Earth Satellite Environment and Orbits
Space System Design, MAE 342, Princeton University
Robert Stengel

- Mission goals
- Near-earth and space environment
- Orbits and orbital decay

Mission Goals

- Make scientific measurements
- Observe earth, planets, moons, asteroids, comets, and the universe
- Aid terrestrial communications
- Aid terrestrial navigation
- Return samples from the solar system

NASA Science Missions

Earth’s Orbit About the Sun

Luna-16 Tether Physics and Survivability (TiPS) Experiment

Earth’s Orbit About the Sun

Equinoxes and Solstices for the Northern Hemisphere:
The Southern Hemisphere experiences the opposite seasons.
Spherical Model of the Earth

- Spherical model of earth’s surface, earth-fixed (rotating) coordinates

\[
\mathbf{R}_E = \begin{bmatrix}
\cos L_E \cos \lambda_E \\
\cos L_E \sin \lambda_E \\
\sin L_E
\end{bmatrix} R
\]

- Celestial longitude measured from First Point of Aries

\[
\lambda_c = \lambda_E + \Omega (t - t_{epoch}) = \lambda_E + \Omega \Delta t
\]

Ellipsoidal Model of the Earth

- Oblate ellipsoid with cross section satisfying the equation

\[
\frac{x^2}{R_x^2} + \frac{y^2}{R_y^2} + \frac{z^2}{R_z^2} = 1
\]

Equatorial Radius, \( R_x = 6,378,137 \text{m} \)
Polar Radius, \( R_z = 6,356,752 \text{m} \)

- Surface radius at latitude \( L_o \)

\[
x_o^2 = R_x^2 \cos^2 L_o; \quad z_o^2 = R_z^2 \sin^2 L_o
\]

\[
R_o^2 = \frac{R_x^2 R_z^2}{\left[1 - \left(\frac{R_y}{R_x}\right)^2 \cos^2 L_o\right]}
\]

where

\[
k^2 = 2(1-e^2) = \text{Eccentricity} = 0.081819
\]

\[
e = \frac{R_y - R_x}{R_x} = \text{Flattening} = 1/298.257
\]

Spherical Gravitational Model of the Earth

- Used to calculate 2-body/central-force field orbits

\[
\mathbf{f}_i = -m \frac{\mu}{r_i^3} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (\text{vector})
\]

\[
\mathbf{f}_i = -m \frac{\mu}{r_i^2} \quad (\text{scalar})
\]

\[
\mu_{\text{Earth}} = 3.986 \times 10^5 \text{km}^3/\text{s}^2
\]
Oblate Ellipsoidal Gravitational Model of the Earth

\[ f_I = m \mu r_I^3 \left( 1 + \frac{3}{2} J_2 \left( \frac{R_o}{r_I} \right)^2 \left( 1 - \frac{5}{3} \left( \frac{z}{r_I} \right)^2 \right) \right) \]

\[ J_2 = 1.0826 \times 10^{-3} \]

- Oblateness perturbs orbital parameters
  - Orbital plane is precessed
  - Perigee is advanced or regressed

Diurnal Variations in the Atmosphere

- Factor of 10 change in density
- \( \pm 20\% \) change in temperature
- Large effect of solar activity

Earth’s High-Altitude Atmosphere

- Temperature of the Atmosphere
- Density of the Atmosphere

- Atmosphere not well-represented as a continuum at high altitude
- Different scale heights for different species

<table>
<thead>
<tr>
<th>Altitude</th>
<th>Molecules/cc</th>
<th>Mean Free Path</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea Level</td>
<td>2 x 10^19</td>
<td>7 x 10^-6 cm</td>
</tr>
<tr>
<td>600 km</td>
<td>2 x 10^7</td>
<td>10 km</td>
</tr>
</tbody>
</table>

Atmospheric Constituents

- Constituents at minimum and maximum solar activity
- Different scale heights for different species
Atmospheric Ionization Profiles

- Scale heights of electrons, ions, and neutrals vary greatly
- Ionospheric electric field (set by heavy oxygen atoms) dominates gravity field for lighter ions, e.g., hydrogen and helium

Magnetosphere and Van Allen Belts

- Trapped Energetic Ions and Electrons
- Light ions form the base population of the magnetosphere

Spacecraft That Defined the Magnetosphere and Van Allen Belts

- see Piacenco for discussion of mechanics and dynamics
  - plasma frequency
  - Debye length
  - spacecraft charging and ram-wake effects
  - motion of charged particles in a dipole field
  - trapped radiation

Micrometeoroids and Space Debris

- Thousands of nuts, bolts, and other fragments in orbit
- 1999 estimate: 2 million kg, 110,000 objects > 1 cm
- January 2007: Chinese anti-satellite test destroyed old satellite and added >1,335 remnants larger than a golf ball
- U.S. plans to attempt to shoot down a failed spy satellite within the next few weeks -- more debris
Satellites for Detecting Micrometeoroids and Space Debris

**Long-Duration Exposure Facility (~5.7 yr in orbit)**

![Long-Duration Exposure Facility](image)

Distribution of Micrometeoroids and Space Debris (from LDEF)

![Distribution of Micrometeoroids](image)

**Effect of Impact Angle on Relative Specific Energy**

<table>
<thead>
<tr>
<th>Impact Angle, deg</th>
<th>Satellite Velocity, km/s</th>
<th>Debris Velocity, km/s</th>
<th>Relative Velocity, km/s</th>
<th>Relative Specific Energy, km</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>7.5</td>
<td>7.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>45</td>
<td>7.5</td>
<td>7.5</td>
<td>10.6</td>
<td>5734</td>
</tr>
<tr>
<td>0</td>
<td>7.5</td>
<td>7.5</td>
<td>15</td>
<td>11468</td>
</tr>
</tbody>
</table>

**In-plane Parameters of an Elliptical Orbit**

- Dimensions of the orbit
  - $p = \frac{h^2}{\mu} = \"The parameter\" or semi-latus rectum$
  - $h = \text{Angular momentum about center of mass}$
  - $e = \sqrt{1 + 2 \frac{E}{\mu}} = \sqrt{1 - \frac{b^2}{a^2}} = \text{Eccentricity}$
  - $E = \text{Specific energy}$
  - $a = \frac{p}{1 - e^2} = \text{Semi-major axis}$
  - $r_a = a(1 + e) = \text{Apogee radius}$
  - $r_p = a(1 - e) = \text{Perigee radius}$
**In-plane Parameters of an Elliptical Orbit**

- Position and velocity of the satellite
  \[ r = \frac{\rho}{1 + e \cos \theta} = \text{Radius of the satellite} \]
  \[ \theta = \text{True anomaly} \]
  \[ V = \sqrt{\frac{\mu}{2a - r}} = \text{Velocity of the satellite} \]
- Period of the orbit
  \[ P = 2\pi \sqrt{\frac{a^3}{\mu}} = \text{Period of the satellite orbit} \]

**Orbital Lifetime of a Satellite**

- Aerodynamic drag causes orbit to decay
  \[ \frac{dV}{dt} = - \frac{C_D \rho V^2 S/2}{m} = -B^* \rho V^2 S/2 \]
  \[ B^* = C_D S/m \]
- Air density decreases exponentially with altitude
  \[ p = \rho_{SL} e^{-h/h_{scale}} \]
  \( \rho_{SL} = \text{air density at sea level}; \ h_{scale} = \text{atmospheric scale height} \]
- Drag is highest at perigee
  - Air drag “circularizes” the orbit
    - Large change in apogee
    - Small change in perigee
    - Until orbit is ~circular
    - Final trajectory is a spiral

**Typical Satellite Periods**

- Period of the orbit
  \[ P = 2\pi \sqrt{\frac{a^3}{\mu}} = \text{Period of the satellite orbit} \]
- Typical periods for circular orbits
<table>
<thead>
<tr>
<th>Altitude, km</th>
<th>Period, min</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>88.5</td>
</tr>
<tr>
<td>400</td>
<td>92.6</td>
</tr>
<tr>
<td>800</td>
<td>100.9</td>
</tr>
<tr>
<td>1600</td>
<td>118.2</td>
</tr>
<tr>
<td>3200</td>
<td>155.5</td>
</tr>
<tr>
<td>6400</td>
<td>239.6</td>
</tr>
<tr>
<td>12800</td>
<td>440.5</td>
</tr>
<tr>
<td>25600</td>
<td>948.5</td>
</tr>
</tbody>
</table>

**Variation of \( a \) over time**

\[ \int_{a_0}^{a} \frac{e^{-\frac{(a-R)/h_0}{\sqrt{\mu B^*}}}}{\sqrt{a}} da = -\sqrt{\mu B^*} \rho_{SL} \int dt \]

**Time, \( t_{decay} \), to reach earth’s surface (\( a = R \)) from starting altitude, \( h_0 \)**

\[ t_{decay} = \frac{h_{scale}}{\sqrt{\mu B^*} \rho_{SL}} \left( e^{h_0/h_{scale}} - 1 \right) \]
NRL Starshine 1 Orbital Decay (2003)

Orientation of an Elliptical Orbit

Impulsive ΔV Orbital Maneuver

- If rocket burn time is short compared to orbital period (e.g., seconds compared to hours), impulsive ΔV approximation can be made
  - Change in position during burn is ~ zero
  - Change in velocity is ~ instantaneous

- Velocity impulse at apogee

Orbit Change due to Impulsive ΔV

- Maximum energy change accrues when ΔV is aligned with the instantaneous orbital velocity vector
  - Energy change → Semi-major axis change
  - Maneuver at perigee raises or lowers apogee
  - Maneuver at apogee raises or lowers perigee

- Optimal transfer from one circular orbit to another involves two impulses [Hohmann transfer]

- Other maneuvers
  - In-plane parameter change
  - Orbital plane change

http://www.azinet.com/starshine/descript.htm
Effect of Launch Latitude on Orbital Parameters

- Launch latitude establishes minimum orbital inclination (without “dogleg” maneuver)
- Time of launch establishes line of nodes
- Argument of perigee established by
  - Launch trajectory
  - On-orbit adjustment

Geo-Synchronous Ground Track

Typical Satellite Orbits

- Typical launch inclinations
- GPS Constellation
- 26,600 km
- Sun-Synchronous Orbit
- GPS Constellation
- Orb Plane
- 26.6° Earth Rotation Per Orbit
- Orbit Plane
- 26.6° Earth Rotation Per Orbit
- Mission Orbit
- Transfer Orbit
- Parking Orbit
- Molniya Orbit
- 1,507 x 39,305 km
- i = 63.4°
Communications Satellite and Delta II Launcher

Satellite Systems

- **Power and Propulsion**
  - Solar cells
  - “Kick” motor/ payload assist module
  - Attitude-control/orbit-adjustment/station -keeping thrusters
  - Batteries, fuel cells
  - Pressurizing bottles
  - De-orbit/ “graveyard” systems

- **Structure**
  - Skin, frames, ribs, stringers, bulkheads
  - Propellant tanks
  - Heat/solar/micrometeoroid shields, insulation
  - Articulation/deployment mechanisms
  - Gravity-gradient tether
  - Re-entry system (e.g., sample return)

- **Electronics**
  - Payload
  - Control computers
  - Control sensors and actuators
  - Control-wheel gyros
  - Radio transmitters and receivers
  - Radar transponders
  - Antennas

Typical Satellite Mass Breakdown

<table>
<thead>
<tr>
<th>Item</th>
<th>Range (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure (total)</td>
<td>15–22</td>
</tr>
<tr>
<td>Primary structure</td>
<td>12–15</td>
</tr>
<tr>
<td>Secondary structure</td>
<td>2–5</td>
</tr>
<tr>
<td>Fasteners</td>
<td>1–2</td>
</tr>
<tr>
<td>Power</td>
<td>12–30</td>
</tr>
<tr>
<td>Thermal control</td>
<td>4–8</td>
</tr>
<tr>
<td>Harness</td>
<td>4–10</td>
</tr>
<tr>
<td>Avionics</td>
<td>3–7</td>
</tr>
<tr>
<td>Guidance &amp; control</td>
<td>5–10</td>
</tr>
<tr>
<td>Communication</td>
<td>2–6</td>
</tr>
<tr>
<td>Payload</td>
<td>7–55</td>
</tr>
</tbody>
</table>

- Satellite without on-orbit propulsion
- “Kick” motor/ PAM can add significant mass
- Total mass: from a few kg to > 30,000 kg

Angular Attitude of Satellite Configurations

- Randomly oriented satellites
  - Angular attitude is free to vary

- Spinning satellites
  - Angular attitude maintained by gyroscopic moment and magnetic coil
  - Asymmetric distribution of mass, solar cells, and instruments
Attitude-Controlled Satellite Configurations

- **Dual-spin satellites**
  - Angular attitude maintained by gyroscopic moment and thrusters
  - Axisymmetric distribution of mass and solar cells
  - Instruments and antennas do not spin

- **Attitude-controlled satellites**
  - Angular attitude maintained by 3-axis control system
  - Non-symmetric distribution of mass, solar cells and instruments

Next Time: Satellites and Space Probes