Flying Qualities Criteria
Robert Stengel, Aircraft Flight Dynamics
MAE 331, 2014

Learning Objectives

- MIL-F-8785C criteria
- CAP, C*, and other longitudinal criteria
- ϕ/β, ωp/ωn and other lateral-directional criteria
- Pilot-vehicle interactions
- Flight control system design

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http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

Flying Qualities Research Moves with the Times

Chapter 21, Airplane Stability and Control, Abzug and Larrabee

- What are the principal subject and scope of the chapter?
- What technical ideas are needed to understand the chapter?
- During what time period did the events covered in the chapter take place?
- What are the three main "takeaway" points or conclusions from the reading?
- What are the three most surprising or remarkable facts that you found in the reading?
Design for Satisfactory Flying Qualities

- Satisfy procurement requirement (e.g., Mil Standard)
- Satisfy test pilots (e.g., Cooper-Harper ratings)
- Avoid pilot-induced oscillations (PIO)
- Minimize time-delay effects
- Time- and frequency-domain criteria

Short-Period “Bullseye” or “Thumbprint”

Robert Harper, Cornell Aero Lab
Cooper-Harper Handling Qualities Rating Scale

MIL-F-8785C Identifies Satisfactory, Acceptable, and Unacceptable Response Characteristics

Damping Ratio

<table>
<thead>
<tr>
<th>Category A and C Flight Phases</th>
<th>Category B Flight Phases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>Minimum</td>
</tr>
<tr>
<td>1</td>
<td>0.95</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
</tr>
</tbody>
</table>

* May be reduced at altitudes above 20,000 feet if approved by the procuring activity.

Short-period angle-of-attack response to elevator input

Frequency Response

Step Response
Military Flying Qualities Specifications, MIL-F-8785C

- Specifications established during WWII
- US Air Force and Navy coordinated efforts beginning in 1945
- First version appeared in 1948, last in 1980
- Distinctions by flight phase, mission, and aircraft type
- Replaced by Military Flying Qualities Standard, MIL-STD-1797A, with procurement-specific criteria

MIL-F-8785C Aircraft Types

I. Small, light airplanes, e.g., utility aircraft and primary trainers
II. Medium-weight, low-to-medium maneuverability airplanes, e.g., small transports or tactical bombers
III. Large, heavy, low-to-medium maneuverability airplanes, e.g., heavy transports, tankers, or bombers
IV. Highly maneuverable aircraft, e.g., fighter and attack airplanes
MIL-F-8785C Flight Phase

A. Non-terminal flight requiring rapid maneuvering precise tracking, or precise flight path control
   • air-to-air combat
   • ground attack
   • in-flight refueling (receiver)
   • close reconnaissance
   • terrain following
   • close formation flying
B. Non-terminal flight requiring gradual maneuvering
   • climb, cruise
   • in-flight refueling (tanker)
   • descent
C. Terminal flight
   • takeoff (normal and catapult)
   • approach
   • wave-off/go-around
   • landing

MIL-F-8785C Levels of Performance

1. Flying qualities clearly adequate for the mission flight phase
2. Flying qualities adequate to accomplish the mission flight phase, with some increase in pilot workload or degradation of mission effectiveness
3. Flying qualities such that the aircraft can be controlled safely, but pilot workload is excessive or mission effectiveness is inadequate
Principal MIL-F-8785C Metrics

• Longitudinal flying qualities
  – static speed stability
  – phugoid stability
  – flight path stability
  – short period frequency and its relationship to command acceleration sensitivity
  – short period damping
  – control-force gradients

• Lateral-directional flying qualities
  – natural frequency and damping of the Dutch roll mode
  – time constants of the roll and spiral modes
  – rolling response to commands and Dutch roll oscillation
  – sideslip excursions
  – maximum stick and pedal forces
  – turn coordination

Longitudinal Criteria
Long-Period Flying Qualities Criteria
(MIL-F-8785C)

- Static speed stability
  - No tendency for aperiodic divergence
    - Phugoid oscillation $\rightarrow$ 2 real roots, 1 that is unstable
  - Stable control stick position and force gradients
    - e.g., Increasing "pull" position and force with decreasing speed

- Flight path stability [Phase C]
  1. $(\Delta \gamma/\Delta V)_{SS} < 0.06$ deg/kt
  2. $(\Delta \gamma/\Delta V)_{SS} < 0.15$ deg/kt
  3. $(\Delta \gamma/\Delta V)_{SS} < 0.24$ deg/kt

Steady-State Response to Elevator

\[
\Delta V_{SS} = a \Delta \delta E_{SS} \\
\Delta \gamma_{SS} = c \Delta \delta E_{SS}
\]

Ratio

\[
\frac{\Delta \gamma_{SS}}{\Delta V_{SS}} = \frac{c}{a} \quad \text{(with appropriate scaling)}
\]
Long-Period Flying Qualities Criteria
(MIL-F-8785C)

• Phugoid stability
  1. Damping ratio $\geq 0.04$
  2. Damping ratio $\geq 0$
  3. “Time to double”, $T_2 \geq 55$ sec

\[
T_{2,ph} = -\frac{0.693}{\zeta_{ph} \omega_{n,ph}}
\]

Short Period Criteria

• Important parameters
  – Short-period natural frequency
  – Damping ratio
  – Lift slope
  – Step response
    • Over-/under-shoot
    • Rise time
    • Settling time
    • Pure time delay
  – Pitch angle response
  – Normal load factor response
  – Flight path angle response (landing)
Short-Period Approximation
Transfer Functions

- Elevator to pitch rate

\[
\frac{\Delta q(s)}{\Delta \delta E(s)} = \frac{k_q(s - z_q)}{s^2 + 2\zeta_n SP s + \omega_n^2} = \frac{k_q(s + 1/T)}{s^2 + 2\zeta_n SP s + \omega_n^2}
\]

- Pure gain or phase change (< 90°) in feedback control cannot produce instability

- Elevator to pitch angle
- Integral of prior example

\[
\frac{\Delta \theta(s)}{\Delta \delta E(s)} = \frac{k_q(s - z_q)}{s^2 + 2\zeta_n SP s + \omega_n^2}
\]

- Pure gain or phase change (< 45°) in feedback control cannot produce instability
Normal Load Factor

\[ \Delta n_z = \frac{V_N}{g} (\Delta \alpha - \Delta q) = -\frac{V_N}{g} \left( \frac{L_\alpha}{V_N} \Delta \alpha + \frac{L_{SE}}{V_N} \Delta E \right) \]

- Therefore, with negligible \( L_{SE} \) (aft tail/canard effect)

\[ \frac{\partial \Delta n_z(s)}{\partial \Delta \delta E(s)} = \frac{1}{g} \left( \frac{L_\alpha}{V_N} \frac{\partial \Delta \alpha(s)}{\partial \Delta \delta E(s)} + L_{SE} \right) \approx \left( \frac{L_\alpha}{g} \right) \frac{\partial \Delta \alpha(s)}{\partial \Delta \delta E(s)} \]

- Elevator to angle of attack \((L_{SE} = 0)\)

\[ \frac{\Delta \alpha(s)}{\Delta \delta E(s)} \approx \frac{k_\alpha}{s^2 + 2 \Sigma \omega_n \omega_{nSP} s + \omega_{nSP}^2} \]

Control Anticipation Parameter, \( CAP \)

Inner ear senses angular acceleration about 3 axes

Initial Angular Acceleration

\[ \Delta \dot{q}(0) = \left( M_{SE} - \frac{M_\alpha}{V_N + L_\alpha} \right) \Delta \delta E_{SS} \]

Desired Normal Load Factor

\[ \Delta n_{SS} = \frac{V_N}{g} \Delta q_{SS} = -\left( \frac{V_N}{g} \right) \left( \frac{M_{SE} L_\alpha / V_N - M_\alpha L_{SE} / V_N}{M_\alpha L_\alpha / V_N + M_\alpha} \right) \Delta \delta E_{SS} \]
Control Anticipation Parameter, \( \textit{CAP} \)

Inner ear cue should aid pilot in anticipating commanded normal acceleration

\[
\text{CAP} = \frac{\Delta \dot{q}(0)}{\Delta n_{SS}} = \frac{-\left( M_{\delta E} - \frac{M_\alpha}{V_N + L_\alpha} \right) \left( M_q \frac{L_\alpha / V_N}{\alpha} + M_\alpha \right)}{\left( L_\alpha M_{\delta E} - L_{\delta E} M_\alpha \right) / g}
\]

with \( L_{\delta E} = 0 \)

\[
\text{CAP} = \frac{-\left( M_q \frac{L_\alpha}{V_N} + M_\alpha \right)}{L_\alpha / g} \approx \frac{\omega_{n_{SP}}^2}{\frac{n_z}{\alpha}}
\]

MIL-F-8785C
Short-Period
Flying Qualities Criterion

\( \text{CAP} = \text{constant along Level Boundaries} \)
Control Anticipation Parameter vs. Short-Period Damping Ratio  
(MIL-F-8785C, Category A)

\[ CAP = -\left( \frac{M_q L_\alpha}{V_N} + M_\alpha \right) \frac{L_\alpha}{g} \approx \frac{\omega_{nSP}^2}{n_z / \alpha} \]

\[ C^* = \Delta n_{pilot} + \frac{V_{crossover}}{g} \Delta q \]
\[ = l_{pilot} \Delta \dot{q} + \Delta n_{cm} \left( \Delta q - \Delta \dot{\alpha} \right) + \frac{V_{crossover}}{g} \Delta q \]

- Below \( V_{crossover} \) \( \Delta q \) is pilot’s primary control objective
- Above \( V_{crossover} \) \( \Delta n_{pilot} \) is the primary control objective

Fighter Aircraft: \( V_{crossover} \approx 125 \text{ m/s} \)
**Gibson Dropback Criterion for Pitch Angle Control**

- Step response of pitch rate should have overshoot for satisfactory pitch and flight path angle response.

\[
\frac{\Delta q(s)}{\Delta \delta E(s)} = \frac{k_q \left( s + \frac{1}{T_{\theta_2}} \right)}{s^2 + 2\xi_{SP} \omega_{nSP} s + \omega_{nSP}^2} = \frac{k_q \left( s + \frac{\omega_{nSP}}{\xi_{SP}} \right)}{s^2 + 2\xi_{SP} \omega_{nSP} s + \omega_{nSP}^2}
\]

- Criterion is satisfied when

\[
\zeta_q \Delta = \frac{1}{T_{\theta_2}} = -\left( \frac{\omega_{nSP}}{\xi_{SP}} \right)
\]

---

- **Shift to Lecture on Advanced Problems in Lateral-Directional Dynamics**
- **Return to Lateral-Directional Flying Qualities Criteria**
Early Lateral-Directional Flying Qualities Criteria

Lateral-Directional Flying Qualities Parameters

• Lateral Control Divergence Parameter, \( LCDP \)
• \( \phi/\beta \) Effect
• \( \omega_\phi/\omega_d \) Effect
Lateral Control Divergence Parameter (LCDP)

• Aileron deflection produces yawing as well as rolling moment
  – “Favorable yaw” aids the turn command
  – “Adverse yaw” opposes it
• Equilibrium response to constant aileron input

\[
\frac{\Delta \phi_s}{\Delta \delta A_s} = \frac{N_\beta + N_r \frac{Y_\beta}{V_N}}{L_{\delta A}} - \frac{\left( L_\beta + L_r \frac{Y_\beta}{V_N} \right) N_{\delta A}}{\frac{g}{V_N} \left( L_\beta N_r - L_r N_\beta \right)}
\]

• Large-enough \( N_{\delta A} \) effect can reverse the sign of the response
  – Can occur at high angle of attack
  – Can cause “departure from controlled flight”
• Lateral Control Divergence Parameter provides simplified criterion

\[
\frac{(N_\beta) L_{\delta A} - (L_\beta) N_{\delta A}}{L_{\delta A}} = N_\beta - \frac{N_{\delta A}}{L_{\delta A}} L_\beta
\]

\[\text{LCDP} \equiv C_{n_\beta} - \frac{C_{n_{\delta A}}}{C_{l_{\delta A}}} C_{l_\beta}\]

\[\omega_\phi/\omega_d\] Effect

• Aileron-to-roll-angle transfer function

\[
\frac{\Delta \phi(s)}{\Delta \delta A(s)} = \frac{k_\phi \left( s^2 + 2 \zeta_\phi \omega_\phi s + \omega_\phi^2 \right)}{(s - \lambda_S)(s - \lambda_R) \left( s^2 + 2 \zeta_{nDR} \omega_{nDR} s + \omega_{nDR}^2 \right)}
\]

– \( \omega_\phi \) is the “natural frequency” of the complex zeros
– \( \omega_d = \omega_{nDR} \) is the natural frequency of the Dutch roll mode
• Conditional instability may occur with closed-loop control of roll angle, even with a perfect pilot
\( \omega_\varphi/\omega_d \) Effect is Important in Roll Angle Control

\[
\frac{\Delta \phi(s)}{\Delta \delta A(s)} = \frac{k_\varphi (s^2 + 2\zeta_\varphi \omega_d s + \omega_d^2)}{(s - \lambda_2)\left((s - \lambda_2)(s^2 + 2\zeta_\delta \omega_d s + \omega_d^2)\right)}
\]

- As feedback gain increases, Dutch roll roots go to numerator zeros
- If zeros are over poles, conditional instability results

\( \varphi/\beta \) Effect

- \( \varphi/\beta \) measures the degree of rolling response in the Dutch roll mode
  - Large \( \varphi/\beta \): Dutch roll is primarily a rolling motion
  - Small \( \varphi/\beta \): Dutch roll is primarily a yawing motion
- Eigenvectors, \( e_i \), indicate the degree of participation of the state component in the \( i^{th} \) mode of motion

\[
\det(sI - F) = (s - \lambda_1)(s - \lambda_2)\ldots(s - \lambda_n)
\]

\[
(\lambda_i I - F)e_i = 0
\]
Eigenvectors

• Eigenvectors, $e_i$, are solutions to the equation

$$\left( \lambda_i I - F \right) e_i = 0, \quad i = 1, n$$

or

$$\lambda_i e_i = Fe_i, \quad i = 1, n$$

• For each eigenvalue, the corresponding eigenvector can be found (within an arbitrary constant) from

$$\text{Adj}\left( \lambda_i I - F \right) = \begin{pmatrix} a_1 e_i & a_2 e_i & \ldots & a_n e_i \end{pmatrix}, \quad i = 1, n$$

MATLAB

$$(V,D) = \text{eig}(F)$$

$V$: Modal Matrix (i.e., Matrix of Eigenvectors)

$D$: Diagonal Matrix of Corresponding Eigenvalues

\[ \phi/\beta \text{ Effect} \]

• With $\lambda_i$ chosen as a complex root of the Dutch roll mode, the corresponding eigenvector is

$$e_{\text{DR}^+} = \begin{bmatrix} e_r \\
 e_\beta \\
 e_p \\
 e_\phi \end{bmatrix}_{\text{DR}^+} = \begin{bmatrix} (\sigma + j\omega)_r \\
 (\sigma + j\omega)_\beta \\
 (\sigma + j\omega)_p \\
 (\sigma + j\omega)_\phi \end{bmatrix}_{\text{DR}^+} = \begin{bmatrix} (AR e^{i\theta})_r \\
 (AR e^{i\theta})_\beta \\
 (AR e^{i\theta})_p \\
 (AR e^{i\theta})_\phi \end{bmatrix}_{\text{DR}^+}$$

• $\phi/\beta$ is the magnitude of the ratio of the $\phi$ and $\beta$ eigenvectors

$$\phi/\beta = \left| \frac{(AR)_\phi}{(AR)_\beta} \right| = \left( \frac{V_N}{g} \right) \left[ \left( \zeta_{\text{DR}^+} \omega_{n_{\text{DR}^+}} + \frac{Y_\beta}{V_N} + \frac{L_\beta}{L_r} \right)^2 + \left( \omega_{n_{\text{DR}^+}} \sqrt{1 - \zeta_{\text{DR}^+}^2} \right) \right]^{1/2}$$
### Criteria for Lateral-Directional Modes (MIL-F-8785C)

#### TABLE VII. Maximum roll-mode time constant, seconds.

<table>
<thead>
<tr>
<th>Flight Phase Category</th>
<th>Class</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>I, IV</td>
<td>1.0</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>II, III</td>
<td>1.4</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>All</td>
<td>1.4</td>
<td>3.0</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>I, II-C, IV</td>
<td>1.0</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>II-L, III</td>
<td>1.4</td>
<td>3.0</td>
<td></td>
</tr>
</tbody>
</table>

#### TABLE VIII. Spiral stability - minimum time to double amplitude.

<table>
<thead>
<tr>
<th>Flight Phase Category</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A &amp; C</td>
<td>12 sec</td>
<td>8 sec</td>
<td>4 sec</td>
</tr>
<tr>
<td>B</td>
<td>20 sec</td>
<td>8 sec</td>
<td>4 sec</td>
</tr>
</tbody>
</table>
Minimum Dutch Roll Natural Frequency and Damping (MIL-F-8785C)

<table>
<thead>
<tr>
<th>Flight Phase Level</th>
<th>Category</th>
<th>Class</th>
<th>( \min \zeta_d )</th>
<th>( \min \zeta_d \omega_0 \text{ rad/sec} )</th>
<th>( \min \omega_0 \text{ rad/sec} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (CO and OA)</td>
<td>I</td>
<td>0.4</td>
<td>-</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>II, III</td>
<td>0.19</td>
<td>0.35</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>All</td>
<td>0.08</td>
<td>0.15</td>
<td>0.4**</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>I, II-C</td>
<td>0.08</td>
<td>0.15</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>II-L, III</td>
<td>0.08</td>
<td>0.10</td>
<td>0.4**</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>All</td>
<td>0.02</td>
<td>0.05</td>
<td>0.4**</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>All</td>
<td>0</td>
<td>-</td>
<td>0.4**</td>
<td></td>
</tr>
</tbody>
</table>

* The governing damping requirement is that yielding the larger value of \( \zeta_d \), except that a \( \zeta_d \) of 0.7 is the maximum required for Class III.

** Class III airplanes may be excepted from the minimum \( \zeta_d \) requirement, subject to approval by the procuring activity, if the requirements of 3.3.2 through 3.3.2.4, 3.3.5 and 3.3.9.4 are met.
YF-16 Test Flight Zero

• High-speed taxi test; no flight intended
• Pilot-induced oscillations from overly sensitive roll control
• Tail strike
• Pilot elected to go around rather than eject

Pilot-Induced Roll Oscillation

\[
\frac{\Delta \phi(s)}{\Delta \delta A(s)}_{\text{pilot in loop}} = \left( \frac{K_p}{T_p} \right) \left( \frac{s + 1}{T_p} \right)
\]

\[
\frac{k_\phi}{\left( s - \lambda_S \right) \left( s - \lambda_R \right) \left( s^2 + 2\zeta_{DR}\omega_{n_{DR}} s + \omega_{n_{DR}}^2 \right)}
\]

Aileron-to-Roll Angle Root Locus
Pilot-Aircraft Nichols Chart
Inverse Problem of Lateral Control

- Given a flight path, what is the control history that generates it?
  - Necessary piloting actions
  - Control-law design
- Aileron-rudder interconnect (ARI) simplifies pilot input

Next Time:
Maneuvering at High Angles and Rates

Flight Dynamics
681-785
Airplane Stability and Control
Chapter 8
Large Aircraft Flying Qualities

- High wing loading, W/S
- Distance from pilot to rotational center
- Slosh susceptibility of large tanks
- High wing span -> short relative tail length
  - Higher trim drag
  - Increased yaw due to roll, need for rudder coordination
  - Reduced rudder effect
- Altitude response during approach
  - Increased non-minimum-phase delay in response to elevator
  - Potential improvement from canard
- Longitudinal dynamics
  - Phugoid/short-period resonance
- Rolling response (e.g., time to bank)
- Reduced static stability
- Off-axis passenger comfort in BWB turns
Criteria for Oscillations and Excursions
(MIL-F-8785C)

3.1.2.2 Roll rate oscillations. Following a yaw-control-free step roll control command, the roll rate at the first minimum following the first peak shall be of the same sign and not less than the following percentage of the roll rate at the first peak:

<table>
<thead>
<tr>
<th>Level</th>
<th>Flight Phase Category</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AA C</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>AA C</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0</td>
</tr>
</tbody>
</table>

3.1.2.3 Sideslip excursions. Following a yaw-control-free step roll control command, the ratio of the sideslip increment, $\Delta \beta$, to the parameter $k$ (6.2.6) shall be less than the values specified herein. The roll command shall be held fixed until the bank angle has changed at least 90 degrees.

<table>
<thead>
<tr>
<th>Level</th>
<th>Flight Phase Category</th>
<th>Adverse Sideslip</th>
<th>Proverse Sideslip</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>6 degrees</td>
<td>2 degrees</td>
</tr>
<tr>
<td></td>
<td>B &amp; C</td>
<td>10 degrees</td>
<td>3 degrees</td>
</tr>
<tr>
<td>2</td>
<td>All</td>
<td>15 degrees</td>
<td>4 degrees</td>
</tr>
</tbody>
</table>
Flight Testing Videos

TSR2 Test Flight
http://www.youtube.com/watch?v=GXdJxjvQZW4

Neil Armstrong, Test Pilot
http://www.youtube.com/watch?v=t6DdlPoPOE4

NASA Dryden(now Armstrong) Flight Research Center
http://www.youtube.com/watch?v=j85jlc1Zfk4

Avro Arrow Revisited
https://www.youtube.com/watch?v=S74zf0YZX20