Lateral-Directional Flying Qualities Criteria
Robert Stengel, Aircraft Flight Dynamics
MAE 331, 2016

Learning Objectives

• Lateral Control Divergence Parameter (LCDP)
• $\omega_d/\omega_d$ and $\Phi/\beta$
• Eigenvectors
• Pilot-Vehicle Interactions
• Pilot-Induced Oscillations
• Optimal Control Pilot Model

Design for Satisfactory Flying Qualities

• Satisfy procurement requirement (e.g., Mil Standard)
• Satisfy test pilots (e.g., Cooper-Harper ratings)
• Avoid pilot-induced oscillations (PIO)
• Minimize time-delay effects
• Time- and frequency-domain criteria
Lateral-Directional Criteria

Early Lateral-Directional Flying Qualities Criteria

\( \omega_n/\omega_d \): Numerator frequency to Dutch roll Natural Frequency in \( \phi(s)/\delta A(s) \) transfer function (more later)

\( \zeta_d \): Dutch roll damping frequency
Early Lateral-Directional Flying Qualities Criteria

\[ T_{1/2} = \frac{0.693}{\zeta_d \phi_n} \]: Time to one-half amplitude, Dutch roll oscillation

\[ |\phi/v| = V_N |\phi/\beta| \]: Ratio of roll to sideslip angle in Dutch roll oscillation

Lateral-Directional Flying Qualities Parameters

• Lateral Control Divergence Parameter, \( LCDP \)
• \( \omega_\phi/\omega_d \) Effect
• \( \phi/\beta \) Effect
Lateral Control Divergence Parameter (LCDP)

• Aileron deflection produces yawing as well as rolling moment
  – “Favorable yaw” aids the turn command
  – “Adverse yaw” opposes it
• Equilibrium response to constant aileron input

\[
\Delta \phi = \frac{\left(N_\beta + N_r \frac{Y_\beta}{V_N}\right) L_{\delta_A} - \left(L_\beta + L_r \frac{Y_\beta}{V_N}\right) N_{\delta_A}}{\Delta \delta A} \cdot \frac{g}{V_N} \left(L_\beta N_r - L_r N_\beta \right)
\]

• Large-enough \( N_{\delta_A} \) effect can reverse the sign of the response
  – Can occur at high angle of attack
  – Can cause “departure from controlled flight”

Lateral Control Divergence Parameter (LCDP)

• Large-enough \( N_{\delta_A} \) effect can reverse the sign of the response
  – Can occur at high angle of attack
  – Can cause “departure from controlled flight”
• Lateral Control Divergence Parameter provides simplified criterion

\[
\left(\frac{N_\beta}{L_{\delta_A}}\right) L_{\delta_A} - \left(\frac{L_\beta}{L_{\delta_A}}\right) N_{\delta_A} = N_\beta - \frac{N_{\delta_A}}{L_{\delta_A}} L_\beta
\]

\[
\text{LCDP} \equiv C_{n_\beta} - \frac{C_{n_{\delta_A}}}{C_{l_{\delta_A}}} C_{l_\beta}
\]
Aileron-to-roll-angle transfer function

\[
\frac{\Delta \phi(s)}{\Delta \delta A(s)} = \frac{k_\phi \left(s^2 + 2\xi_\phi \omega_\phi s + \omega_\phi^2\right)}{(s - \lambda_S)(s - \lambda_R)\left(s^2 + 2\xi_{DR} \omega_{nDR} s + \omega_{nDR}^2\right)}
\]

- \(\omega_\phi\) is the “natural frequency” of the complex zeros
- \(\omega_d = \omega_{nDR}\) is the natural frequency of the Dutch roll mode
- Conditional instability may occur with closed-loop control of roll angle, even with a perfect pilot

\(\omega_\phi/\omega_d\) Effect is Important in Roll Angle Control

\[
\Delta \phi(s) = \frac{k_\phi \left(s^2 + 2\xi_\phi \omega_\phi s + \omega_\phi^2\right)}{(s - \lambda_S)(s - \lambda_R)\left(s^2 + 2\xi_{DR} \omega_{nDR} s + \omega_{nDR}^2\right)}
\]

- As feedback gain increases, Dutch roll roots go to numerator zeros
- Zeros over poles: conditional instability results
- Poles over zeros: no instability
• \( \varphi/\beta \) measures the degree of rolling response in the Dutch roll mode
  - Large \( \varphi/\beta \): Dutch roll is primarily a rolling motion
  - Small \( \varphi/\beta \): Dutch roll is primarily a yawing motion
• Eigenvectors, \( \mathbf{e}_p \), indicate the degree of participation of the state component in the \( p^{th} \) mode of motion

\[
\det(s \mathbf{I} - \mathbf{F}) = (s - \lambda_1)(s - \lambda_2)\ldots(s - \lambda_n)
\]
\[
(\lambda_i \mathbf{I} - \mathbf{F}) \mathbf{e}_i = 0
\]

**Eigenvectors**

Eigenvectors, \( \mathbf{e}_p \), are solutions to the equation

\[
(\lambda_i \mathbf{I} - \mathbf{F}) \mathbf{e}_i = 0, \quad i = 1, n
\]

or

\[
\lambda_i \mathbf{e}_i = \mathbf{F} \mathbf{e}_i, \quad i = 1, n
\]

For each eigenvalue, the corresponding eigenvector can be found (within an arbitrary constant) from

\[
\text{Adj}(\lambda_i \mathbf{I} - \mathbf{F}) = \begin{pmatrix} a_1 \mathbf{e}_i & a_2 \mathbf{e}_i & \ldots & a_n \mathbf{e}_i \end{pmatrix}, \quad i = 1, n
\]

**MATLAB**

\[
(\mathbf{V}, \mathbf{D}) = \text{eig}(\mathbf{F})
\]

\( \mathbf{V} \): Modal Matrix (i.e., Matrix of Normalized Eigenvectors)
\( \mathbf{D} \): Diagonal Matrix of Corresponding Eigenvalues
**φ/β Effect**

With $\lambda_i$ chosen as a complex root of the Dutch roll mode, the corresponding eigenvector is

$$
e_{DR^+} = 
\begin{bmatrix}
  e_r \\
  e_\beta \\
  e_p \\
  e_\phi \\
\end{bmatrix}_{DR^+} = 
\begin{bmatrix}
  (\sigma + j\omega)_r \\
  (\sigma + j\omega)_\beta \\
  (\sigma + j\omega)_p \\
  (\sigma + j\omega)_\phi \\
\end{bmatrix}_{DR^+} = 
\begin{bmatrix}
  (AR e^{j\phi})_r \\
  (AR e^{j\phi})_\beta \\
  (AR e^{j\phi})_p \\
  (AR e^{j\phi})_\phi \\
\end{bmatrix}_{DR^+}$$

$\phi/\beta$ is the magnitude of the ratio of the $\varphi$ and $\beta$ eigenvectors

$$\frac{\phi}{\beta} = \left| \begin{array}{c}
  (AR)_\varphi \\
  (AR)_\beta \\
\end{array} \right| = \left( \frac{V_N}{g} \right) \left[ \left( \zeta_{DR} \omega_{n_{DR}} + \frac{Y_\beta}{V_N} + \frac{L_\beta}{L_r} \right)^2 + \left( \omega_{n_{DR}} \sqrt{1 - \zeta_{DR}^2} \right) \right]^{\frac{1}{2}}$$

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**φ/β Effect for the Business Jet Example**

*Roll/Sideslip Angle ratio in the Dutch roll mode*

$$
e_{DR^+} = 
\begin{bmatrix}
  |e_r| \\
  |e_\beta| \\
  |e_p| \\
  |e_\phi| \\
\end{bmatrix}_{DR^+} = 
\begin{bmatrix}
  0.525 \\
  0.416 \\
  0.603 \\
  0.433 \\
\end{bmatrix}_{DR^+}$$

$$\frac{\phi}{\beta} = 1.04$$
Criteria for Lateral-Directional Modes
(MIL-F-8785C)

Maximum Roll-Mode Time Constant

<table>
<thead>
<tr>
<th>Flight Phase Category</th>
<th>Class</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>I, IV, II, III</td>
<td>1.0</td>
<td>1.4</td>
<td>3.0</td>
</tr>
<tr>
<td>B</td>
<td>All</td>
<td>1.4</td>
<td>3.0</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>I, II-C, IV, II-L, III</td>
<td>1.0</td>
<td>1.4</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Criteria for Lateral-Directional Modes
(MIL-F-8785C)

Minimum Spiral-Mode Time to Double

<table>
<thead>
<tr>
<th>Flight Phase Category</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A &amp; C</td>
<td>12 sec</td>
<td>8 sec</td>
<td>4 sec</td>
</tr>
<tr>
<td>B</td>
<td>20 sec</td>
<td>8 sec</td>
<td>4 sec</td>
</tr>
</tbody>
</table>
## Minimum Dutch Roll Natural Frequency and Damping (MIL-F-8785C)

<table>
<thead>
<tr>
<th>Flight Phase Level</th>
<th>Category</th>
<th>Class</th>
<th>$\zeta_d$</th>
<th>$\omega_{nd}$ rad/sec.</th>
<th>$\omega_{nd}$ rad/sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A (CO and GA)</td>
<td>IV</td>
<td>0.4</td>
<td>-</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>I, IV</td>
<td>0.19</td>
<td>0.35</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>II, III</td>
<td>0.19</td>
<td>0.35</td>
<td>0.4**</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>All</td>
<td>0.08</td>
<td>0.15</td>
<td>0.4**</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>I, II-C, IV</td>
<td>0.08</td>
<td>0.15</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>II-L, III</td>
<td>0.08</td>
<td>0.10</td>
<td>0.4**</td>
</tr>
<tr>
<td>2</td>
<td>All</td>
<td>All</td>
<td>0.02</td>
<td>0.05</td>
<td>0.4**</td>
</tr>
<tr>
<td>3</td>
<td>All</td>
<td>All</td>
<td>0</td>
<td>-</td>
<td>0.4**</td>
</tr>
</tbody>
</table>

* The governing damping requirement is that yielding the larger value of $\zeta_d$, except that a $\zeta_d$ of 0.7 is the maximum required for Class III.

** Class III airplanes may be excepted from the minimum $\omega_{nd}$ requirement, subject to approval by the procuring activity, if the requirements of 3.3.87 through 3.3.2.4.1, 3.3.5 and 3.3.9.4 are met.

### Pilot-Vehicle Interactions
YF-16 Test Flight Zero

- High-speed taxi test; no flight intended
- **Pilot-induced oscillations** from overly sensitive roll control
- Pilot elected to go around rather than eject

\[
\frac{\Delta \phi(s)}{\Delta \delta A(s)}_{\text{pilot in loop}} = \left( \frac{K_p}{T_p} \right) \left( \frac{1}{s + 1/T_p} \right) \left[ \frac{k_\phi \left( s^2 + 2\zeta_\phi \omega_\phi s + \omega_\phi^2 \right)}{(s - \lambda_S)(s - \lambda_R) \left( s^2 + 2\zeta_{DR}\omega_{n_{DR}} s + \omega_{n_{DR}}^2 \right)} \right]
\]

Pilot-Induced Roll Oscillation

Pilot Transfer Function

Aircraft Transfer Function

Aileron-to-Roll Angle
Root Locus

Pilot-Aircraft Nichols Chart

YF-16
**Inverse Problem of Lateral Control**

- Given a flight path, what is the control history that generates it?
  - Adapted Optimal Control Pilot Model generates necessary piloting actions
- Aileron-rudder interconnect (ARI) simplifies pilot input

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**Fig. 1 Block diagram of the pilot-aircraft model.**

- **Pilot Input**
- **Aircraft Model**
- **Disturbances**
- **Display**
- **Delay**
- **State Estimation**
- **Control**
- **Neuromuscular Lag**
- **Observation Error**

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**Grumman F-14 Tomcat**

<table>
<thead>
<tr>
<th>Yaw Angle</th>
<th>Desired Roll Angle</th>
<th>Lateral-Stick Command</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of attack ($\alpha$) = 10 deg; ARI off</td>
<td>(\alpha = 30) deg; ARI off</td>
<td>(\alpha = 30) deg; ARI on</td>
</tr>
</tbody>
</table>

Figure 54: Adapted High-\(\alpha\) and Low-\(\alpha\) Pitching Procedures Using Lateral-Stick Control.

Kleinman, Baron, Levison, 1970

Stengel, Broussard, 1978
Next Time:
Maneuvering at High Angle and Angular Rate

Learning Objectives

High angle of attack and angular rates
Asymmetric flight
Nonlinear aerodynamics
Inertial coupling
Spins and tumbling

Supplemental Material
3.3.2.2 Roll rate oscillations. Following a yaw-control-free step roll control command, the roll rate at the first minimum following the first peak shall be of the same sign and not less than the following percentage of the roll rate at the first peak:

<table>
<thead>
<tr>
<th>Level</th>
<th>Flight Phase Category</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A &amp; C</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>A &amp; C</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0</td>
</tr>
</tbody>
</table>

3.3.2.3 Sidle slip excursions. Following a yaw-control-free step roll control command, the ratio of the sideslip increment, $\Delta \beta$ to the parameter $k$ (6.2.6) shall be less than the values specified herein. The roll command shall be held fixed until the bank angle has changed at least 90 degrees.

<table>
<thead>
<tr>
<th>Level</th>
<th>Flight Phase Category</th>
<th>Adverse Sideslip (Right roll command causes right sideslip)</th>
<th>Proverse Sideslip (Right roll command causes left sideslip)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>6 degrees</td>
<td>2 degrees</td>
</tr>
<tr>
<td></td>
<td>B &amp; C</td>
<td>10 degrees</td>
<td>3 degrees</td>
</tr>
<tr>
<td>2</td>
<td>All</td>
<td>15 degrees</td>
<td>$\frac{1}{3}$ degrees</td>
</tr>
</tbody>
</table>