Launch Vehicle Design: Propulsion

Launch Goals

- Deliver payload to desired position and velocity, safely, reliably, and on time
  - Low earth orbit: \( \sim 24,000 \text{ ft/s} \approx 7.3 \text{ km/s} \)
  - Escape: \( \sim 36,000 \text{ ft/s} \approx 11 \text{ km/s} \)
- Minimize launch cost and propellant use
- Minimize hazard to infrastructure and damage to environment

Launch Vehicle Systems

- Propulsion and Power
  - Main engines
  - Attitude-control thrusters
  - Retro-rockets
  - Uillage rockets
  - Turbo-pumps
  - Batteries, fuel cells
  - Pressurizing bottles
  - Escape/destroy systems
- Electronics
  - Guidance and control computers
  - Sensors and actuators
  - Radio transmitters and receivers
  - Radar transponders
  - Antennas
- Structure
  - Skin, frames, ribs, stringers, bulkheads
  - Propellant tanks
  - Fins, control surfaces
  - Inter-stage adapters, fairings
  - Heat shields, insulation
- Reusable launchers/orbiters
  - Wings, parachutes
  - Landing gear
  - Orbital maneuvering units
  - Robot arms
  - Life support systems (manned vehicle)

Launch Vehicle Design: Propulsion

Space System Design, MAE 342, Princeton University

Robert Stengel

Launch goals

Vehicle systems

Propulsion

Staging

From the Earth to the Moon (Jules Verne, 1865)

- Half-mile long cannon
- Acceleration = 28,000 “g”s
- Deceleration after cannon exit: probably greater
- Aerodynamic heating might melt the capsule
- But, ... some of the fiction matched some of the facts of Project Apollo
  - Size of capsule
  - Number of astronauts
  - Required velocity
  - Time of flight
  - Launch from US at latitude similar to that of Cape Canaveral
  - Recovery at sea

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http://www.princeton.edu/~stengel/MAE342.html
**Chemical (Thermal) Rockets**

- **Liquid/Gas Propellant**
  - Monopropellant
    - Catalytic ignition
    - Chemical decomposition
  - Bipropellant
    - Separate oxidizer and fuel
    - Hypergolic (spontaneous) ignition
    - External ignition
    - Storage
      - Ambient temperature and pressure
      - Cryogenic
      - Pressurized tank
  - Throttling
    - Start/stop cycling

**Liquid-Fuel Rocket Motor**

- Need for high-pressure propellant feed
  - Pressurized propellant tanks
  - Turbopump

**Solid-Fuel Rocket Motor**

- **Solid Propellant**
  - Mixed oxidizer and fuel
  - External ignition
  - Burn to completion
- **Hybrid Propellant**
  - Liquid oxidizer, solid fuel
  - Throttling
  - Start/stop cycling

**Solid-Fuel Rocket Motor**

- Thrust is proportional to burning area
- Rocket grain patterns affect thrust profile
- Propellant chamber must sustain high pressure and temperature
- Environmentally unfriendly exhaust gas
Hybrid-Fuel Rocket Motor

- **SpaceShipOne motor**
  - Nitrous oxide
  - Hydroxy-terminated polybutadiene (HTPB)
- **Issues**
  - Hard start
  - Blow back
  - Complete mixing of oxidizer and fuel toward completion of burn

Rocket Thrust

\[
\text{Thrust} = m \frac{V_{\text{exhaust}}}{g_o} + A_{\text{exit}} (P_{\text{exit}} - P_{\text{ambient}})
\]

\[
c_{\text{eff}} = \frac{\text{Thrust}}{m} = \text{Effective exhaust velocity}
\]

\[
m = \text{Mass flow rate of on-board propellant}
\]

Specific Impulse

\[
I_{sp} = \text{Thrust} \times \text{mass flow} = \frac{c_{\text{eff}}}{g_o}, \quad \text{Units} = \frac{m}{s} \times \frac{m}{s^2} = \frac{s}{g}
\]

- \(g_o\) is a normalizing factor for the definition
- Chemical rocket specific impulse (vacuum)
  - Solid propellants: < 295 s
  - Liquid propellants: < 510 s

Typical Values of Specific Impulse

- Chamber pressure = 7 MPa (low by modern standards)
- Expansion to exit pressure = 0.1 MPa

<table>
<thead>
<tr>
<th>Liquid-Fuel Rockets</th>
<th>V_{sp}, kg m^3/s \times 10^3</th>
<th>I_{sp}, s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopropellant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrogen Peroxide</td>
<td>165</td>
<td>238</td>
</tr>
<tr>
<td>Hydrazine</td>
<td>199</td>
<td>201</td>
</tr>
<tr>
<td>Nitromethane</td>
<td>255</td>
<td>290</td>
</tr>
<tr>
<td>Bipropellant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kerosene Oxidizer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel</td>
<td>V_{sp}, kg m^3/s \times 10^3</td>
<td>I_{sp}, s</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>301</td>
<td>307</td>
</tr>
<tr>
<td>Oxygen</td>
<td>320</td>
<td>394</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>268</td>
<td>369</td>
</tr>
<tr>
<td>UDMH</td>
<td>390</td>
<td>109</td>
</tr>
</tbody>
</table>

Solid-Propellant Rockets

<table>
<thead>
<tr>
<th>V_{sp}, kg m^3/s \times 10^3</th>
<th>I_{sp}, s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double-Base</td>
<td></td>
</tr>
<tr>
<td>JPL 540A</td>
<td>196</td>
</tr>
<tr>
<td>TRX-H609</td>
<td>245</td>
</tr>
<tr>
<td>PBAN (SSV)</td>
<td>260</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>V_{sp}, kg m^3/s \times 10^3</th>
<th>I_{sp}, s</th>
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<tr>
<td>JPL 540A</td>
<td>231</td>
</tr>
<tr>
<td>TRX-H609</td>
<td>245</td>
</tr>
<tr>
<td>PBAN (SSV)</td>
<td>260</td>
</tr>
</tbody>
</table>

Hybrid-Fuel Rocket

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Oxidizer</th>
<th>V_{sp}, kg m^3/s \times 10^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>HTPB</td>
<td>N2O</td>
<td>250</td>
</tr>
<tr>
<td>N2O</td>
<td>HTPB</td>
<td>250</td>
</tr>
</tbody>
</table>
Specific Impulse

- Specific impulse is a product of characteristic velocity, $c^*$, and rocket thrust coefficient, $C_F$

$$I_{sp} = \frac{\text{Thrust}}{m g_o} = \frac{C_F c^*}{g_o} = \frac{V_{exhaust}}{g_o} \quad \text{when } C_F = 1, p_e = p_{ambient}$$

- Characteristic velocity is related to:
  - combustion chamber performance
  - propellant characteristics
- Thrust coefficient is related to:
  - nozzle shape
  - exit/ambient pressure differential

Rocket Characteristic Velocity, $c^*$

$$c^* = \frac{p_c A_t}{m} = \text{exhaust velocity if } C_F = 1$$

Rocket Thrust Coefficient, $C_F$

$$C_r = \frac{\text{Thrust}}{p_c A_t}$$

where

- Thrust = $\lambda m v_e A_t (p_e - p_{ambient})$
- $\lambda$ = reduction ratio (function of nozzle shape)

$$C_F = \lambda \Gamma \left( \frac{2}{\gamma} \frac{V_e}{c^*} \frac{p_{ambient}}{p_e} \left( \frac{\gamma}{\gamma - 1} \right) \left( \frac{p_e}{p_{ambient}} \right)^{\gamma - 1} \right)$$

- typically, $0.5 < C_F < 2$
Mixture Ratio, $r$

- Stoichiometric mixture: complete chemical reaction of propellants
- Specific impulse maximized with lean mixture ratio, $r$ (i.e., below stoichiometric maximum)

\[ r = \frac{\dot{m}_{\text{oxidizer}}}{\dot{m}_{\text{fuel}}} \quad \dot{m}_{\text{fuel}} = \frac{\dot{m}_{\text{total}}}{1 + r}; \quad \text{"leaner" } r < \text{"richer"} \]

Effect of Pressure Ratio on Mass Flow

- In choked flow, mass flow rate is maximized
- Choked flow occurs when

\[ \frac{p_e}{p_c} \leq \left( \frac{2}{\gamma + 1} \right)^{\gamma / (\gamma - 1)} \approx 0.53 \]

Rocket Nozzles

- Expansion ratio, $A_e/A_t$ chosen to match exhaust pressure to average ambient pressure
  - Ariane rockets: Viking V for sea level, Viking IV for high altitude
- Rocket nozzle types
  - DeLaval nozzle
  - Isentropic expansion nozzle
  - Spike/plug nozzles
  - Expansion-deflection nozzle

Shock Diamonds

- When $p_e \neq p_a$ exhaust flow is over- or underexpanded
- Effective exhaust velocity < maximum value
The Rocket Equation

- **Ideal velocity increment of a rocket stage, $\Delta V_j$ (gravity and aerodynamic effects neglected)**

$$
\frac{dV}{dt} = \frac{\text{Thrust}}{m} = \frac{\dot{m} c_{\text{eff}}}{m} = -\frac{\dot{m}}{m} I_{\text{sp}} g_o
$$

$$
\int_{V_i}^{V_f} dV = -I_{\text{sp}} g_o \int \frac{\dot{m}}{m} dm = -I_{\text{sp}} g_o \ln m_{i_j}^{m_j}
$$

$$
(V_f - V_i) = \Delta V_i = I_{\text{sp}} g_o \ln \left( \frac{m_i}{m_f} \right) = I_{\text{sp}} g_o \ln \mu
$$

- **Ideal velocity increment of a 2-stage rocket**

$$
\Delta V_i = \left( I_{\text{sp}} \right)_1 g_o \ln \left( \frac{m_1}{m_f} \right) + \left( I_{\text{sp}} \right)_2 g_o \ln \left( \frac{m_f}{m_f/2} \right) = g_o \left[ \left( I_{\text{sp}} \right)_1 \ln \mu_1 + \left( I_{\text{sp}} \right)_2 \ln \mu_2 \right]
$$

- $\mu_j$ is the mass ratio of the $j^{th}$ stage
Vehicle Mass Components

• Initial and final masses of a single-stage rocket

\[ m_i = m_{\text{payload}} + m_{\text{structure/ engine}} + m_{\text{propellant}} \]
\[ m_f = m_{\text{payload}} + m_{\text{structure/ engine}} \]

Ideal Velocity Increment for Single Stage With Various Specific Impulses

<table>
<thead>
<tr>
<th>Mass Ratio</th>
<th>Isp = 220 s</th>
<th>Isp = 275 s</th>
<th>Isp = 400 s</th>
<th>Isp = 500 s</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.5</td>
<td>1.9</td>
<td>2.7</td>
<td>3.4</td>
</tr>
<tr>
<td>3</td>
<td>2.4</td>
<td>3</td>
<td>4.3</td>
<td>5.3</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3.8</td>
<td>5.4</td>
<td>6.8</td>
</tr>
<tr>
<td>5</td>
<td>3.5</td>
<td>4.3</td>
<td>6.3</td>
<td>7.9</td>
</tr>
</tbody>
</table>

• Single stage to orbit with payload (\( \Delta V \approx 7.3 \text{ km/s} \))? Not easy.

\[ \mu_{\text{required}} = e^{\Delta V_I / I_{sp} g_0} \]

Ratios Characterizing a Rocket Stage

\[ \mu = \frac{m_{\text{initial}}}{m_{\text{final}}} \]
\[ \lambda = \frac{m_{\text{payload}}}{m_{\text{initial}}} \]
\[ \eta = \frac{m_{\text{structure/ engine}}}{m_{\text{initial}}} \]
\[ \varepsilon = \frac{m_{\text{propellant}}}{m_{\text{initial}}} = \frac{\mu - 1}{\mu} \]
\[ \lambda + \eta + \varepsilon = 1 \]

• Payload is what’s left after propellant and structure are subtracted

Required Mass Ratio for Various Velocity Increments

Ideal Velocity Increment, km/s

<table>
<thead>
<tr>
<th>Ideal Velocity Increment, km/s</th>
<th>Required Mass Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isp = 240 s</td>
<td>Isp = 400 s</td>
</tr>
<tr>
<td>7</td>
<td>19.6</td>
</tr>
<tr>
<td>8</td>
<td>29.9</td>
</tr>
<tr>
<td>9</td>
<td>45.7</td>
</tr>
<tr>
<td>10</td>
<td>69.9</td>
</tr>
<tr>
<td>11</td>
<td>106.9</td>
</tr>
<tr>
<td>12</td>
<td>163.5</td>
</tr>
</tbody>
</table>

• ... and there are velocity losses due to gravity and aerodynamic drag
Ideal Velocity Increment for a Multiple-Stage Rocket

- Ideal velocity increment of an \( n \)-stage rocket
  \[
  \Delta V_I = g_o \sum_{j=1}^{n} \left( \frac{L_{sp}}{L_{sp}^j} \right) \ln \mu_j
  \]

- Optimal ideal velocity increment with equal specific impulses
  \[
  \Delta V_I = \frac{L_{sp} g_o \ln(\mu_1 \cdot \mu_2 \cdot \ldots \cdot \mu_n)}{L_{sp} g_o \ln(\mu_{overall})} = \frac{L_{sp} g_o \ln(\mu^n)}{L_{sp} g_o \ln(\mu_{overall})}
  \]

Required Mass Ratios for Multiple-Stage Rockets

- Staging reduces mass ratios to achievable values
- With equal specific impulses for each stage

<table>
<thead>
<tr>
<th>Required Mass Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal Velocity Increment, km/s</td>
</tr>
<tr>
<td>Isp = 240 s</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
</tbody>
</table>

Scout Launch Vehicle

- Liftoff mass = 16,450 kg
- 4 solid-rocket stages

<table>
<thead>
<tr>
<th>Typical Figures for Scout</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage</td>
</tr>
<tr>
<td>1 (Altair)</td>
</tr>
<tr>
<td>2 (Castor)</td>
</tr>
<tr>
<td>3 (Ariane)</td>
</tr>
<tr>
<td>4 (Altair)</td>
</tr>
</tbody>
</table>

- Overall mass ratio = 34
- Overall payload ratio = 0.00425 = 0.425% (67-kg payload)

Overall Payload Ratio of a Multiple-Stage Rocket

\[
\lambda_{overall} = \left( \frac{m_{payload}}{m_{initial}} \right)_n = \left( \frac{m_{payload}}{m_{initial}} \right)_n \cdot \left( \frac{m_{payload}}{m_{initial}} \right)_{n-1} \cdot \ldots \cdot \left( \frac{m_{payload}}{m_{initial}} \right)_1
\]

\[
= \lambda_1 \cdot \lambda_2 \cdot \ldots \lambda_n
\]

- Feasible design goal: Choose stage mass ratios to maximize overall payload ratio
Payload Ratios of a Two-Stage Rocket

- For equal specific impulses
  \[
  \Delta V_i = I_{sp} g_o \left[ \ln \mu_i + \ln \mu_2 \right] = I_{sp} g_o \left[ \ln \mu_{overall} \right]
  \]

- Payload ratios for different structural ratios
  \[
  \lambda_i = \frac{1}{\mu_i} - \eta_i = \frac{1 - \mu_i \eta_i}{\mu_i} \quad \lambda_2 = \frac{1 - \mu_2 \eta_2}{\mu_2}
  \]

Maximum Payload Ratio of a Two-Stage Rocket

- Overall payload ratio
  \[
  \lambda_{overall} = \lambda_1 \lambda_2 = \left( \frac{1 - \mu_1 \eta_1}{\mu_1} \right) \left( \frac{1 - \mu_2 \eta_2}{\mu_2} \right)
  \]

- Condition for a maximum with respect to first stage mass ratio
  \[
  \frac{\partial \lambda_{overall}}{\partial \mu_1} = \left( -\eta_1 + \frac{\mu_{overall} \eta_2}{\mu_2} \right) \frac{\mu_{overall}}{\mu_1} = 0
  \]

- Stage mass ratios and overall payload ratio
  \[
  \mu_1 = \sqrt{\frac{\eta_2}{\eta_1}} \quad \mu_2 = \sqrt{\frac{\eta_1}{\eta_2}}
  \]
  \[
  \lambda_{overall} = \frac{1}{\mu_{overall}} - 2 \sqrt{\frac{\eta_1 \eta_2}{\mu_{overall}^2}} + \eta_1 \eta_2
  \]

- Also see
  [http://www.princeton.edu/~stengel/Prop.pdf](http://www.princeton.edu/~stengel/Prop.pdf)

Maximum Payload Ratio of a Two-Stage Rocket

- Specific impulse
  \[
  \Delta V_i = I_{sp} g_o \ln \mu = I_{sp} g_o \ln \left( \frac{m_{final} + m_{propellant}}{m_{final}} \right) = I_{sp} g_o \ln \left( 1 + \frac{m_{propellant}}{m_{final}} \right)
  \]
  \[
  = I_{sp} g_o \ln \left( 1 + \frac{\text{Density}_\text{propellant} \times \text{Volume}_\text{propellant}}{m_{final}} \right)
  \]
  \[
  = g_o \left( I_{sp} \rho \right) \frac{V_{propellant}}{m_{final}}
  \]

- Volumetric specific impulse
  \[
  I_{sp_{vol}} \equiv VI_{sp} = I_{sp} \rho \text{ propellant}
  \]
Volumetric Specific Impulse

- For fixed volume and final mass, increasing volumetric specific impulse increases ideal velocity increment.

<table>
<thead>
<tr>
<th>Density, g/cc</th>
<th>Isp, s, SL</th>
<th>VelSp, s, SL</th>
<th>Isp, s, vac</th>
<th>VelSp, s, vac</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOX/Kerosene</td>
<td>1.3</td>
<td>265</td>
<td>345</td>
<td>304</td>
</tr>
<tr>
<td>LOX/LH2 (Saturn V)</td>
<td>0.28</td>
<td>360</td>
<td>101</td>
<td>424</td>
</tr>
<tr>
<td>LOX/LH2 (Shuttle)</td>
<td>0.28</td>
<td>390</td>
<td>109</td>
<td>455</td>
</tr>
<tr>
<td>Shuttle Solid Booster</td>
<td>1.35</td>
<td>242</td>
<td>327</td>
<td>262</td>
</tr>
</tbody>
</table>

Saturn V Specific Impulses, vacuum (sea level):
- 1st Stage, 5 F-1 LOX-Kerosene Engines: 304 s (265 s)
- 2nd Stage, 5 J-2 LOX-LH2 Engines: 424 s (~360 s)
- 3rd Stage, 1 J-2 LOX-LH2 Engine: 424 s (~360 s)

Strap-On Boosters

- High volumetric specific impulse is desirable for first stage of multi-stage rocket.
- Strap-on solid rocket boosters are a cost-effective way to increase mass and payload ratios.

Atlas Evolution

Delta Evolution

Implementing a Low Risk Evolution Process
Space Shuttle Solid Rocket Boosters

Next Time:
Launch Vehicle Design:
Trajectories and Aerodynamics