Preliminaries

- **Office Hours**
  - Tues – Thurs, 1:30-3pm

- **Assistants in Instruction:**
  - Will Coogan
  - Yibin Zhang
    - Office hours: TBD
    - Precepts, tutorials: TBD

- **MATLAB/SimuLink, STK-AGI, CREO**

- **Course Home Page, Syllabus, and Links**
  - [www.princeton.edu/~stengel/MAE342.html](http://www.princeton.edu/~stengel/MAE342.html)

- **Wednesday afternoon “Lab Sessions” following regular class: TBD**

- **GRADING**
  - Class participation: 10%
  - Assignments: 45%
  - Term Paper: 45%
Text and References

- Principal textbook:

- Supplemental references
  - Various technical reports and papers (e.g., NASA and AIAA pubs)
  - Books on reserve at Engineering Library (paper and on-line)
  - Web pages
    - [http://blackboard.princeton.edu/](http://blackboard.princeton.edu/)
    - [http://www.princeton.edu/~stengel/MAE342.html](http://www.princeton.edu/~stengel/MAE342.html)

First Half of the Term

- Overview and Preliminaries
- Orbital Mechanics
- Planetary Defense
- Spacecraft Guidance
- Spacecraft Environment
- Chemical/Nuclear Propulsion Systems
- Electric Propulsion Systems
- Launch Vehicles
- Spacecraft Structures
- Spacecraft Configurations
- Spacecraft Dynamics
- Spacecraft Control
Second Half of the Term

- System Engineering & Integration
- Sensors & Actuators
- Electrical Power Systems
- Thermal Control
- Telecommunications
- Telemetry, Command, Data Handling & Processing
- Spacecraft Mechanisms
- Electromagnetic Compatibility
- Space Robotics
- Human Factors of Spaceflight
- Product Assurance
- Ground Segment

Electronic Devices in Class

- Silence all cellphones and computer alarms
- Don’t check e-mail or send text, tweets, etc.
- If you must make a call or send a message, you may leave the room to do so
- Tablets/laptops for class-related material ONLY
**Collaborative Learning**

- Significant student participation in most classes, Q&A
- Slides will be available before each class
- Discussion of slides by students
- Randomly assigned teams for assignments during first half
- Project-oriented teams during second half
- Single grade for each team

**Written Assignment Reporting Format**

- Assignments evolve toward Technical Reports
  - Abstract
  - Introduction
  - Methods and Results
  - Conclusion
  - References
- Write-ups should present explanations, not just numbers, graphs, or computer code
- Orderliness and neatness count
- Don’t forget your name, date, and assignment title and number
Overview

Spacecraft Mission Objectives and Requirements

Fortescue, et al, 2011
Functional Requirements of Spacecraft Subsystems

1. Desired dynamic state (attitude, position, velocity) must be maintained
2. Desired orbits for the mission must be maintained
3. Payload must be operable
4. Payload must be held together and mounted on the spacecraft structure
5. Payload must operate reliably over some specified period
6. Adequate power must be provided
7. Operation should be largely autonomous, but …
   a) Data must be communicated to the ground
   b) Ground control must be maintained
Early History of Space Systems

• 1926-1945: Goddard rockets; V-2 and its precursors
  – Development of rocket technology
  – Development of guidance and control systems

• 1945-1949: Learning from the V-2; Altitude sounding
  – V-2/WAC-Corporal to 250-mi altitude (Project Bumper)
  – Development of military missiles
German A-4 (V-2) Rocket

- Liquid-fuel rocket
- 6,084 built; 1000+ test flights; 3,225 launched in combat
- Gyroscopes and accelerometer for guidance
- Air and jet vanes for pitch, yaw, and roll control torques
- Aft tail for aerodynamic stability

Project Bumper (V-2/WAC Corporal)

- 8 flights, 4 failures; Mach 9, 400-km apogee
- Engineering development
- High-altitude photography
- Atmospheric temperature profile
- Cosmic radiation
Post-WWII History of Space Systems

• 1945-1957: Payload design; animals in space
  – Sounding rockets
  – Aerobee, Viking → Vanguard
  – IRBMs and ICBMs received major emphasis in US and USSR

• 1957-1961: Unmanned satellites; animals in orbit; manned spaceflight about the Earth
  – 1957: Sputnik 1
  – 1958: Van Allen belts (Explorer 1); NACA -> NASA
  – 1959: Luna 1-3
  – 1961: Gagarin orbit; Ham and Shepard sub-orbit; Enos orbit

Sounding Rockets

• Several minutes of high-altitude flight
  – "Weightlessness"
  – "Above the atmosphere"
  – Near-vacuum
  – High-altitude measurements

• Simplicity, low cost
• Recoverable payloads
• Wide range of flight conditions
Sputnik 1
(October 4, 1957)

- 84 kg, 58-cm diameter
- 96-min, elliptical orbit
- 1,440 orbits
- Measurements
  - Gravity
  - Ionospheric effects
  - Internal temperature and pressure
  - Micrometeoroid detection

R-7 (Semyorka) Launch Vehicle
(October 4, 1957)

- R-7 (Semyorka) launch vehicle
  - 1-1/2-stage ICBM
  - 4 strap-on booster rockets
  - 1 core-stage rocket
  - Liquid oxygen and kerosene
  - Lift-off thrust: 3.9 MN
  - Gross weight: 267 metric tons
Project Vanguard (1957-1959)

• 3 stages
  – 1st-stage based on Viking; gimballed motor for control
  – 2nd-stage based on Aerobee; reaction-control thrusters
  – Solid-fuel 3rd stage; spin stabilized

• 11 launches, 3 successful satellites
• Vanguard 1 solar-powered, still in orbit (launched March 17, 1958)

• Measurements
  – Gravity, Ionospheric effects, Internal temperature
  – Optical scanner, Magnetic field, Solar X-rays
  – Micrometeorites
Explorer 1
(January 31, 1958)

Juno 1 Launched Explorer 1
(January 31, 1958)

• Juno lineage from V-2
  – Jupiter
  – Redstone
Earth Satellite “Firsts”

- **Communications satellites**
  - 1960: Echo 1
  - 1961: First amateur radio satellite (OSCAR 1)
  - 1962: Telstar 1
  - 1963: Geosynchronous satellite (Syncom 1)

- **Earth Satellite “Firsts”**
  - 1960: Weather satellite: TIROS-1
  - 1972: Earth observation satellite: Landsat 1 (ERTS 1)
  - 1962: Navigation satellite: Transit
Lunar Probe “Firsts”

• 1959:
  – Lunar flyby (Luna 1, Pioneer 4)
  – Lunar impact (Luna 2)
  – Pictures of “The Far Side” (Luna 3)

• 1966:
  – Lunar soft landing (Luna 9, Surveyor 1)
  – Lunar orbit (Lunar Orbiter 1)

Typical Lunar Probe Instrumentation, 1959

Radios for telemetry and command
Cosmic ray counter
Magnetometers
Temperature
Pressure
Micrometeorite sensors
Sodium cloud release

*Sodium cloud release at high altitude, Wallops Island, 2009*
https://www.youtube.com/watch?v=Lb45uBaj2Mc
Lunar Probe “Firsts”

- **1967:**
  - High-resolution photos (Ranger 7)
- **1969:** *Apollo 11*
- **1970:**
  - Robotic sample return (Luna 16)
  - Robotic lunar rover (Luna 17)

---

Inner-Planet Probe “Firsts”

- **1962:** Venus flyby (Mariner 2)
- **1964:** Mars flyby (Mariner 4)
- **1970:** Venus lander (Venera 8)
- **1971:** Mars orbit (Mars 2 Orbiter)
Inner-Planet Probe “Firsts”

• 1973: Mercury flyby (Mariner 10)
• 1975: Mars landing (Viking 1)
• 1978: Venus orbit (Pioneer Venus 1)

Outer-Planet Probe “Firsts”

• 1972: Jupiter flyby (Pioneer 10)
• 1973: Saturn flyby (Pioneer 11)
• 1986: Uranus flyby (Voyager 2)
• 1989: Neptune flyby (Voyager 2)
Lagrange-Point “Firsts”

• 1978: Solar observatory at L1 (ISEE-3); later rendezvoused with a comet as ICE (1983)
• 2001: Astronomical observatory at L2 (Wilkinson Microwave Anisotropy Probe)

Comet and Asteroid Rendezvous “Firsts”

• 1999: Comet sample return (Stardust)
• 2005: Asteroid landing (Muses-C/Hayabusa)
• 2014: Comet 67P Flyby/Landing (Rosetta/Philae)
Manned Spacecraft Launch Vehicles

- **1961:** Gagarin orbit (Vostok); Shepard sub-orbit (Mercury/Redstone)
- **1962:** Glenn orbit (Mercury/Atlas)
- **1964:** USSR 3-person crew in orbit (Voshkod)
- **1965:** US 2-person crew in orbit (Gemini/Titan II)

Manned Spacecraft

- **1961:** Gagarin orbit (Vostok); Shepard sub-orbit (Mercury/Redstone)
- **1962:** Glenn orbit (Mercury/Atlas)
- **1963:** X-15 rocket plane reaches 100-km altitude
- **1964:** USSR 3-person crew in orbit (Voshkod)
- **1965:** US 2-person crew in orbit (Gemini/Titan II)
Manned Flight to the Moon

- **1961-1972: Apollo Program**
  - 6 lunar landings and returns

- **1961-1974: Soviet Lunar Program**
  - 4 launches (unmanned), none successful
Space Stations

- 1971-1982: Salyut
- 1973-1974: Skylab
- 1975: Soyuz-Apollo docking
- 1986-2001: Mir
- 1998-present: International Space Station

Space Shuttle

1981-2011:
- 5-7 astronauts
- 50,000-lb payload
- 135 missions flown
- 5 operational vehicles; 2 destroyed
  - 1986: Challenger accident
  - 2003: Columbia accident
Project Constellation

- **Orion**: Crew Spacecraft
- **Ares 1**: Crew Launch Vehicle
  - First (only) unmanned launch: 2009
  - 5-segment Shuttle Solid-Rocket Booster
- **Ares 5**: Cargo Launch Vehicle
- Manned return to the Moon
- Project cancelled in 2011
- Development of Orion continues
- Ares 5 morphed into the Space Launch System (SLS)

Saturn V - Space Shuttle – Ares - SLS Size Comparison
Planetary Defense Term Project

- Design of spacecraft to protect against asteroid/comet impact that would extinguish life on Earth
- Detection, characterization, intercept, and deflection of a “Doomsday Rock”
- **Design Teams** to address distant and near-Earth intercepts
- Single final report written “with one voice” by the class as a *Working Group*

Assignment #1
Report on the Book, *Project Icarus*

- Teams will discuss segments of the 1979 book during Lab Session, Feb 10<sup>th</sup>, including the following:
  - *Overview*
  - *Main points*
  - *Conclusions to be drawn*

- *Team members TBD*
Next Time: Orbital Mechanics

Supplemental Material
**Math Review**

- **Scalars and Vectors**
- **Sums and Multiplication**
- **Inner Product**
- **Derivatives and Integrals**

### Scalars and Vectors

- **Scalar**: usually lower case: \( a, b, c, \ldots, x, y, z \)
- **Vector**: usually bold or with underbar: \( \mathbf{x} \) or \( \mathbf{x} \)
  - Ordered set
  - Column of scalars
  - Dimension = \( n \times 1 \)

**Transpose**: interchange rows and columns

\[
\mathbf{X}^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}
\]

\[
\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} ; \quad \mathbf{y} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}
\]

3-D Vector

**3-D Vector**
Multiplication of Vector by Scalar

Multiplication of vector by scalar is associative, commutative, and distributive

\[ ax = xa = \begin{bmatrix} ax_1 \\ ax_2 \\ ax_3 \end{bmatrix} \]

\[ a(x + y) = (x + y)a = (ax + ay) \]

\[ \text{dim}(x) = \text{dim}(y) \]

\[ a^T = \begin{bmatrix} ax_1 & ax_2 & ax_3 \end{bmatrix} \]

- Could we add \((x + a)\)?
- Only if \(\text{dim}(x) = (1 \times 1)\)

MATLAB allows it as an “overloaded function” [https://en.wikipedia.org/wiki/Function_overloading](https://en.wikipedia.org/wiki/Function_overloading)

Addition

Conformable vectors and matrices are added term by term

\[ \mathbf{x} = \begin{bmatrix} a \\ b \end{bmatrix} ; \quad \mathbf{z} = \begin{bmatrix} c \\ d \end{bmatrix} \]

\[ \mathbf{x} + \mathbf{z} = \begin{bmatrix} a + c \\ b + d \end{bmatrix} \]
**Inner (Dot) Product**

Inner (dot) product of vectors produces a scalar result

\[ \mathbf{x}^T \mathbf{x} = \mathbf{x} \cdot \mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = (1 \times m)(m \times 1) = (1 \times 1) \]

\[ = (x_1^2 + x_2^2 + x_3^2) \]

---

**Derivatives and Integrals of Vectors**

Derivatives and integrals of vectors are vectors of derivatives and integrals

\[ \frac{dx}{dt} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} \quad \int x \, dt = \begin{bmatrix} \int x_1 \, dt \\ \int x_2 \, dt \\ \int x_3 \, dt \end{bmatrix} \]

\[ \mathbf{x}(t) = \begin{bmatrix} 7 \\ 8t \\ 9t^2 \end{bmatrix} ; \quad \frac{d\mathbf{x}(t)}{dt} = \begin{bmatrix} 0 \\ 8 \\ 18t \end{bmatrix} \]

\[ \mathbf{x}(t) = \begin{bmatrix} \frac{7}{8t} \\ \frac{8t}{9t^2} \end{bmatrix} ; \quad \int \mathbf{x}(t) \, dt = \begin{bmatrix} \frac{7t + x_1(0)}{8t^2} + x_2(0) \\ \frac{8t^2 + x_1(0)}{9t^3} + x_3(0) \end{bmatrix} \]
MATLAB Code for Math Review

% MAE 345 Lecture 1 Math Review
% Rob Stengel

clear
disp(' ')
disp('======================================')
disp('>>>MAE 345 Lecture 1 Math Review<<<')
disp('======================================')
disp(['Date and Time are ', num2str(datestr(now))]);
disp(' ')

% Scalars and Vectors
a = 4               % Scalar
x = [1; 2; 3]       % Column Vector
y = [4; 5; 6; 7]    % Column Vector

% Vector Transpose
xT = x'
yT = y'

% Multiplication by Scalar
w = a * x
v = x * a
wT = a * xT

% Vector Addition
zz = [8; 9; 10]
u = x + zz

% Inner (Dot) Product
zzz = x' * x

% Symbolic Toolbox
disp(' ')
disp('Symbolic Toolbox')
syms x y z z1 z2 z3 z4

y = x * x            % Define Function
z = diff(y)          % Differentiate Function
z1 = int(y)          % Integrate Function
z2 = [x; y; z]       % Column Vector
z3 = diff(z2)        % Derivative of Column Vector
z4 = int(z2)         % Integral of Column Vector
MATLAB Command Window
Output for Math Review

======================================
>>>MAE 345 Lecture 1 Math Review<<<
======================================
Date and Time are 24-May-2013 12:31:13

a =  4
x =
  1
  2
  3
y =
  4
  5
  6
  7

xT = 1     2     3
yT = 4     5     6     7

w =
  4
  8
12

v =
  4
  8
12

wT = 4     8    12

zz =
  8
  9
10

u =
  9
 11
13

zzz = 14

Symbolic Toolbox

y = x^2
z = 2*x
z1 = x^3/3
z2 =
x
x^2
2*x
z3 =
  1
  2*x
  2
z4 =
x^2/2
x^3/3
x^2

Math Review

• Matrix and Transpose
• Sums and Multiplication
• Matrix Products
• Identity Matrix
• Matrix Inverse
• Transformations
Matrix and Transpose

- **Matrix:**
  - Usually bold capital or capital: \( \mathbf{F} \) or \( \mathbf{F} \)
  - Dimension = \((m \times n)\)

- **Transpose:**
  - Interchange rows and columns

\[
\mathbf{A} = \begin{bmatrix}
  a & b & c \\
  d & e & f \\
  g & h & k \\
  l & m & n \\
\end{bmatrix} \quad \text{4x3}
\]

\[
\mathbf{A}^T = \begin{bmatrix}
  a & d & g & l \\
  b & e & h & m \\
  c & f & k & n \\
\end{bmatrix} \quad \text{3x4}
\]

Matrix Products

**Matrix-vector product** transforms one vector into another

\[
\mathbf{y} = \mathbf{A} \mathbf{x} = \begin{bmatrix}
  a & b & c \\
  d & e & f \\
  g & h & k \\
  l & m & n \\
\end{bmatrix} \begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
\end{bmatrix} = \begin{bmatrix}
  ax_1 + bx_2 + cx_3 \\
  dx_1 + ex_2 + fx_3 \\
  gx_1 + hx_2 + kx_3 \\
  lx_1 + mx_2 + nx_3 \\
\end{bmatrix}
\]

\((n \times 1) = (n \times m)(m \times 1)\)

**Matrix-matrix product** produces a new matrix

\[
\mathbf{A} = \begin{bmatrix}
  a_1 & a_2 \\
  a_3 & a_4 \\
\end{bmatrix} ; \quad \mathbf{B} = \begin{bmatrix}
  b_1 & b_2 \\
  b_3 & b_4 \\
\end{bmatrix} ; \quad \mathbf{AB} = \begin{bmatrix}
  (a_1b_1 + a_2b_3) & (a_1b_2 + a_2b_4) \\
  (a_3b_1 + a_4b_3) & (a_3b_2 + a_4b_4) \\
\end{bmatrix}
\]

\((n \times m) = (n \times l)(l \times m)\)
Numerical Example 1

\[ y = Ax = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -5 & 7 \\ 4 & 1 & 8 \\ -9 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \]

\[(n \times 1) = (n \times m)(m \times 1)\]

\[= \begin{bmatrix} (2x_1 + 4x_2 + 6x_3) \\ (3x_1 - 5x_2 + 7x_3) \\ (4x_1 + x_2 + 8x_3) \\ (-9x_1 - 6x_2 - 3x_3) \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \]

Numerical Example 2

\[A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} ; \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \]

\[AB = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \]

\[x_A = A\mathbf{x}_B ; \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_B \]

\[x_B = B\mathbf{x}_o ; \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_o \]

\[x_A = A\mathbf{x}_B = AB\mathbf{x}_o ; \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_A = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_o \]
Square Matrix Identity and Inverse

- **Identity matrix**: no change when it multiplies a conformable vector or matrix

\[
I_3 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

- A non-singular square matrix multiplied by its inverse forms an identity matrix

\[
AA^{-1} = A^{-1}A = I
\]

Matrix Inverse Example

- **Transformation** \(x_2 = Ax_1\)

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}_2 =
\begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}_1
\]

- **Inverse Transformation** \(x_1 = A^{-1}x_2\)

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}_1 =
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}_2
\]
Consequently, ...

\[ \mathbf{A} \mathbf{A}^{-1} = \mathbf{A}^{-1} \mathbf{A} = \mathbf{I} \]

\[
\begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}^{-1}
= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[ \mathbf{x}_2 = \mathbf{A} \mathbf{x}_1 = \mathbf{A} \mathbf{A}^{-1} \mathbf{x}_2 = \mathbf{x}_2 \]

**Computation of \((n \times n)\)**

**Matrix Inverse**

\[ \mathbf{y} = \mathbf{A} \mathbf{x}; \quad \mathbf{x} = \mathbf{A}^{-1} \mathbf{y} \]

\[ \text{dim}(\mathbf{x}) = \text{dim}(\mathbf{y}) = (n \times 1); \quad \text{dim}(\mathbf{A}) = (n \times n) \]

\[
[\mathbf{A}]^{-1} = \frac{\text{Adj}(\mathbf{A})}{|\mathbf{A}|} = \frac{\text{Adj}(\mathbf{A})}{\det \mathbf{A}} (n \times n) (1 \times 1)
\]

\[
= \frac{\mathbf{C}^T}{\det \mathbf{A}}; \quad \mathbf{C} = \text{matrix of cofactors}
\]

**Cofactors** are signed minors of \( \mathbf{A} \)

**ijth minor of \( \mathbf{A} \) is the determinant of \( \mathbf{A} \) with the ith row and jth column removed**
MATLAB Code for Math Review

Use of Symbolic Variables

% MAE 345 Lecture 2 Math Review
% Rob Stengel

clear
disp(' ')
disp('======================================')
disp('>>>MAE 345 Lecture 2 Math Review<<<')
disp('======================================')
disp(['Date and Time are ', num2str(datestr(now))]);
disp(' ')

% Matrix
syms A AT a b c d e f g h k l m n
A   =   [a b c;d e f;g h k;l m n]   % Matrix
AT  =   A'                          % Matrix Transpose

% Matrix-Vector Product
syms x x1 x2 x3 y1 y2 y3 y4
x   =   [x1;x2;x3]
y   =   [y1;y2;y3;y4]
y   =   A * x

% Matrix-Matrix Product
syms A a1 a2 a3 a4 B b1 b2 b3 b4 AB
A   =   [a1 a2;a3 a4]
B   =   [b1 b2;b3 b4]
AB  =   A * B

% Example 1
syms A
A   =   [2 4 6;3 -5 7;4 1 8;-9 -6 -3]
y   =   A * x

% Example 2
A   =   [1 2;3 4]
B   =   [5 6;7 8]
AB  =   A * B

syms xA xB x0
x0   =   [x1;x2]
xA   =   A * xB
xB   =   B * x0
xA   =   A * B * x0
MATLAB Code for Math Review

```matlab
% Matrix Identity and Inverse
I3 = eye(3)
x = I3 * x
syms A Ainv
A = [a b c; d e f; g h k]
Ainv = inv(A)
I3 = simplify(A * Ainv)
I3 = simplify(Ainv * A)
```

```matlab
% Matrix Inverse Example
syms A Th cTh sTh Ainv
A = [cTh 0 sTh; 0 1 0; -sTh 0 cTh]
Ainv = inv(A)
detA = det(A)
cTh = cos(Th)
sTh = sin(Th)
Th = pi / 4
syms A Ainv
A = [cos(Th) 0 sin(Th); 0 1 0; -sin(Th) 0 cos(Th)]
Ainv = inv(A)
```

```matlab
% Consequently, ...
I3 = A * Ainv
```

```matlab
% Computation of (n x m) Inverse
detA = det(A)
AdjA = Ainv * detA
```

MATLAB Command Window
Output for Math Review

```
>> MAE 345 Lecture 2 Math Review<<<
Date and Time are 03-Sep-2013 13:49:40
```

```
A =
[ a, b, c]
[ d, e, f]
[ g, h, k]
[ l, m, n]
AT =
[ conj(a), conj(d), conj(g), conj(l)]
[ conj(b), conj(e), conj(h), conj(m)]
[ conj(c), conj(f), conj(k), conj(n)]
x =
x1
x2
x3
y =
y1
y2
y3
y4
y =
a*x1 + b*y2 + c*z3
d*x1 + e*x2 + f*y3
g*x1 + h*x2 + k*x3
l*x1 + m*x2 + n*x3
```
A =
\[
\begin{pmatrix}
  a & b & c \\
  d & e & f \\
  g & h & k 
\end{pmatrix}
\]
Ainv =
\[
\begin{pmatrix}
  (f*h - e*k)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k + b*d*k),
  -(c*h - b*k)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k + b*d*k),
  -(b*f - c*e)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k + b*d*k)
\\
  -(f*g - d*k)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k + b*d*k),
  (c*g - a*k)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k + b*d*k),
  (a*f - c*d)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k + b*d*k)
\\
  -(d*h - e*g)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k + b*d*k),
  (a*h - b*g)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k + b*d*k),
  (c*h - b*k)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k + b*d*k)
\end{pmatrix}
\]
I3 =
\[
\begin{pmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 1 
\end{pmatrix}
\]