and hence, the sign of \( \lambda \), can never change. Thus, the solution is always on the lower boundary, as would be predicted by the solutions shown in Ref. 2.

This can also be shown by freezing the final time and fixing the final altitude in the first formulation. The solution obtained in this manner is discussed in Ref. 3 and is shown to provide the same results as those just given.

Summary

The optimal control problem, to determine the drag coefficient history inside of upper and lower bounds, which produces maximum or minimum values of dynamic pressure at specified initial and final conditions for a ballistic re-entry vehicle, has been formulated. The results indicate that, if the final time is fixed, then the optimal solutions may contain jumps in the control variable from one boundary to the other. If the final time is fixed, the optimal solutions will not contain jumps. These results are consistent with results published earlier which indicate that jumps in the drag coefficient should not occur, if the altitude is used as the independent variable. If the final state of a re-entry vehicle trajectory segment is to be determined by a timer, however, then a design scheme which attempts to determine trajectories with minimum or maximum dynamic pressure must consider possible jumps in the drag coefficient history.

Acknowledgments

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References


In-Flight Simulation with Pilot-Center of Gravity Offset and Velocity Mismatch

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Nomenclature

\( a \) = body-axis acceleration vector  
\( a_f \) = lateral acceleration component  
\( C_f \) = feedback gain matrix  
\( C_r \) = forward gain matrix  
\( F \) = fundamental (stability) matrix  
\( G \) = control effect matrix  

**\( e \) = gravitational acceleration vector  
**\( g \) = gravitational acceleration magnitude  
**\( H \) = Earth-to-body-axis transformation matrix  
**\( L \) = roll stability and control derivatives  
**\( N \) = yaw stability and control derivatives  
**\( p \) = roll rate  
**\( r \) = yaw rate  
**\( T \) = similarity transformation matrix  
**\( t \) = time  
**\( U \) = control transformation matrix  
**\( V \) = velocity magnitude  
**\( v \) = body-axis velocity vector  
**\( x \) = body-axis position vector  
**\( x \) = axial position  
**\( y \) = side force vector  
**\( Y \) = side-force stability and control derivatives  
**\( z \) = vertical position  
**\( \alpha \) = sideslip angle  
**\( \beta \) = control vector  
**\( \delta A \) = aileron angle  
**\( \delta B \) = rudder angle  
**\( \delta S \) = side-force panel angle  
**\( \phi \) = roll angle  
**\( \omega \) = body-axis angular rate vector  
**\( \omega \) = cross-product equivalent matrix

Introduction

Two objectives for in-flight simulation of one aircraft by another are matching the modal characteristics of the model and duplicating its response to command inputs. Six-degree-of-freedom in-flight simulators, such as the Princeton Variable-Response Research Aircraft (VRA) and the USAF/CalSPAN Total In-Flight Simulator (TIFS), can provide "perfect" model following (in the sense of Erzeberger) for rigid-body modes and responses. However, if there is airspeed mismatch or difference in the pilot’s location relative to the rotational center, then perfect following of the model’s state variables is no guarantee of an acceptable simulation. The pilot’s acceleration cues, which are central to flying qualities evaluation, are certain to be different from those of the model. In such instances, it becomes necessary to modify the simulation control logic so that accelerations at the pilot’s station are matched at the expense of mismatch in cues which are secondary to the simulated piloting task. If the model is realized explicitly in the simulator’s control system, airspeed and pilot location mismatch effects can be compensated by transforming the outputs of the system’s model equations before they are transmitted to the model-following logic. If implicit model following (or "response feedback") is used, it is necessary to transform the model itself prior to generating the implicit model-following control gains.

A solution for airspeed and pilot station mismatch is found by performing a sequential similarity transformation on the linear differential equations which describe model dynamics. For lateral acceleration matching, the simplest procedure is to accommodate pilot station effects by transforming model sideslip angle, while velocity mismatch is compensated through yaw rate transformation. Normal acceleration matching can be accomplished through similar transformations of model angle of attack and pitch rate, and it is not discussed further. In either case, the similarity transformation preserves model eigenvalues while matching acceleration cues. The model transformation for lateral acceleration matching is developed, and an example based on VRA simulation of the Space Shuttle is presented.

Implicit Model Following

Given the linear-time-invariant dynamic model of the aircraft to be simulated,

\[
\Delta x_M = F_M \Delta x_M + G_M \Delta b_M \quad \Delta x_M (0) = \Delta x_0
\]  

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*Associate Professor, Dept. of Mechanical and Aerospace Engineering, Associate Fellow AIAA.
the objective is to find feedback and forward gain matrices, \( C_s \) and \( C_p \), which cause the simulation aircraft's modes and responses to mimic the model. The simulation aircraft's rigid-body motions are represented by

\[
\Delta x_s = F_s \Delta x_s + G_s \Delta \delta_s \quad \Delta x_s (0) = \Delta x_c \quad (2)
\]

The perturbation state vectors (\( \Delta x_m \) and \( \Delta x_s \)) are \( n \)-dimensional, the model control (\( \Delta \delta_m \)) has \( m \) components and the simulator control (\( \Delta \delta_s \)) has \( m \) components. For the complete aircraft equations, \( n \geq m \).

With identical initial conditions, the model and simulator motions are the same when \( \Delta x_m = \Delta x_s \); this establishes the criterion for "perfect" model following. In the equivalent stability derivative (ESD) approach, the state rates are identically matched \( \dot{x} \); using linear-quadratic implicit model following, the state rates are matched in a least-squares sense. The two methods produce identical control structures and gain matrices when the "perfect" model-following criterion is satisfied and there are no added constraints on control usage or parameter insensitivity.

The stability-axis state vectors of the lateral-directional model and the simulator each take the form,

\[
\Delta x^T = (\Delta \alpha \Delta \beta \Delta \phi \Delta \delta)
\]

and their fundamental matrices contain the aircraft's dimensional stability derivatives. Neglecting unsteady aerodynamic effects,

\[
F_s = \begin{bmatrix}
N_s & N_p' & N_p &= 0 \\
Y'_s & Y'_p & Y'_p & V'_p / g' \\
L_s & L_p & L_p &= 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

Note that the fourth rows of \( F_s \) and \( F_p \) are necessarily identical. Consequently, a three-component control vector, \( \Delta \delta = [\Delta \alpha \Delta \beta \Delta \phi] \), can provide "perfect" model following subject to control actuator limitations. The corresponding control effect matrix is

\[
G_s = \begin{bmatrix}
N_s & N_s & N_s & N_s \\
Y_s / V & Y_s / V & Y_s / V & Y_s / V \\
L_s & L_s & L_s & L_s \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

The model's control vector may not contain \( \Delta S F \), in which case \( \Delta \delta_m \) has two components, and \( G_m \) is red fined accordingly.

Because the fourth rows of the model and simulator equations are identical, they can be neglected in determining the "perfect" model-following control law. The model and simulator state rates are equal when

\[
F_s \Delta x_s + G_s \Delta \delta_s = F_p \Delta x_p + G_p \Delta \delta_p
\]

where the prime denotes a truncated (three-row) matrix. When the model and simulator are matched, \( \Delta x_m = \Delta x_s \), and the ESD control law is derived from Eq. (5):

\[
\Delta \delta_p = G_s^{-1} (F_s - F_p) \Delta x_s + G_s \Delta \delta_s
\]

This "perfect" model-following control law is seen to include state feedback (through \( C_s \)) and command-input interconnect (through \( C_p \)). The elements of the closed-loop simulator matrices are equivalent to the stability derivatives of the model:

\[
\Delta x_s = (F_s + G_s C_p) \Delta x_s + G_s C_p \Delta \delta_m = F_M \Delta x_M + G_M \Delta \delta_M
\]

Lateral Acceleration Matching

The body-axis acceleration vector at the pilot's station is

\[
\alpha_p = \alpha_p \Delta \omega + \Delta \omega \times \Delta x_p - \Delta x_p \Delta \omega
\]

Recognizing that stability axes are a special set of body axes (nominally oriented to the velocity vector), neglecting coupling between longitudinal and lateral-directional motions, and assuming that the pilot's station is on the aircraft's plane of symmetry, perturbations in lateral acceleration are described by

\[
V_M \Delta \beta_M + x_M \Delta \phi_M - z_M \Delta \phi_M - (z_M - z_0) \Delta \phi = \Delta \phi
\]

With differing velocity or pilot offset, one or more simulator state variables must be altered to provide the \( \Delta \delta_p \) match. This is most readily accomplished by transforming a different state variable for each type of acceleration mismatch.

Consider first the model transformation for pilot-center of gravity (c.g.) offset at identical airspeeds. Three states (\( \Delta r \), \( \Delta p \), and \( \Delta \phi \)) are to be matched identically. Defining \( k = x_p - x_M \), model and simulator lateral accelerations are matched when

\[
V_M \Delta \beta_M + x_M \Delta \phi_M - z_M \Delta \phi_M = V_M \Delta \beta_M + k \Delta \phi
\]

Rearranging Eq. (10) and integrating once provides \( \Delta \beta_M \), and the model state vector which the simulator should follow to compensate for differing pilot-c.g. offsets is

\[
\begin{bmatrix}
\Delta r \\
\Delta \beta \\
\Delta p \\
\Delta \phi
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\Delta r \\
\Delta \beta \\
\Delta p \\
\Delta \phi
\end{bmatrix}
\]

or \( \Delta x_M = T_M \Delta x_s \); \( T_M \) is a similarity transformation matrix; therefore, the model eigenvalues can be preserved in writing Eq. (1) with \( \Delta x_M \) as the state vector:

\[
\Delta x_M = T_M \Delta x_p + T_M \Delta \delta_M = F_M \Delta x_M + G_M \Delta \delta_M
\]

This "perfect" model-following control law is seen to include state feedback (through \( C_s \)) and command-input interconnect (through \( C_p \)). The elements of the closed-loop simulator matrices are equivalent to the stability derivatives of the model:

\[
\Delta x_s = (F_s + G_s C_p) \Delta x_s + G_s C_p \Delta \delta_m = F_M \Delta x_M + G_M \Delta \delta_M
\]

\[
Lateral\ Acceleration\ Matching
\]

The body-axis acceleration vector at the pilot's station is

\[
\alpha_p = \dot{\alpha}_p - H_p \Delta \omega + \Delta \omega \times \Delta x_p - \Delta x_p \Delta \omega
\]

Recognizing that stability axes are a special set of body axes (nominally oriented to the velocity vector), neglecting coupling between longitudinal and lateral-directional motions, and assuming that the pilot's station is on the aircraft's plane of symmetry, perturbations in lateral acceleration are described by

\[
\Delta \alpha_p = V(\Delta \beta + \Delta r) + g \Delta \phi + (x_p - x_M) \Delta \phi
\]

With differing velocity or pilot offset, one or more simulator state variables must be altered to provide the \( \Delta \delta_p \) match. This is most readily accomplished by transforming a different state variable for each type of acceleration mismatch.

Consider first the model transformation for pilot-center of gravity (c.g.) offset at identical airspeeds. Three states (\( \Delta r \), \( \Delta p \), and \( \Delta \phi \)) are to be matched identically. Defining \( k = x_p - x_M \), model and simulator lateral accelerations are matched when

\[
V_M \Delta \beta_M + x_M \Delta \phi_M - z_M \Delta \phi_M = V_M \Delta \beta_M + k \Delta \phi
\]

Rearranging Eq. (10) and integrating once provides \( \Delta \beta_M \), and the model state vector which the simulator should follow to compensate for differing pilot-c.g. offsets is

\[
\begin{bmatrix}
\Delta r \\
\Delta \beta \\
\Delta p \\
\Delta \phi
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\Delta r \\
\Delta \beta \\
\Delta p \\
\Delta \phi
\end{bmatrix}
\]

or \( \Delta x_M = T_M \Delta x_s \); \( T_M \) is a similarity transformation matrix; therefore, the model eigenvalues can be preserved in writing Eq. (1) with \( \Delta x_M \) as the state vector:

\[
\Delta x_M = T_M \Delta x_p + T_M \Delta \delta_M = F_M \Delta x_M + G_M \Delta \delta_M
\]

Fig. 1 Comparison of SSV and VRA response, with lateral acceleration matching.
Table 1: Implicit model-following gains for VRA simulation of SSV

<table>
<thead>
<tr>
<th>Cₚ</th>
<th>Cᵧ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δᵣₛ</td>
<td>Δᵣₛ</td>
</tr>
<tr>
<td>Δᵣₛ</td>
<td>Δᵣₛ</td>
</tr>
</tbody>
</table>

without lateral acceleration matching

<table>
<thead>
<tr>
<th>Δᵣₛ</th>
<th>Δᵣₛ</th>
<th>Δᵣₛ</th>
<th>Δᵣₛ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.071</td>
<td>-1.057</td>
<td>0.057</td>
<td>0.033</td>
</tr>
<tr>
<td>-0.018</td>
<td>-0.556</td>
<td>-0.014</td>
<td>0.383</td>
</tr>
<tr>
<td>0.034</td>
<td>-0.139</td>
<td>-0.295</td>
<td>-0.021</td>
</tr>
</tbody>
</table>

with lateral acceleration matching

<table>
<thead>
<tr>
<th>Δᵣₛ</th>
<th>Δᵣₛ</th>
<th>Δᵣₛ</th>
<th>Δᵣₛ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.165</td>
<td>-0.483</td>
<td>0.081</td>
<td>-0.014</td>
</tr>
<tr>
<td>-0.041</td>
<td>0.928</td>
<td>-0.020</td>
<td>0.003</td>
</tr>
<tr>
<td>0.019</td>
<td>-0.238</td>
<td>-0.299</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Next, consider the model transformation for velocity mismatch with no pilot-c.g. offset. With Δᵦ, Δᵦᵧ, and Δᵦᵧ mismatched identically,

\[ Vₛ(Δᵦₘ + Δᵦₛ) = Vₘ(Δᵦₘ + Δᵦₛ) \]  

(13)

or

\[ Δᵦₛ = (Vₘ/Vₛ)Δᵦₘ + (Vₘ/Vₛ)Δᵦₛ \]  

(14)

Substituting for Δᵦₘ using Eqs. (1), (3), and (5),

\[ Δᵦₛ = \begin{pmatrix} \frac{Yₛ}{Vₛ}Δᵦₘ - \frac{Yₘ}{Vₘ}Δᵦₘ + \left( \frac{Yₘ}{Vₘ}Δᵦₘ + \frac{Yₘ}{Vₘ}Δᵦᵧ \right) \left( \frac{Yₘ}{Vₘ}Δᵦₘ + \frac{Yₘ}{Vₘ}Δᵦᵧ \right) + \frac{Gₘ}{Vₘ}Δᵦₘ \end{pmatrix} \]

(15)

The model state transformation for velocity mismatch incorporates Eq. (15) for the modeled yaw rate in \( Tₛ \) and \( Uₛ \), and \( Δᵦₛ = TₛΔᵦₘ + UₛΔᵦᵧ \). The transformed model's dynamic equation is

\[ Δₓₖₛ = TₛFₛTₛ⁻¹Δᵦₖₛ + Tₛ(Gₛ - FₛTₛ⁻¹Uₛ)Δᵦₘ + UₛΔᵦᵧ \]  

(16)

Since \( Uₛ \) contains only side-force control terms, it often would be negligible. In this case, applying the velocity mismatch transformation to the offset-corrected model [Eq. (12)] leads to

\[ Δₓₖₛ = TₛTₛ⁻¹Δᵦₖₛ + Tₛ(Gₛ - FₛTₛ⁻¹Uₛ)Δᵦₘ + UₛΔᵦᵧ \]  

(17)

with \( Δₓₖₛ = TₛTₖΔᵦₛ \). This is the model which should be followed by the in-flight simulator for lateral acceleration matching.

Velocity Mismatch Example

An example based upon VRA simulation of the Space Shuttle vehicle (SSV) is presented, considering only the velocity mismatch between the two vehicles. The SSV flight condition is \( M = 1.5 \) at an altitude of 18,300 m (60,000 ft); the VRA is assumed to perform the in-flight simulation at an airspeed of 105 knots and an altitude of 1500 m (5000 ft).

Open-loop response of the SSV to a 1°-differential elevon step input results in a maximum lateral acceleration of 0.05 g at \( t = 4 \) s. Without lateral acceleration matching, the in-flight simulator can match the state response with negligible error; however, as a result of the velocity mismatch, \( Δᵦₛ \) is 0.24 g at \( t = 4 \) s and growing steadily. The VRA can match lateral acceleration precisely (as well as \( Δᵦ, Δᵦᵧ, \) and \( Δᵦᵧ \)) by following a transformed yaw rate (Fig. 1). Response in the first few seconds after the pilot's command is of greatest importance, and roll control is the primary SSV piloting task at this flight condition; therefore, the difference in VRA and SSV yaw responses should not have a major effect on flying qualities assessment.

The effect of \( Δᵦₛ \) matching on ESD gains is of interest; Table 1 presents \( Cₚ \) and \( Cᵧ \) for the two cases. Feedback gains are changed substantially, but \( Δᵦₛ \) has no significant effect on control interconnects. The most notable effects are on sideslip and roll angle feedback to rudder and side-force panel, as might be expected.

Conclusion

Similarity transformations which preserve modal characteristics and acceleration cues in in-flight simulation are presented. A velocity-mismatch example illustrates that acceleration matching is achieved at the expense of mismatching in cues which are secondary to the simulated piloting task, while primary cues are preserved. The approach is applicable for both implicit and explicit model following, and it can be easily extended to the longitudinal case.

Acknowledgments

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