At the End of the Course, you should be able to:

- Understand aircraft configuration aerodynamics, performance, stability, and control
- Estimate an aircraft’s aerodynamic characteristics from geometric and inertial properties
- Analyze linear and nonlinear dynamic systems
- Recognize airplane modes of motion and their significance
- Compute aircraft motions
- Appreciate historical development of aviation
Syllabus, First Half

- Introduction, Math Preliminaries
- Point Mass Dynamics
- Aerodynamics of Airplane Configurations
- Cruising Flight Performance
- Gliding, Climbing, and Turning Performance
- Nonlinear, 6-DOF Equations of Motion
- Aircraft Control Devices and Systems
- Linearized Equations of Motion
- Longitudinal Dynamics
- Lateral-Directional Dynamics

Details, reading, homework assignments, and references at
http://blackboard.princeton.edu/

Syllabus, Second Half

- Analysis of Linear Systems
  - Time Response
  - Transfer Functions and Frequency Response
  - Root Locus Analysis
- Advanced Problems in Longitudinal Dynamics
- Advanced Problems in Lateral-Directional Dynamics
- Flying Qualities Criteria
- Maneuvering at High Angles and Rates
- Aeroelasticity and Fuel Slosh
- Problems of High Speed and Altitude
You’re interested in MAE 331 because …?

Details

- Lecture: 3–4:20, J-201, Tue & Thu, E-Quad
  - 5-min break at mid-point
- Precept: 7–8:20, D-221, Mon
- Engineering, science, & math
- Case studies, historical context
- 8 homework assignments
- Office hours: 1:30–2:30, MW, D-202, or any time my door is open. e-mail ahead, if possible
- Assistant in Instruction: Pierre-Yves Taunay, Office hours: TBD
Details

• Lecture slides
  - pdfs from all 2014 lectures are available now at http://www.princeton.edu/~stengel/MAE331.html
  - pdf for current (2016) lecture on Blackboard morning of class or day before

• GRADING
  - Assignments: 50%
  - Term Paper: 30%
  - Class participation: 10%
  - Quick Quiz (10, 5 min): 10%

Text and References

• Science, Engineering, and Math:
  - Flight Dynamics, RFS, Princeton University Press, 2004

• Case Studies, Historical Context

• Supplemental references
  - References at end of FD chapters & Appendix
  - On-line: Virtual reference book
**Flight Dynamics Book and Computer Code**

- All programs are accessible from the *Flight Dynamics* web page
  - [http://www.princeton.edu/~stengel/FlightDynamics.html](http://www.princeton.edu/~stengel/FlightDynamics.html)
- ... or directly
- **ERRATA** for the book are listed there
- 6-degree-of-freedom nonlinear simulation of a business jet aircraft (MATLAB)
  - [http://www.princeton.edu/~stengel/FDcodeB.html](http://www.princeton.edu/~stengel/FDcodeB.html)
- Linear system analysis (MATLAB)
- Paper airplane simulation (MATLAB)
- Performance analysis of a business jet aircraft (Excel)
  - [http://www.princeton.edu/~stengel/Example261.xls](http://www.princeton.edu/~stengel/Example261.xls)

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**Quick Quizzes**

*First 5 Minutes of 10 Classes*

- One question about the lectures and reading assignments from the previous week
- Largely qualitative but may require simple calculations
- Be sure to bring a pencil, paper, and calculator to class
Homework Assignments

- Groups of 2 or 3 students for all assignments
- Team members for each assignment will be different
  - chosen using a spreadsheet and random number generator (TBD)
- Each member of each team will receive the same grade as the others
- Identify who did what on each assignment
- Submit via [http://blackboard.princeton.edu/](http://blackboard.princeton.edu/)

Flight Tests Using Balsa Glider and Cockpit Flight Simulator

- In your Cessna 172:
  - Takeoff from Princeton Airport
  - Fly over Carnegie Lake
  - Land at Princeton Airport
- Compare actual flight of the glider with trajectory simulation
Assignment #1

due: September 23, 2016

• Document the physical characteristics and flight behavior of a balsa glider
  - Everything that you know about the physical characteristics of the glider
  - Everything that you know about the flight characteristics of the glider

• 2-person team, joint write-up

• Team assignments on http://blackboard.princeton.edu/

Luke Nash’s Biplane Glider
Flight #1 (MAE 331, 2008)
Frame Grab Sequence of Luke Nash’s Flight

• Can determine height, range, velocity, flight path angle, and pitch angle from sequence of digital photos (*QuickTime*)

Trajectory Estimation from Photo Sequence

Smaller, fixed-interval time steps Interpolation and differencing
Stability and Control Case Studies

![F-100](image)

![Ercoupe](image)

![Electra](image)

**Reading Assignments**

- **Do Flight Dynamics reading before class**
- **Case Studies/Historical Context:**
  - **Airplane Stability and Control**
  - 10-minute synopses by groups of 3 students
    - Principal subject/scope of chapter
    - Technical ideas needed to understand chapter
    - When did the events occur?
    - 3 main “takeaway points” or conclusions
    - 3 most surprising or remarkable facts
  - 1st synopsis: Sept 29th, team members TBD
Goals for Airplane Design

- Shape of airplane determined by purpose
- Safety, handling, performance, functioning, and comfort
- Agility vs. sedateness
- Control surfaces adequate to produce needed moments (i.e., torques)
- Tradeoffs, e.g., center of mass location
  - too far forward increases unpowered control-stick forces
  - too far aft degrades static stability

Configuration Driven By The Mission and Flight Envelope
Inhabited Air Vehicles

Uninhabited Air Vehicles (UAV)
Introduction to Flight Dynamics

Airplane Components

- Vertical stabilizer ("fin")
- Rudder
- Elevator
- Horizontal stabilizer
- Aileron
- Flap
- Fuselage
- Wing
- Landing gear
Airplane Rotational Degrees of Freedom

Airplane Translational Degrees of Freedom
Phases of Flight

Flight of a Paper Airplane
Flight of a Paper Airplane
Example 1.3-1, *Flight Dynamics*

- Equations of motion integrated numerically to estimate the flight path
- Red: Equilibrium flight path
- Black: Initial flight path angle = 0
- Blue: plus increased initial airspeed
- Green: loop

Flight of a Paper Airplane
Example 1.3-1, *Flight Dynamics*

- Red: Equilibrium flight path
- Black: Initial flight path angle = 0
- Blue: plus increased initial airspeed
- Green: loop
Assignment #2

- Compute the trajectory of a balsa glider
- Simulate using equations of motion
- Compare to the actual flight of the glider (HW #1)
- Similar to the flight of a paper airplane
- 2-person team assignment

Gliding Flight
(b) Glide aerodynamic characteristics.
Dynamic Systems

**Dynamic Process:** Current state depends on prior state

- $x =$ dynamic state
- $u =$ input
- $w =$ exogenous disturbance
- $p =$ parameter
- $t$ or $k =$ time or event index

**Observation Process:** Measurement may contain error or be incomplete

- $y =$ output (error-free)
- $z =$ measurement
- $n =$ measurement error

All of these quantities are multidimensional
They can be expressed as vectors

---

**Notation for Scalars and Vectors**

- **Scalar:** usually lower case: $a$, $b$, $c$, …, $x$, $y$, $z$

  
  \[
  a = 12; \quad b = 7; \quad c = a + b = 19; \quad x = a + b^2 = 12 + 49 = 61
  \]

- **Vector:** usually bold or with underbar: $\mathbf{x}$ or $\mathbf{x}$
  
  - Ordered set
  - Column of scalars
  - Dimension = $n \times 1$

\[
\mathbf{a} = \begin{bmatrix} 2 \\ -7 \\ 16 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}
\]
Matrices and Transpose

- **Matrix**: usually bold capital or capital: \( \mathbf{F} \) or \( \mathbf{F} \)
  - Dimension = \((m \times n)\)

\[
\mathbf{x} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}; \quad \mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \\ l & m & n \end{bmatrix}
\]

\((3 \times 1) \quad (4 \times 3)\)

- **Transpose**: interchange rows and columns

\[
\mathbf{x}^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \quad \mathbf{A}^T = \begin{bmatrix} a & d & g & l \\ b & e & h & m \\ c & f & k & n \end{bmatrix}
\]

Multiplication

- **Operands must be conformable**
- **Multiplication of vector by scalar** is associative, commutative, and distributive

\[
a \mathbf{x} = \mathbf{x} a = \begin{bmatrix} ax_1 \\ ax_2 \\ ax_3 \end{bmatrix}
\]

\[
a (\mathbf{x} + \mathbf{y}) = (\mathbf{x} + \mathbf{y}) a = (a\mathbf{x} + a\mathbf{y})
\]

\[
\dim(x) = \dim(y)
\]

\[
a \mathbf{x}^T = \begin{bmatrix} ax_1 & ax_2 & ax_3 \end{bmatrix}
\]
Addition

- Conformable vectors and matrices are added term by term

\[
\mathbf{x} = \begin{bmatrix} a \\ b \end{bmatrix}; \quad \mathbf{z} = \begin{bmatrix} c \\ d \end{bmatrix}
\]

\[
\mathbf{x} + \mathbf{z} = \begin{bmatrix} a + c \\ b + d \end{bmatrix}
\]

Inner Product

- Inner (dot) product of vectors produces scalar result

\[
\mathbf{x}^T \mathbf{x} = \mathbf{x} \bullet \mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = (x_1^2 + x_2^2 + x_3^2)
\]

- Length (or magnitude) of vector is square root of dot product

\[
= (x_1^2 + x_2^2 + x_3^2)^{1/2}
\]
Vector Transformation

- **Matrix-vector product** transforms one vector into another
- **Matrix-matrix product** produces a new matrix

\[
y = Ax = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -5 & 7 \\ 4 & 1 & 8 \\ -9 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
\]

\[(n \times 1) = (n \times m)(m \times 1)\]

Derivatives and Integrals of Vectors

Derivatives and integrals of vectors are vectors of derivatives and integrals

\[
\frac{dx}{dt} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix}
\]

\[
\int x \, dt = \begin{bmatrix} \int x_1 \, dt \\ \int x_2 \, dt \\ \int x_3 \, dt \end{bmatrix}
\]
Matrix Inverse

Transformation

\[ x_2 = Ax_1 \]

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}_2 =
\begin{bmatrix}
  \cos \theta & 0 & -\sin \theta \\
  0 & 1 & 0 \\
  \sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}_1
\]

Inverse Transformation

\[ x_1 = A^{-1}x_2 \]

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}_1 =
\begin{bmatrix}
  \cos \theta & 0 & \sin \theta \\
  0 & 1 & 0 \\
  -\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}_2
\]

Matrix Identity and Inverse

- **Identity matrix**: no change when it multiplies a conformable vector or matrix

\[
I_3 =
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 1
\end{bmatrix}
\]

\[ y = Iy \]

- **A non-singular square matrix** multiplied by its inverse forms an identity matrix

\[
AA^{-1} = A^{-1}A = I
\]
Mathematical Models of Dynamic Systems are Differential Equations

Continuous-time dynamic process: Vector Ordinary Differential Equation

\[ \dot{x}(t) \triangleq \frac{dx(t)}{dt} = f[x(t), u(t), w(t), p(t), t] \]

Output Transformation

\[ y(t) = h[x(t), u(t)] \]

Measurement with Error

\[ z(t) = y(t) + n(t) \]

Next Time:

Learning Objectives
Point-Mass Dynamics
Aerodynamic/Thrust Forces

Reading:
Flight Dynamics
Introduction, 1–27
The Earth’s Atmosphere, 29–34
Kinematic Equations, 38–53
Forces and Moments, 59–65
Introduction to Thrust, 103–107
Supplemental Material

Helpful Resources

• Web pages
  – http://blackboard.princeton.edu/
  – http://www.princeton.edu/~stengel/MAE331.html
  – http://www.princeton.edu/~stengel/FlightDynamics.html

• Princeton University Engineering Library (paper and online)

• NACA/NASA pubs
  – http://ntrs.nasa.gov/search.jsp
<table>
<thead>
<tr>
<th>Course Learning Objectives</th>
<th>ABET Criterion 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding of the dynamics and control of aircraft.</td>
<td>a</td>
</tr>
<tr>
<td>Ability to estimate aerodynamic coefficients and stability derivatives from aircraft geometry and flight envelope.</td>
<td>a, c</td>
</tr>
<tr>
<td>Facility in analyzing mathematical descriptions of the rigid-body motions of flying vehicles.</td>
<td>a</td>
</tr>
<tr>
<td>Ability to estimate the performance, stability, and control characteristics of aircraft.</td>
<td>b</td>
</tr>
<tr>
<td>Development of appreciation for flight-testing methods and results.</td>
<td>b, k</td>
</tr>
<tr>
<td>Ability to apply systems-engineering approach to the analysis, design, and testing of aircraft.</td>
<td>b, c</td>
</tr>
<tr>
<td>Demonstration of ability to work in multidisciplinary teams.</td>
<td>d</td>
</tr>
<tr>
<td>Demonstration of computational problem-solving, through thorough knowledge, application, and development of analytical software</td>
<td>e, k</td>
</tr>
<tr>
<td>Appreciation of the historical context within which airplanes have evolved to present-day configurations.</td>
<td>f, h, i, j</td>
</tr>
<tr>
<td>Competence in presenting ideas.</td>
<td>g</td>
</tr>
</tbody>
</table>