose stage mass ratios to maximize overall payload ratio*

Scout Launch Vehicle

- *Liftoff mass = 16,450 kg*



Typical Figures for Scout

Stage	Isp, s, vac (SL)	Mass Ratio	Payload Ratio	Structural Ratio	Impact Range, km
1 (Algol)	284 (238)	2.08	0.358	0.123	~60
2 (Castor)	262 (232)	2.33	0.277	0.152	~250
3 (Antares)	295	2.53	0.207	0.189	~2500
4 (Altair)	280	2.77	0.207	0.154	Orbit

- *Overall mass ratio = 34*
- *Overall payload ratio = 0.00425 = 0.425% (67-kg payload)*

Payload Ratios of a Two-Stage Rocket

- For equal specific impulses

$$\Delta V_I = I_{sp} g_o \left[\ln \mu_1 + \ln \mu_2 \right]$$
$$= I_{sp} g_o \left[\ln \mu_1 \mu_2 \right] = I_{sp} g_o \left[\ln \mu_{overall} \right]$$

- Payload ratios for different structural ratios

$$\lambda_1 = \frac{1}{\mu_1} - \eta_1 = \frac{1 - \mu_1 \eta_1}{\mu_1}; \quad \lambda_2 = \frac{1 - \mu_2 \eta_2}{\mu_2}$$

Propulsion and Staging Considerations
http://www.princeton.edu/~stengel/Prop.pdf

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Maximum Payload Ratio of a Two-Stage Rocket

- Overall payload ratio

$$\lambda_{overall} = \lambda_1 \lambda_2 = \frac{(1 - \mu_1 \eta_1)(1 - \mu_2 \eta_2)}{\mu_{overall}}$$

- Condition for a maximum with respect to first stage mass ratio

$$\frac{\partial \lambda_{overall}}{\partial \mu_1} = \frac{\left(-\eta_1 + \frac{\mu_{overall} \eta_2}{\mu_1^{\,2}} \right)}{\mu_{overall}} = 0$$

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Maximum Payload Ratio of a Two-Stage Rocket

Stage mass ratios

$$\mu_1 = \sqrt{\mu_{overall} \, \eta_2 / \eta_1}; \quad \mu_2 = \sqrt{\mu_{overall} \, \eta_1 / \eta_2}$$

Optimal payload ratio

$$\lambda_{overall} = \frac{1}{\mu_{overall}} - 2\sqrt{\eta_1 \eta_2 / \mu_{overall}} + \eta_1 \eta_2$$

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Next Time:

Apollo & the Space Race:
... The Heavens and the Earth, Ch 10, Part 3 Conclusion
A Man on the Moon, Ch 1 to 3, Part 1
Interplanetary Travel:
Understanding Space, Ch 6, Sec 7.1, 7.2

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Supplemental Material

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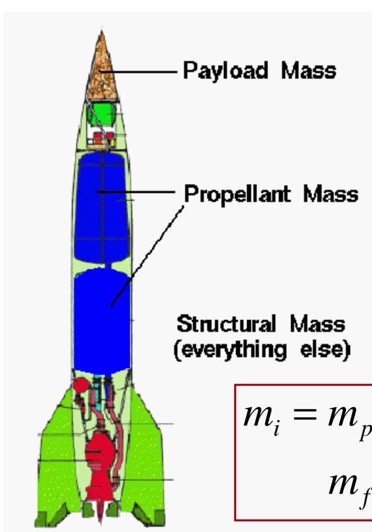
Vehicle Mass Components



Initial and final masses of a single-stage rocket

$$m_i = m_{payload} + m_{structure/engine} + m_{propellant}$$
$$m_f = m_{payload} + m_{structure/engine}$$

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Typical Values of Specific Impulse

- *Chamber pressure = 7 MPa (low by modern standards)*
- *Expansion to exit pressure = 0.1 MPa*

Liquid-Fuel Rockets

Monopropellant		Isp, s	VIsp, kg-s/m^3 x 10^3
Hydrogen Peroxide		165	238
Hydrazine		199	201
Nitromethane		255	290

Bipropellant

Fuel	Oxidizer	Isp, s	VIsp, kg-s/m^3 x 10^3
Kerosene	Oxygen	301	307
	Flourine	320	394
	Red Fuming Nitric Acid	268	369
Hydrogen	Oxygen	390	109
	Flourine	410	189
UDMH	Nitrogen Tetroxide	286	339

Solid-Propellant Rockets

Double-Base	Isp, s	VIsp, kg-s/m^3 x 10^3
AFU	196	297
ATN	235	376
JPN	250	405
Composite		
JPL 540A	231	383
TRX-H609	245	431
PBAN (SSV)	260	461

Hybrid-Fuel Rocket

Fuel	Oxidizer	Isp, s
HTPB	N2O	250

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*Rocket Characteristic Velocity, c**

$$c* = \frac{1}{\Gamma}\sqrt{\frac{R_o T_c}{M}}$$

where

$$\Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$R_o = universal\ gas\ constant = 8.3\times10^3\ kg\ m^2/s^2\ °K$$

$$T_c = chamber\ temperature, °K$$

$$M = exhaust\ gas\ mean\ molecular\ weight$$

$$\gamma = ratio\ of\ specific\ heats\ (\sim 1.2 - 1.4)$$

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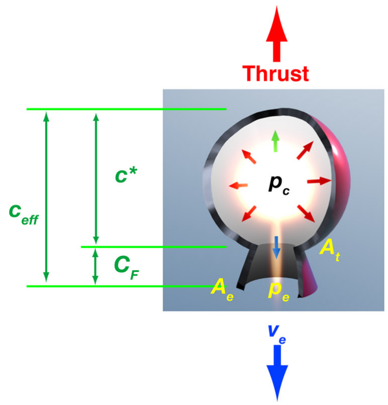
Rocket Characteristic Velocity, c*

$$c^* = \frac{p_c A_t}{\dot{m}} = \text{exhaust velocity if } C_F = 1$$



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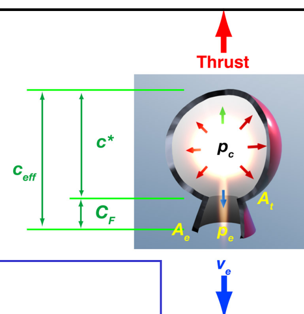
Rocket Thrust Coefficient, C_F



$$C_F = \frac{Thrust}{p_c A_t}$$

where

$$Thrust = \lambda\, \dot{m}\, v_e + A_e\left(p_e - p_{ambient}\right)$$

$$\lambda = reduction\ ratio\ (function\ of\ nozzle\ shape)$$

$$C_F = \lambda \Gamma \sqrt{\left(\frac{2\gamma}{\gamma-1}\right)\left[1-\left(\frac{p_e}{p_c}\right)^{(\gamma-1)/\gamma}\right]} + \left(\frac{p_e - p_{ambient}}{p_c}\right)\frac{A_e}{A_t}$$

- **typically, $0.5 < C_F < 2$**

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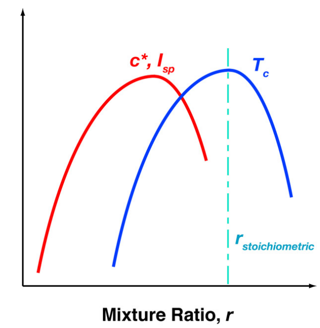
Mixture Ratio, r

- Stoichiometric mixture: complete chemical reaction of propellants
- Specific impulse maximized with lean mixture ratio, r (i.e., below stoichiometric maximum)



$$r = \frac{\dot{m}_{oxidizer}}{\dot{m}_{fuel}}; \quad \dot{m}_{fuel} = \frac{\dot{m}_{total}}{1+r}; \quad \text{"}leaner\text{"} < r < \text{"}richer\text{"}$$

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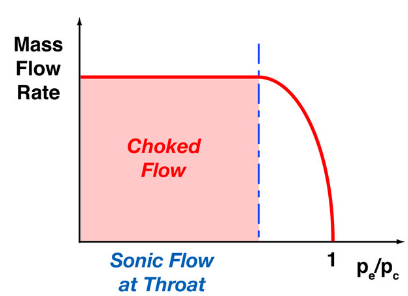
Effect of Pressure Ratio on Mass Flow



In choked flow, mass flow rate is maximized

$$\dot{m} = \frac{\Gamma p_c A_t}{\sqrt{R_o T_c / M}}$$

Choked flow occurs when

$$\frac{p_e}{p_c} \leq \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1} \approx 0.53$$

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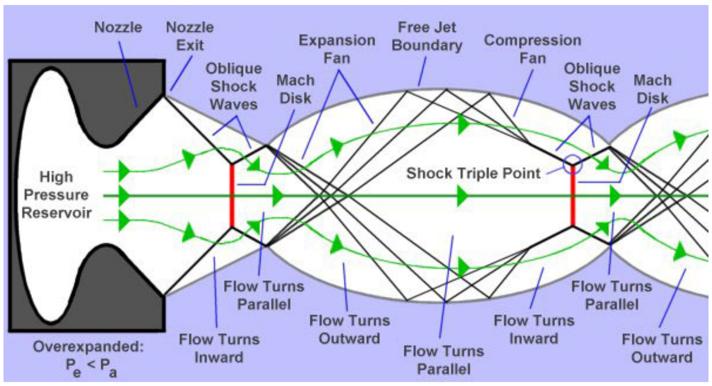
Shock Diamonds

When $p_e \neq p_a$, exhaust flow is over- or underexpanded
Effective exhaust velocity < maximum value



https://www.youtube.com/watch?v=qiMSko4HBe8

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Volumetric Specific Impulse

Specific impulse

$$\Delta V_I = I_{sp} g_o \ln \mu = I_{sp} g_o \ln\left(\frac{m_{final} + m_{propellant}}{m_{final}}\right) = I_{sp} g_o \ln\left(1 + \frac{m_{propellant}}{m_{final}}\right)$$

$$= I_{sp} g_o \ln\left(1 + \frac{Density_{propellant} \bullet Volume_{propellant}}{m_{final}}\right)$$

$$\approx g_o I_{sp}\left(\frac{\rho_{propellant} \bullet Volume_{propellant}}{m_{final}}\right) = g_o\left(I_{sp}\rho_{propellant}\right)\frac{Volume_{propellant}}{m_{final}}$$

Volumetric specific impulse portrays propellant density as well as performance

$$VI_{sp} \triangleq I_{sp}\rho_{propellant}$$

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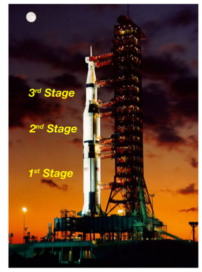
Volumetric Specific Impulse



- *For fixed volume and final mass, increasing volumetric specific impulse increases ideal velocity increment*

	Density, g/cc	Isp, s, SL	VIsp, s, SL	Isp, s, vac	VIsp, s, vac
LOX/Kerosene	1.3	265	345	304	395
LOX/LH2 (Saturn V)	0.28	360	101	424	119
LOX/LH2 (Shuttle)	0.28	390	109	455	127
Shuttle Solid Booster	1.35	242	327	262	354

- *Saturn V Specific Impulses, vacuum (sea level)*
  - *1st Stage, 5 F-1 LOX-Kerosene Engines: 304 s (265 s)*
  - *2nd Stage, 5 J-2 LOX-LH2 Engines: 424 s (~360 s)*
  - *3rd Stage, 1 J-2 LOX-LH2 Engine: 424 s (~360 s)*

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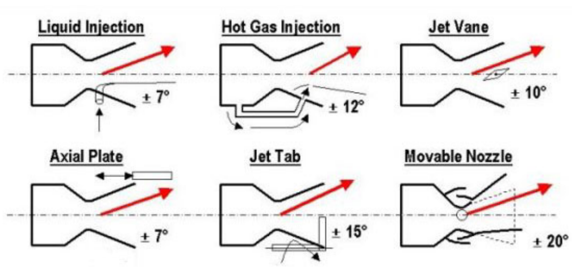
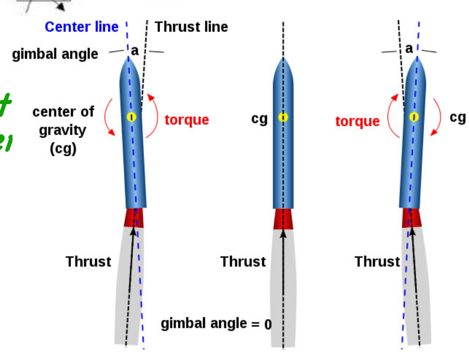
Thrust Vectoring



- *Maintain average thrust direction through center of mass*
- *Provide pitch and yaw control*



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Strap-On Boosters

- *High volumetric specific impulse is desirable for first stage of multi-stage rocket*
- *Strap-on solid rocket boosters are a cost-effective way to increase mass and payload ratios*

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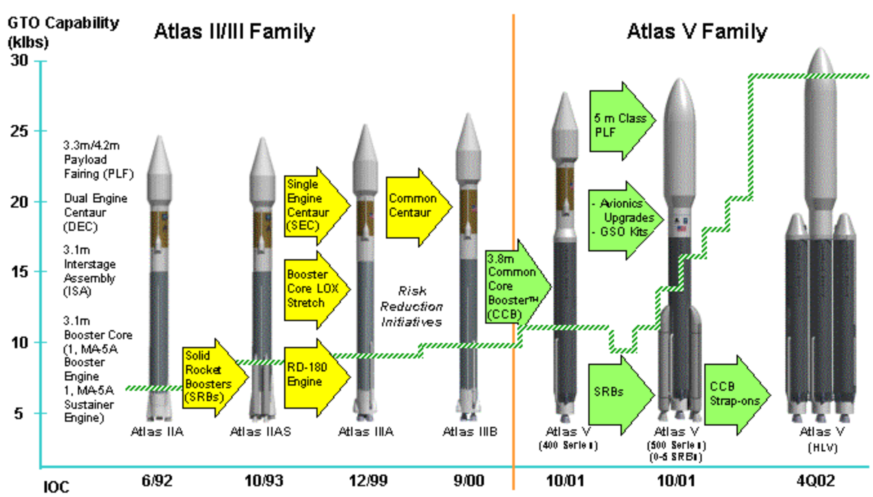
Atlas Evolution



Implementing a Low Risk Evolution Process

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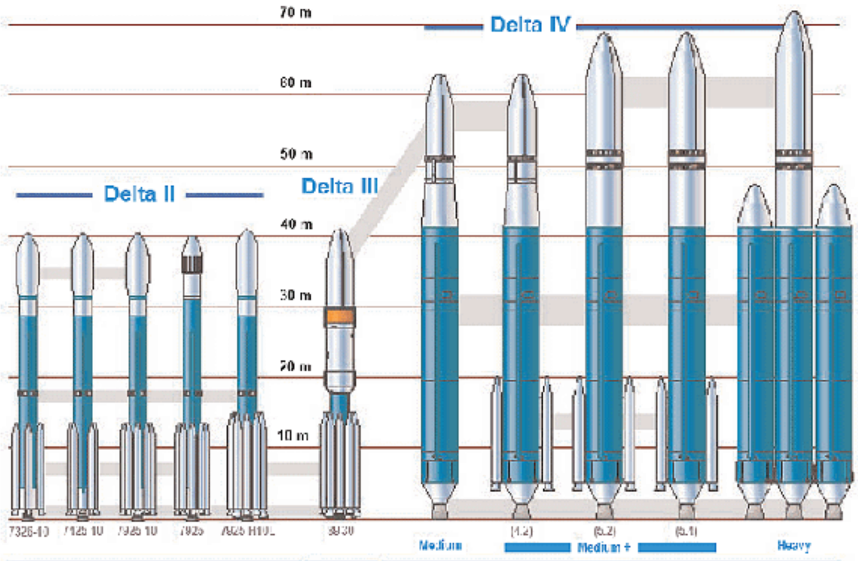
Delta Evolution



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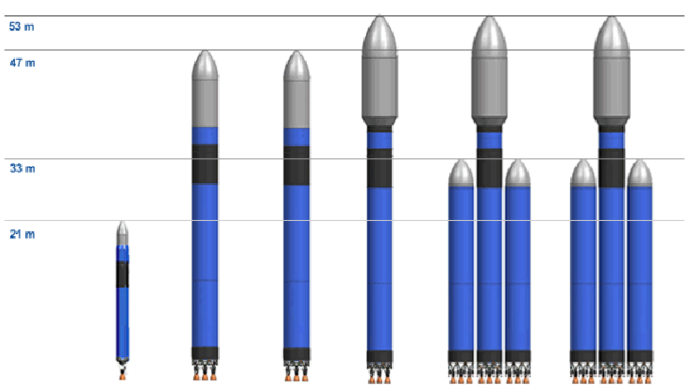
Falcon Evolution



	Falcon 1	Falcon 5	Falcon 9	Falcon 9	Falcon 9 - S5	Falcon 9 - S9
LEO	570 kg	4,100 kg	9,300 kg	8,700 kg	16,500 kg	24,750 kg
GTO	--	1,050 kg	3,400 kg	3,100 kg	6,400 kg	9,650 kg
Fairing diameter	1.5 m	3.6 m	3.6 m	5.2 m	5.2 m	5.2 m
Price in millions *	$6.7	$18	$27	$35	$51	$78

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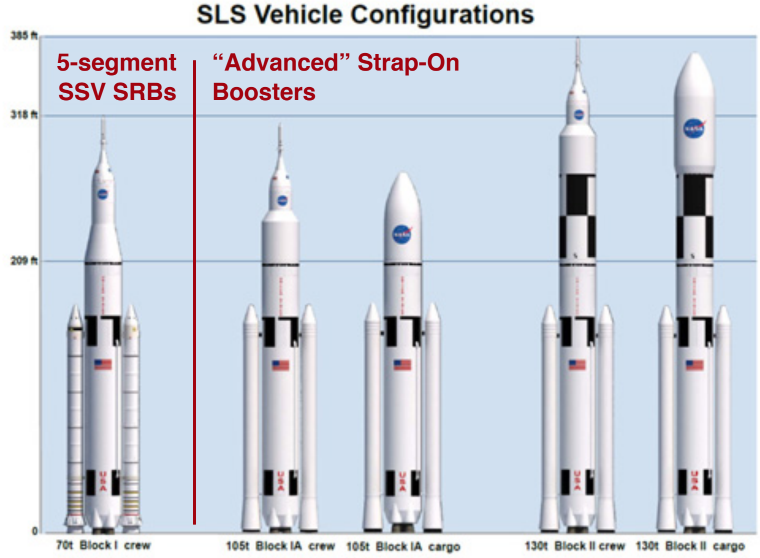
Space Launch System Configurations



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