Maneuvering at High Angles and Angular Rates
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Learning Objectives
• High angle of attack and angular rates
• Asymmetric flight
• Nonlinear aerodynamics
• Inertial coupling
• Spins and tumbling

Tactical Airplane Maneuverability

• Maneuverability parameters
  – Stability
  – Roll rate and acceleration
  – Normal load factor
  – Thrust/weight ratio
  – Pitch rate
  – Transient response
  – Control forces
• Dogfights
  – Preferable to launch missiles at long range
  – Dogfight is a backup tactic
  – Preferable to have an unfair advantage
• Air-combat sequence
  – Detection
  – Closing
  – Attack
  – Maneuvers, e.g.,
    • Scissors
    • High yo-yo
  – Disengagement
Coupling of Longitudinal and Lateral–Directional Motions

Longitudinal Motions can Couple to Lateral–Directional Motions

- Linearized equations have limited application to high-angle/high-rate maneuvers
  - Steady, non-zero sideslip angle (Sec. 7.1, \(FD\))
  - Steady turn (Sec. 7.1, \(FD\))
  - Steady roll rate

\[
\mathbf{F} = \begin{bmatrix}
F_{Lon} & F_{Lon}^{Lat-Dir} \\
F_{Lat-Dir} & F_{Lat-Dir}^{Lon}
\end{bmatrix}
\]

\[F_{Lon}^{Lat-Dir}, F_{Lon}^{Lat-Dir} \neq 0\]
Stability Boundaries Arising From Asymmetric Flight

Stability Boundaries with Nominal Sideslip, $\beta_o$, and Roll Rate, $p_o$
Pitch-Yaw Coupling Due To Steady Roll Rate, $p_o$

- Combine 2nd-order short period and Dutch roll modes
  - Body axes
  - Constant roll rate = $p_o$, rad/s

**State vector**

\[
\Delta x(t) = \begin{bmatrix} \Delta x_{Lon} \\ \Delta x_{LD} \end{bmatrix} = \begin{bmatrix} \Delta w \\ \Delta q \\ \Delta v \\ \Delta r \end{bmatrix}
\]

- Normal velocity, m/s
- Pitch rate, rad/s
- Side velocity, m/s
- Yaw rate, rad/s

**Control input vector**

\[
\Delta u(t) = \begin{bmatrix} \Delta \delta E \\ \Delta \delta A \\ \Delta \delta R \end{bmatrix}
\]

- Elevator, deg or rad
- Ailerons, deg or rad
- Rudder, deg or rad

4th-order dynamic model

\[
\begin{bmatrix} \Delta x_{Lon} \\ \Delta x_{LD} \end{bmatrix} = \begin{bmatrix} F_{Lon} & F_{Lon}^{LD} \\ F_{LD} & F_{LD} \end{bmatrix} \begin{bmatrix} \Delta x_{Lon} \\ \Delta x_{LD} \end{bmatrix} + \begin{bmatrix} G_{Lon} \\ G_{LD} \end{bmatrix} \Delta u
\]

Time Response to Elevator Step Input

- **When $p_o = 0^\circ$/s**
  - Elevator input produces longitudinal response but no lateral-directional response

- **At $p_o = 60^\circ$/s**
  - Short-period (faster) mode dominates longitudinal response
  - Dutch-roll (slower) mode dominates lateral directional response

- **At $p_o = 120^\circ$/s**
  - Both modes are evident in both responses
  - Fast mode is even faster
  - Slow mode is even slower
Pitch-Yaw Coupling Due To Steady Roll Rate, $p_o$

- 4th-order stability matrix
  - Body axes
  - Negligible $v_p$, $u_o \sim V_N$
  - Negligible coupling aerodynamic effects
- Constant roll rate is only source of coupling

\[
\begin{bmatrix}
    F_{Lon}^{Lm} & F_{Lon}^{LD}
\end{bmatrix} = \begin{bmatrix}
    Z_w & u_o \\
    M_w & M_q
\end{bmatrix}
\]

\[
\begin{bmatrix}
    0 & \frac{(I_{xx} - I_{yy})}{I_{zz}} p_o \\
    \frac{(I_{zz} - I_{xx})}{I_{yy}} p_o & 0
\end{bmatrix}
\]

\[
\Delta s(t) = \begin{bmatrix}
    \Delta W \\
    \Delta q \\
    \Delta v \\
    \Delta r
\end{bmatrix}
\]

**Pitch-Yaw Coupling Due To Steady Roll Rate, $p_o$**

**Characteristic Polynomial**

\[
\Delta_{\text{rolling}}(s) = \left[ \left( s - Z_w \right) \left( s - Y_v \right) \left( s - N_r \right) \right] \left[ \left( s - M_w \right) \left( s - M_q \right) \left( s - u_w \right) \right]
\]

\[
+ p_o^2 \left[ \left( s - Y_v \right) \left( s - N_r \right) \left( s - Z_w \right) \right] \left( \frac{I_{xx} - I_{yy}}{I_{zz}} \right) \left( \frac{I_{zz} - I_{yy}}{I_{xx}} \right) - u_w M_w \left( \frac{I_{xx} - I_{yy}}{I_{zz}} \right) - u_v N_v \left( \frac{I_{zz} - I_{yy}}{I_{xx}} \right)
\]

\[
- p_o^4 \left( \frac{I_{zz} - I_{yy}}{I_{xx}} \right) \left( \frac{I_{zz} - I_{yy}}{I_{zz}} \right)
\]

- Coupling effect is proportional to $p_o^2$ and $p_o^4$
- Effect on roots is independent of the sign of $p_o$
- Cannot use Evans’ s root-locus rules with $k = p_o^2$, as $k^2$ also appears
- Can compute effect of $p_o^2$ on roots using MATLAB’s `eig`

\[
\Delta_{\text{rolling}}(s) = \left[ \Delta_{SP}(s) \Delta_{AE}(s) \right] + p_o^2 \left[ \text{fcn}\left( s, M_w, M_q, Z_w, Y_v, I_{xx}, I_{yy}, I_{zz}, u_w, u_v, N_v, N_r \right) \right] - p_o^4 \left( \frac{I_{zz} - I_{yy}}{I_{xx}} \right) \left( \frac{I_{zz} - I_{yy}}{I_{zz}} \right)
\]

Thunderbird F-16 Barrel Roll
http://www.youtube.com/watch?v=ovSOT1ncbU
Effect of Steady Roll Rate, $p_o$, on Pitching and Yawing Roots

- Factor $\Delta_{\text{rolling}}(s)$ for various values of $p_o^2$
- $p_o^2 = \text{root locus gain, } k$
- Faster mode gets faster
- Slower mode gets slower and may become unstable

Steady Roll Rate, $p_o$, Effect Expressed by Root Locus or Parameter Plot

Parameter plot: variation of real and imaginary parts of roots vs. roll rate, $p_o$
Steady-State Response as Well as Stability is Affected by High Roll Rate

\[ f(v, w, p, q, r, \Delta \delta A, \Delta \delta E, \Delta \delta R)_{SS} = 0 \]

- Effects of steady roll rate on nonlinear equilibrium control response
  - Pitch-yaw coupling
  - “p jump” or “p acceleration”
- Multiple equilibria for same control settings
  - Up to 9 possible roll rates for one aileron setting
  - Sensitivity to elevator setting
    - Flight Dynamics, 7.3

The Butterfly Catastrophe*

\[ f_1(v, w, p, q, r, \Delta \delta A, \Delta \delta E, \Delta \delta R)_{SS} = 0 \]
\[ p_{SS} = f_2(v, w, q, r, \Delta \delta A, \Delta \delta E, \Delta \delta R)_{SS} \]

- Surface of equilibrium solutions for roll rate
- Possibility of an unrecoverable spin

* René Thom, 1974

\[ \text{after Mehra, Carroll, 1979} \]
Tumbling and Spins

Tumbling, Spins, and Recovery

- Strong nonlinear effects
- Aircraft-specific control strategy for recovery
Wind Tunnel Spin Testing

- Sidney B. Gates, RAE: "The Spinning of Aeroplanes" (with L.W. Bryant, 1926), neutral and maneuver points, stick force per $g$
- Continued research on stalls and spins at NASA, USAF, and in many other countries

[Image of wind tunnel and diagrams]

NASA Langley Spin Tunnel Testing


http://www.youtube.com/watch?v=u7FCqLPgLgk
http://www.youtube.com/watch?v=tQwMCml55QQ0
http://www.youtube.com/watch?v=VUKTBv1Rll
Tails with Negative Dihedral (Anhedral)

- Horizontal tail below wing's wake
- May have adverse effect on spin characteristics
- F-4 model test

### Yawing Moment at High Angle of Attack

- Dynamic as well as static effects, e.g., *hysteresis*
- Random asymmetric yawing moments *(left or right)*
  - generated by slender nose at zero sideslip angle
  - may exceed rudder control power
Controlling Yawing Moment at High Angle of Attack

- *Sucking, blowing, or movable strakes* to control nose vortices
- *X-29, F/A-18 HARV*
- *Vortex bursting effect on tail*

Control Effectiveness at High Angle of Attack and Deflection Angle

- Assumption of Newtonian flow

**Elevator Effect**  
**Aileron Effect**
Control at High Aerodynamic Angles

Supermaneuverability

- Means of forcing opponent to *overshoot*
- *Pugachev’s Cobra maneuver*, first done in *Sukhoi Su-27*
- Beneficial effect of *thrust-vector control (X-31)*
- *Mongoose maneuver (X-31)*
- Essentially low-speed maneuvers, not where you want to be in air combat (i.e., high energy-state)
Thrust Vector Control

Pitch and Yaw Control (X-31)

Pitch Control (F-22)

Next Time:
Aeroelasticity and Fuel Slosh

Flight Dynamics
418–419, 549–569, 665–678
Airplane Stability and Control
Chapter 19

Learning Objectives
- Aerodynamic effects of bending and torsion
- Modifications to aerodynamic coefficients
- Dynamic coupling
- Fuel shift and sloshing dynamics
Supplemental Material

Stall-Spin Studies of General Aviation Aircraft

http://www.youtube.com/watch?v=TmWB6oyJ9IE