# Valence Candidates and Ambiguous Platforms in Policy Announcement Games

Yuichiro Kamada and Takuo Sugaya*

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**Abstract**

Over the course of election campaigns, candidates often use ambiguous language in the early stage of the campaigns while they sometimes make their policies clear subsequently. We explain this phenomenon by constructing a dynamic model of campaigns. In the model, two candidates obtain opportunities to make their policies unambiguous over the course of a campaign period until the predetermined election date. While there is no incentive to keep policies ambiguous if the two candidates are perfectly symmetric, there is a strategic incentive to keep policies ambiguous if one candidate is slightly stronger than the other. The driving force of the ambiguity is that the strong candidate tries to copy the policy of the weak candidate, but the latter seeks to avoid this to happen by deferring the announcement.

*Kamada: Department of Economics, Harvard University, Cambridge, MA, 02138, e-mail: ykamada@fas.harvard.edu; Sugaya: Department of Economics, Princeton University, Princeton, NJ, 08544, e-mail: tsugaya@princeton.edu; We thank Itay Fainmesser, Drew Fudenberg, Fuhito Kojima, David Laibson, John B. Londregan, Shih-En Lu, Francisco Martínez-Mora, Adam H Meirowitz, Stephen Morris, Kristopher Ramsay, Ken Shepsle, Yuki Takagi, Satoru Takahashi and seminar participants at Harvard University Department of Economics and Department of Government, Princeton University Department of Economics and Department of Political Science, Stony Brook and Urrutia Elejalde Workshop on Information, Dynamics and Political Decision Making for helpful comments. In addition, Josh Gottlieb read through the previous version of this paper and gave us very detailed comments, which significantly improved the presentation of the paper.

## 1 Introduction

In elections such as those for the US presidency, candidates' policy announcements are often ambiguous. Shepsle (1972) quoted Nicholas Biddle, the manager of William Henry Harrison's campaign for the presidency, advising "Let him say not a single word about his principles, or his creed - let him say nothing - promise nothing. Let no Committee, no convention - no town meeting ever extract from him a single word, about what he thinks now, or what he will do hereafter." More recently, at the beginning of US presidential primary election 2008, John Edwards criticized Barack Obama for his abstaining from many votes as a Senator, trying not to clarify his political position. Over the course of the election, However, Barack Obama clarified his policies.

The objective of this paper is two-fold. The first is to explain the phenomenon just described in the previous paragraph. That is, we explain why candidates use ambiguous language in campaigns, and why it is sometimes refined subsequently. The second component of our objective is somewhat ambitious: Despite the obvious importance of election campaigns on the electoral outcome and the fact that the campaigns are dynamic in nature, there are apparently no models of dynamic campaigns in the literature.[1] One reason, it seems, is that there is no obvious way to model campaigns in the way that gives rise to dynamic strategic considerations. Our aim is to fill this gap, by proposing a tractable model in which candidates face dynamic strategic considerations.

We propose a "policy announcement game," in which candidates strategically use ambiguous language which, in equilibrium, is sometimes refined subsequently. In our model, each of two candidates obtains opportunities to announce their policies according to a Poisson process over a campaign period until a predetermined election date. At each opportunity, candidates can either clarify their policies or remain ambiguous. Once a candidate has made her policy clear, she cannot change it afterwards. We first show that, if two candidates are perfectly symmetric, there are no interesting strategic considerations. Specifically, each candidate makes their policy clear as soon as possible. Next we show that, if one candidate is slightly stronger than the other (has more valence), there are rich strategic considerations involved in equilibrium. For example, the weak candidate will not make his policy clear in early stages of the election campaign. This is because if he does so then the strong candidate will simply copy that policy afterwards, so that the weak candidate

[1] By a model of dynamic election campaigns we mean a model with a single election, so in particular we do not include models that have primaries and the general election when we mention "models of dynamic election campaigns."

loses for sure. Depending on the environment, the strong candidate also has an incentive to be ambiguous, expecting a chance to copy the weak candidate's policy near the election date.

Our work shows that *a candidate's valence* leads to *ambiguous language* in dynamic election campaigns. Let us position our work in the literature with respect to these two factors.

In the standard simultaneous-move Hotelling-Downs model with valence candidates, there exists no pure strategy equilibrium: the strong candidate always wants to copy the weak candidate's policy, while the weak candidate does not want to be copied, just as in the "matching pennies" game. There are two approaches to address this issue. The first approach is to assume that the strong candidate is the incumbent and the weak candidate is the entrant (Bernhardt and Ingberman (1985), Berger et al. (2000)). In this approach, a typical result is that the strong candidate positions close to the median and the weak candidate positions at a policy slightly away from the strong candidate's policy, where the distance between two policies is determined by the degree of asymmetry between candidate's valences. The second approach is that of Aragonès and Palfrey (2002), in which they consider the simultaneous-move game seriously and characterize the unique equilibrium in a discrete policy space and consider the limit as the discrete space approximates the standard continuous policy space in a certain sense. They show that the strong candidate assigns a high probability around the median while the weak candidate assigns a small probability to it. Although these two approaches give us understanding of what the equilibrium behavior looks like in the electoral situation with valence candidates, in either of these two models the order of policy announcements is exogenously given by the modelers. We view our work as *endogenizing* the order of policy announcements.[2,3]

The mechanism that generates ambiguous policy announcements in our model is starkly different from those obtained in the existing literature. In the literature on ambiguous policies such as Shepsle (1972) and Aragonès and Postlewaite (2002), it is assumed that candidates choose their policy positions simultaneously once and for all.[4] Ambiguity obtains because voters are assumed to possess convex utility functions and hence they prefer uncertainty, that is, ambiguous policy announcements. On the other hand, ambiguity in our model arises from the dynamic strategic

[2]This answers the question posed by Aragonès and Palfrey (2002), where they ask "What is the correct sequential model"?

[3]Although we consider policy space consisting of two points in the base model, Section 5.3 discusses a continuous policy space comparable to Bernhardt and Ingberman (1985), Berger et al. (2000) and Aragonès and Palfrey (2002).

[4]We will discuss more papers on ambiguity in the literature review section.

interactions in an election campaign: each candidate's strategic concern about the opponent's future play causes ambiguity. Especially, we do not assume convexity; rather, in one of the variants of our model, we show that ambiguity obtains even with concavity.

Before going to the details, let us emphasize that we do not aim to provide a model and results that are definitive. Rather, we view them as suggestive. The whole objective is to formalize ambiguity as a result of valence and dynamic strategic considerations, and perhaps more importantly, to provide a basis for future research on dynamic campaigns by proposing a tractable model to analyze this and other issues.

The paper proceeds as follows. Section 1.1 reviews the literature that we have not yet discussed. In Section 2, we introduce the model of a policy announcement game. Section 3 analyzes the case when there is no campaign phase. Section 4 analyzes the dynamic model. In Subsection 4.1, we establish that if two candidates are perfectly symmetric then both candidates would want to be clear as soon as possible. In Subsection 4.2, we establish that if one candidate is slightly stronger than the other, then there are rich strategic considerations driving the incentive to announce ambiguous policies. Therefore, we identify that the key assumption for ambiguous policies is valence. In Section 5, we compare the dynamic model with the one-shot game, and also discuss other variants of the model such as those with heterogeneous arrival rates, generalized payoff structures, and synchronous announcements, to discuss the robustness of our ambiguity result to model specifications. Section 6 concludes. All the proofs not provided in the main text are provided in the Appendix.

### 1.1 Literature Review

In the Introduction we have already detailed existing work on models with valence candidates, and have mentioned a few papers on ambiguous policy announcements. Ambiguous policy announcements have received much attention in the literature, so in this section we refer to these papers and compare them with our work. We also explain how our model compares to a line of recent papers on revision games and related work.

***Ambiguity papers:***

Page (1976, 1978) propose a theory that attributes ambiguity to candidates' limited resources to make their policy positions precise, and voters' limited capability to understand these positions. In our model, however, voters do not have any inability to understand what the candidates are announcing. Candidates do have a positive probability of not being able to have any chance to make a policy announcement, but we obtain ambiguity even in the limit as this probability shrinks to zero.

Glazer (1990) argues that, if candidates do not have a control over which policy to specify when they intend to make a policy announcement, they may prefer being ambiguous.[5] Ambiguity obtains either when the policy space consists of unequally-dispersed points, or when the median voter is assumed to believe that a policy resulting from an ambiguous announcement is close enough to her bliss point, or when in a sequential announcement model each candidate has a private information about the position of the median so observing the opponent's position gives new information. None of these assumptions drives the conclusion in our model.

Alesina and Cukierman (1990) and Aragonès and Neeman (2000) show that ambiguity occurs if candidates prefer to be less constrained in office after being elected, whereas voters prefer commitment to precise stances. That is, the driving force of ambiguity is different from office-motivation. In contrast, we derive ambiguity from pure office-motivation.

When the selection of candidates consists of more than one step such as the US presidential election with primaries and general elections, Meirowitz (2005) shows that ambiguous policies in the earlier stages arise if voter preferences are unknown at the beginning but are revealed as a result of the earlier stages. In our model, no new information arrives about voter preferences and ambiguous policies are purely the result of the strategic interaction between candidates.[6]

In the base model, we find that ambiguity is likely when the probability distribution of the median voter's position is close to uniform. Although we view our results as suggestive, this result is consistent with the empirical finding by Campbel (1983), who suggests that opinion dispersion has a strong positive effect on ambiguity.

***Revision games and related works:***

To formally model the dynamics of policy announcements, we employ a framework with con-

[5]Glazer (1990) claims that this assumption is a result of uncertainty about the median voter's preferences. However, if we fix a policy space and a candidate's subjective belief about the distribution of the median voter, then a best response against a given opponent's policy position and a best response against an ambiguous policy would differ. Glazer does not consider this difference and simply considers a binary choice between being unambiguous and being ambiguous.

[6]Alesina and Holden (2008) show this result as well. They also show that if candidates have policy motivation and it is uncertain from the viewpoint of voters, and campaign contributions affect the electoral outcomes, then ambiguity is present even without the primaries.

tinuous time, finite horizon, and Poisson revision process. This modeling device is extensively explored recently: The revision games by Kamada and Kandori (2009), Calcagno and Lovo (2010), and Kamada and Sugaya (2010) consider settings where players obtain opportunities to revise their preparation of actions according to Poisson processes, and the finally-revised action profile is prepared at the predetermined deadline.[7] In these papers, revisions of actions are not restricted, in the sense that players can freely choose their actions from a fixed action space at each opportunity to move, as opposed to our assumption that once a candidate makes the policy platform clear, they cannot change it afterwards. The other difference is in the nature of the game analyzed: These papers analyze games in which cooperation and coordination are at issue, while we analyze a constant-sum game. This drives a difference in the effects of heterogeneity in arrival rates, which we discuss in Subsection 5.2.

As for the idea of being ambiguous in expectation of future events, Gale's (1995, 2001) model of "monotone games" also considers a similar problem. In his model, at each period, players can only (weakly) increase their actions. Thus, in effect, in each period players commit to a smaller and smaller subset of their action spaces, and they will never be able to "expand" that subset (thus the revisions are "restricted"). The main difference from our paper is that they analyze "games with positive spillover" and show that collusive outcomes can be achieved, while we analyze a constant sum game, thus his results are not applicable in our context.

## 2 The Model

There are two candidates, $S$ and $W$, interpreted as a "strong candidate" and a "weak candidate," respectively.[8] In the main body of the paper, the model is particularly simple, so as to highlight the complexity introduced by the election phase into an election model. Specifically, the policy platform consists of two points $X := \{0, 1\}$. Notice that this is the minimal environment in which we could potentially have strategic ambiguity. Section 5.3 presents a general version of the model that involves many other cases such as continuous policy space.

Time is continuous and flows from $-T$ to 0 where $T > 0$ is large. Imagine 0 is the fixed election date and the campaign starts at $-T$. For each $-t \in [-T, 0]$, according to the Poisson process

[7] Ambrus and Lu (2010) consider a bargaining model in this fashion.

[8] We will use a female pronoun for $S$ and a male pronoun for $W$.

with arrival rate $\lambda > 0$, each candidate obtains an opportunity to announce their "policy sets" from a subset of $X$. In this section, we assume the Poisson processes are independent between the candidates. This in particular implies that policy announcements are asynchronous. The case of synchronous announcements is discussed in Section 5.4.

The set of candidate $i$'s possible announcements at time $-t$ depends on $i$'s own past policy announcement: if $i$ has already announced $\{0\}$ in the past, then $i$ can only announce $\{0\}$; Similarly, if $i$ has already announced $\{1\}$ in the past, then $i$ can only announce $\{1\}$; However, if $i$ has announced only $\{0, 1\}$ in the past, then $i$ can announce either $\{0\}$, $\{1\}$, or $\{0, 1\}$. We let the policy set at time $-T$ be exogenously given to be $\{0, 1\}$. Let $X_i$ with $i \in \{S, W\}$ be candidate $i$'s most recently announced policy at time 0 (election date). Here, we interpret announcing $\{0, 1\}$ as announcing the "ambiguous policy" while announcing $\{0\}$ or $\{1\}$ is seen as specifying a policy platform. Thus, once the candidates specify their platforms, they cannot change it later. To simplify the exposition, we will occasionally call the act of announcing a policy set either $\{0\}$ or $\{1\}$ "enter."

Let us now specify voters' utility function and behavior rules. If a candidate $w \in \{S, W\}$ wins the election and implements policy $x \in \{0, 1\}$, a voter with position $y \in \{0, 1\}$ obtains the payoff of

$$u(|x - y|) + \delta \cdot \mathbb{I}_{w=S},$$

where $u(0) > u(1)$ and $0 \leq \delta < (u(0) - u(1))/2$. The voters believe that, if candidate $i$ who has specified a policy $x \in \{0, 1\}$, then $x$ is implemented. If candidate $i$ with the ambiguous policy $X_i = \{0, 1\}$ wins, then the voters believe the policies $\{0\}$ and $\{1\}$ are implemented with equal probability $\frac{1}{2}$.[9] The voters are sincere, that is, each voter votes for the candidate who, if elected, maximizes her expected payoff. The candidate with more votes wins (In the case of tie, each candidate wins with probability 1/2). Note that $\delta$ is a utility to have $S$ as a winner, that is, $S$ is stronger than $W$ by nature (valences) for $\delta > 0$.

The result of the election is determined by the voter distribution over the policy space $\{0, 1\}$ and $(X_S, X_W)$. During the campaign, the voter distribution is unknown but the distribution of the median voter is known to follow the probability mass function: $f(0) = p$, $f(1) = 1-p$, where $p < \frac{1}{2}$.

[9]The model is not a knife-edge case with respect to this assumption. At least for an open set of environments, our results are basically unchanged.

The environment just specified is the one where we can apply the median voter theorem, that is, without valence, the candidate who specifies the policy at the position of the median voter wins with a positive probability.[10] The candidate who obtains more votes wins, and obtains the payoff of 1, and the other candidate obtains the payoff of 0. Hence we are assuming purely office-motivated candidates. Each candidate's objective is to maximize the expected payoff, that is, their objective is to maximize the winning probability. We summarize the voters' behaviors and resulting expected payoffs for the candidates given these specifications in Figure 1.

| $(X_S, X_W)$ at date 0 | Voters at 0 vote for | Voters at 1 vote for | S's expected utility | W's expected utility |
|---|---|---|---|---|
| {0,1},{0,1} | S | S | 1 | 0 |
| {0,1},{0} | W | S | 1-p | p |
| {0,1},{1} | S | W | p | 1-p |
| {0},{0,1} | S | W | p | 1-p |
| {0},{0} | S | S | 1 | 0 |
| {0},{1} | S | W | p | 1-p |
| {1},{0,1} | W | S | 1-p | p |
| {1},{0} | W | S | 1-p | p |
| {1},{1} | S | S | 1 | 0 |

Figure 1: Voter behaviors and the expected payoffs

In what follows, we will analyze subgame perfect equilibria of this game.

## 3 The One-Shot Case

To better understand the incentive problems that candidates are facing, let us first demonstrate what would happen if our game were the one-shot simultaneous-move game, that is, $T = 0$. In this case, the game can be represented by the following payoff matrix: Calculating, it turns out that

| $S \setminus W$ | $\{0\}$ | $\{1\}$ | $\{0,1\}$ |
|---|---|---|---|
| $\{0\}$ | $1, 0$ | $p, 1-p$ | $p, 1-p$ |
| $\{1\}$ | $1-p, p$ | $1, 0$ | $1-p, p$ |
| $\{0,1\}$ | $1-p, p$ | $p, 1-p$ | $1, 0$ |

Figure 2: The one-shot game

[10] The voter located in a policy platform with a majority is a median voter.

there is a unique (mixed) Nash equilibrium in this game. In this equilibrium, $S$ and $W$ take $\{0\}$, $\{1\}$ and $\{0,1\}$ independently with probabilities

$$\left(\frac{p^2}{1-p+p^2}, \frac{(1-p)^2}{1-p+p^2}, \frac{p(1-p)}{1-p+p^2}\right) \quad \text{and} \quad \left(\frac{(1-p)^2}{1-p+p^2}, \frac{p^2}{1-p+p^2}, \frac{p(1-p)}{1-p+p^2}\right), \qquad (1)$$

respectively.

This means that *even without the dynamic considerations, candidates face the incentive to be ambiguous.* This would deserve an explanation. Notice first that in equilibrium clearly no candidate can use a pure strategy. So suppose that $S$ mixes between the two unambiguous policies while assigning zero probability to the ambiguous policy. In this case, $\{0,1\}$ dominates both $\{0\}$ and $\{1\}$ for $W$ because he can win whenever the median voter's position and the $S$'s position differ, and if they are the same he would loose for sure. If $W$ takes the ambiguous policy with a positive probability then it becomes also attractive for $S$ too, because if both candidates are ambiguous then $S$ wins for sure. This is the intuition for why both candidates assign positive probabilities to the ambiguous policy.

Thus, in a sense, our model predicts ambiguous policy even without a dynamic component. However, this is only a part of our story. What we will show in the main section (the next section) is that the candidates face complicated dynamic incentive problems in our policy announcement game. Specifically, *the candidates' incentives to announce ambiguous policies change over time.*

# 4 The Dynamic Case

Now we turn to the dynamic model. In the first subsection we consider the case of $\delta = 0$ as a benchmark case. It turns out that there are no strategic incentives to announce the ambiguous policy $\{0,1\}$. Then, in the second subsection, we consider the case of $\delta > 0$, and demonstrate that candidates face complicated strategic considerations that are absent in the model with $\delta = 0$.

## 4.1 The Benchmark Case: Perfectly Symmetric Candidates

Suppose that $\delta = 0$. The following proposition gives us a stark result.

**Proposition 1** *Suppose $\delta = 0$. Then, each candidate announces $\{1\}$ as soon as possible in equilibrium.*

**Proof.** Suppose that at any time $-s > -t$, if each candidate has an opportunity to enter, then he/she enters at 1. Then, at time $-t$, entering at 1 gives the payoff strictly greater than $\frac{1}{2}$, entering at 0 gives $p < \frac{1}{2}$, and not entering gives the payoff of $\frac{1}{2}$ by symmetry. Thus, entering at 1 is a strict best response. Therefore, by continuity of expected payoffs in probability, for sufficiently small $\varepsilon > 0$, it is strictly optimal to enter at 1 for all $-\tau \in (-t - \varepsilon, -t]$. This establishes the desired result.[11] ■

This negative result is very general. In particular, it is straightforward to check that the result holds also in the other versions of models that we will present in Section 5. Hence the assumption of $\delta > 0$ is the key for the ambiguous policy announcements. From the next subsection on, we will demonstrate that the above simple argument breaks down once we introduce asymmetry with respect to candidates' valence and candidates face complicated dynamic incentive problems.

## 4.2 The Cases with Valence Candidates

In this section, we demonstrate that if $\delta > 0$, then there are rich strategic considerations involved in equilibrium, which derives ambiguous policy announcements. Therefore, a small valence matters.

Let us start with the following lemma. It states that, if $S$ has an opportunity to enter after $W$ has entered at $x \in \{0, 1\}$, then $S$ enters at $x$ and wins for sure.[12] On the other hand, if $W$ has an opportunity to enter after $S$ has entered at $x \in \{0, 1\}$, then $W$ is indifferent between announcing $\{0, 1\}$ and entering at $x' \in \{0, 1\} \setminus \{x\}$. $W$ can win if and only if the median voter is at $x'$. Given this, since the median is more likely to be at $\{1\}$ ($p < \frac{1}{2}$), if a candidate enters before the opponent, he/she should enter at $\{1\}$.

**Lemma 3** *In equilibrium, the following is true:*

[11] We hope this is clear enough, but for those who wonder why this is sufficient, note that the following lemma is true:

**Lemma 2** *Suppose that for any $-t$, there exists $\varepsilon > 0$ such that statement $A_s$ is true for all $-s \in (-t - \varepsilon, -t]$ if statement $A_\tau$ is true for any $-\tau > -t$. Then for any $t$, statement $A_t$ is true.*

For completeness, in the Appendix we provide the proof for this which is taken from Calcagno, Kamada, Lovo, and Sugaya (2011).

[12] Remember that by "enter" we mean "clarify the policy," or "announce the policy $\{0\}$ or $\{1\}$."

1. *Given that* $W$ *has entered,* $S$ *enters at the same platform as soon as possible for all* $t$.

2. *Given that* $S$ *has entered,* $W$ *is indifferent between announcing* $\{0,1\}$ *and entering at the different platform for all* $t$.

3. *If a candidate* $i$ *enters before the opponent,* $i$ *enters at* $\{1\}$.

The above lemma pins down the equilibrium behaviors on and off the equilibrium path except when no candidates have yet entered. It also says that if both are still ambiguous and a candidate $i$ enters, then $i$ enters at $\{1\}$. Hence in the following analysis, we consider the incentives to enter at $\{1\}$ in such a situation.

Before presenting the characterization of the equilibrium, let us provide the intuition for our result. For the time being, consider the case with $p = \frac{1}{2}$, which is actually outside of the model (Remember that we set $p < \frac{1}{2}$). Suppose that at time $-t$, both $S$ and $W$ have previously announced $\{0,1\}$. If there is no further revision, $W$'s payoff is 0. So $W$ needs to specify his policy to obtain a positive payoff. Thus $W$ announces $\{0\}$ or $\{1\}$ at some point, if he can. Since $\{0\}$ and $\{1\}$ are symmetric, assume without loss of generality that $W$ announces $\{1\}$ when he clarifies his policy.

On the other hand, $S$ does not have an incentive to specify her policy until $W$ specifies his policy since she gets $\frac{1}{2}$ for sure by specifying her policy while being ambiguous gives her 1 if $W$ cannot enter in future and $\frac{1}{2}$ if $W$ can enter. But after $W$'s announcement, $S$ tries to copy $W$'s choice as soon as possible, which gives $S$ ($W$, respectively) the payoff of 1 (0, respectively).

If $W$ announces $\{1\}$ in an early stage of the campaign, then the probability that $S$ enters afterwards is high. So $W$ wants to defer. But waiting too much is not optimal for $W$ either, since if he does not have a chance to revise his policy set, $W$ gets the payoff 0. So there should exist a "cutoff," $-t^*$, until which $W$ announces $\{0,1\}$ and after which $W$ announces $\{1\}$ when he gets a revision opportunity.

Remember that we did not have this type of strategic considerations when $\delta = 0$. The point where the simple argument in the proof of Proposition 1 breaks down is that the continuation payoff from taking each action is different once we introduce valence. For example, $W$ expects a payoff close to zero if she specifies some policy when the deadline is far away, as opposed to the payoff strictly above $\frac{1}{2}$.

Next, consider the case with $p = 0$. In this case, $S$ would want to commit to $\{1\}$ as soon as possible, because then she can obtain the payoff of 1, which is the highest possible payoff.

The next proposition fully characterizes the form of equilibrium for each $p \in (0, \frac{1}{2})$. The equilibrium strategy of $W$ is to wait until a finite cutoff and to enter as soon as possible after that cutoff. The equilibrium strategy for $S$ depends on the value of $p$. If $p$ is close to $\frac{1}{2}$ (Case 1 of Proposition 4), $S$ does not enter until $W$ enters by the same reason as for $p = \frac{1}{2}$. On the other hand, for small $p$ (Case 2 of Proposition 4), $S$ enters when the deadline is far away while she does not enter when the deadline is close. The intuition for this ambiguity near the deadline is as follows: if $S$ obtains an opportunity at $-t$ when the deadline is close, then the probability that $W$ has a chance to announce his policy afterwards is small. So it is likely that $W$ is ambiguous at the deadline. Thus, staying ambiguous is good for $S$, because by doing so, $S$ gets 1 with a high probability.

**Proposition 4** *Suppose that the previous policy sets are both $\{0, 1\}$ at time $-t$. Then, there exists a unique equilibrium. In this equilibrium, the following is satisfied:*

1. *If $p < \frac{1}{1+e}$, then there exists $t_S$ and $t_W$ such that the following holds:*

    (a) *For $-t < -t_S$, $S$ announces $\{1\}$ as soon as possible and for $-t > -t_S$, she announces $\{0, 1\}$.*

    (b) *For $-t < -t_W$, $W$ announces $\{0, 1\}$ and for $-t > -t_W$, he announces $\{1\}$ as soon as possible.*

2. *Otherwise, the following holds:*

    (a) *For all $-t$, $S$ always announces $\{0, 1\}$.*

    (b) *For $-t < -t^* = \frac{1}{\lambda}$, $W$ announces $\{0, 1\}$ and for $-t > -t^*$, he announces $\{1\}$ as soon as possible.*

In Figure 3, we depict the values of $t_S$ and $t_W$ for various values of $p$. In this figure, we depict $t_S$ and $t_W$ that appear in Proposition 4 for different values of $p$. For example, $p = .2$ corresponds to part 1 of the proposition. In this case, there are two points that the curves in the figure intersects with $p = .2$ line, so as a result the time spectrum is divided in three regions: In the left-most region,

$S$ enters while $W$ does not enter. In the middle region, both candidates enter. Finally, in the right-most region, $S$ does not enter while $W$ enters. When $p = .4$, there is only once intersection, and and as a result the time spectrum is divided into two regions: In the left region, no candidate enters. In the right region, $S$ does not enter while $W$ enters.

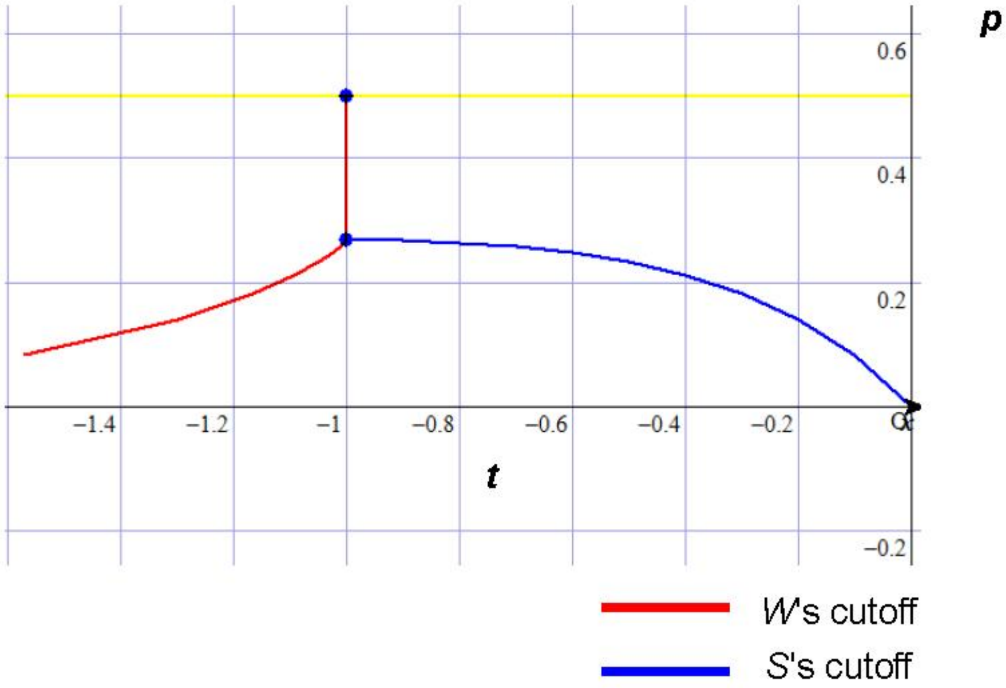



Figure 3: Cutoffs for the base model

Notice that this particular model predicts that when the distribution of voters is ex ante very skewed ($p$ is very small), $S$ enters as soon as possible, so if $T$ is large then there would be almost no ambiguity in equilibrium. This hinges on our assumption that even if $W$ enters after $S$, $S$ does not incur any loss. In Section 5.3, we show that if there is a small loss then $S$ prefers to become ambiguous until some point in time that does not depend on the horizon length $T$, hence the model is consistent with ambiguity even if the distribution of voters is ex ante very skewed.

# 5 Discussions

This section provides discussions of our model. In the first subsection we compare the outcome of our dynamic game with the case of one-shot game analyzed in Section 3, and discuss how the addition of the campaign phase changes the likelihood of eventual ambiguity and the welfare of candidates.

The model in the main section was kept as simple as possible to highlight the complexity added by the fact that candidates face dynamic incentive problems. In the remaining three subsections (5.2-5.4), we extend and modify this model in various directions, to examine the robustness of our prediction that candidates use ambiguous language at early stages of the campaign, and also to discuss the new implications that arise in respective models.

## 5.1 The One-Shot Game versus the Dynamic Game

Let us compare the ex ante probability distribution of the policy profile in our model with that in a one-shot simultaneous-move game with the same payoff structure. Given Proposition 4, the limit ex ante distribution of the policy profile at the election date and expected payoffs as $T \to \infty$ is calculated as follows:

1. If $p > \frac{1}{1+e}$, then in the limit $S$ enters first. In this case $W$ is indifferent between entering and not entering. The expected payoffs are $(1-p, p)$.[13]

2. Otherwise, $W$ announces $\{1\}$ after $-t = -\frac{1}{\lambda}$ and $S$ tries to copy $W$'s policy after $W$ enters. Hence, there are following three cases:

   (a) $W$ cannot enter and $(\{0,1\}, \{0,1\})$ is realized with probability $e^{-\lambda t^*} = e^{-1}$. The payoffs in this case are $(1, 0)$.

   (b) At $-t^*$, $W$ is indifferent. Hence $W$'s expected payoff is equal to $1-p$ times the probability that $S$ cannot enter after $-t^*$, which is $(1-p) \cdot e^{-\lambda t^*}$. In equilibrium, $W$ gets a positive payoff of $1-p$ only when she enters and $S$ cannot enter afterwards. The probability of this event is thus $\frac{(1-p) \cdot e^{-\lambda t^*}}{1-p} = e^{-\lambda t^*} e^{-1}$.

   (c) With the remaining probability $1 - 2e^{-\lambda t^*} = 1 - 2e^{-1}$, both candidates enter.

[13] The first component denotes $S$'s payoff, and the second component denotes $W$'s payoff.

Overall, the probability distribution over outcomes at the election date is

$$(\Pr(\{0,1\},\{0,1\}),\Pr(\{1\},\cdot),\Pr(\{1\},\{1\})) = (e^{-1},e^{-1},1-2e^{-1}).$$

Thus, in equilibrium, the probability that each candidate takes the ambiguous policy is at least $e^{-1}$. We note that this probability is higher than the probability assigned to $\{0,1\}$ in the one-shot case, which is $\frac{p(1-p)}{1-p+p^2}$ for any $p$. That is, we find that ambiguity is more likely in the dynamic game. The basic intuition for this result is that in the dynamic game the only occasion where $S$ becomes unambiguous is when $W$ has already specified his policy, and $W$ tries to minimize the probability for such an occasion. This is why ambiguity is likely in the dynamic game.

Notice that the probability distribution over outcomes at the election date corresponds to correlated (mixed) strategy profile. This is because the sequential nature of the game serves as the correlation device. On the other hand, we have seen in Section 3 that in the unique Nash equilibrium in the one-shot simultaneous-move game, the strategies are given in (1). Therefore, the ex ante distribution of the realized policy platform is different between our model and a one-shot simultaneous-move game.

Now let us move on to the analysis of expected payoffs. In the one shot game, the expected payoff profile is

$$\left(\frac{1-p}{1-p+p^2},\frac{p^2}{1-p+p^2}\right).$$

In the dynamic game, the expected payoff profile is

$$\left(1-(1-p)e^{-1},(1-p)e^{-1}\right).$$

Figure 4 graphs $S$'s expected payoffs for different values of $p$. Notice that the payoff in the one shot game is decreasing in $p$, while the payoff in the dynamic game takes its minimum at $p=\frac{1}{1+e}$. The latter exceeds the former when $p$ is sufficiently large. This can be explained as follows: When $p$ is small, $S$ gives up on matching with $W$, and simply go for the policy $\{1\}$ which guarantees him the payoff of $1-p$. In the one-shot game however, the equilibrium is mixing as we have seen in Section 3, and thus $S$ must be getting the payoff greater than $1-p$ which he can guarantee by taking policy $\{1\}$. This is why when $p<\frac{1}{1+e}$, $S$ is worse off in the dynamic game. However, when $p$ becomes large, the value of committing to $\{1\}$ become small, and hence $S$ tries to match $W$ in the dynamic game. In the dynamic game, the only case where $S$ looses is when he cannot enter after $W$'s entry, and the entry payoff for $W$, which is $1-p$, is deceasing in $p$. Since the probabilities for each policy set profile is independent of $p$ in this region (as $W$ faces a binary choice problem between payoff $1-p$ and 0), $S$'s payoff is increasing in $p$ in this region. Eventually, his payoff exceeds that of the one-shot case.

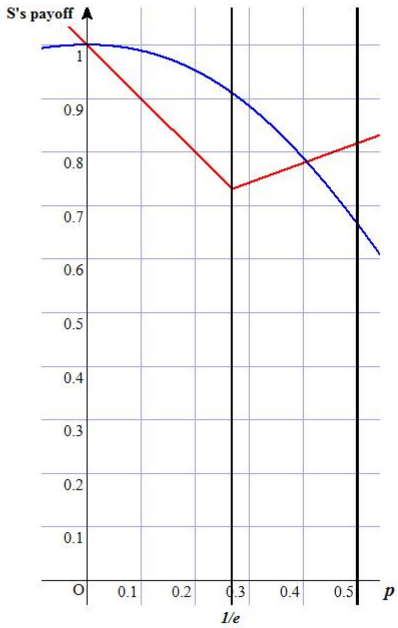



Figure 4: $S$'s expected payoffs in the one-shot game and the dynamic game (blue: $S$'s expected payoff in the one-shot game; red: $S$'s expected payoff in the dynamic game)

We summarize the findings so far in the following proposition:

**Proposition 5** *1. If $p > \frac{1}{1+e}$, the probability of ambiguity is greater in the dynamic game than in the one-shot game.*

*2. There exists $p^* \in (\frac{1}{1+e}, \frac{1}{2})$ such that $S$ is strictly better off in the dynamic game than in the one-shot game if $p > p^*$, and she is strictly worse off if $p < p^*$.*

## 5.2 Heterogeneous Arrival Rates

In this section we discuss the effect of heterogeneous arrival rates. In this section we let the arrival rate for candidate $i$ be $\lambda_i$, and allow for the possibility that $\lambda_S \neq \lambda_W$.

First, it is straightforward to calculate that the basic structure of the equilibrium does not change even if $\lambda_S \neq \lambda_W$: The equilibrium behaviors after some candidate has already entered are the same as before. When currently both candidates are announcing the ambiguous policy, there exist $p^*$ and $t^*$ such that if $p > p^*$, then $W$ enters if and only if if $-t \leq -t^*$, and $S$ never enters. If $p < p^*$, $S$ enters as soon as possible until some cutoff, and $W$ enters after another cutoff, where the former comes after the latter in time.

$p^*$ can be calculated as

$$p^* = \frac{\left(\frac{\lambda_S}{\lambda_W}\right)^{-\frac{\lambda_S}{\lambda_S - \lambda_W}}}{1 + \left(\frac{\lambda_S}{\lambda_W}\right)^{-\frac{\lambda_S}{\lambda_S - \lambda_W}}},$$

and the expected payoffs when $p > p^*$ are

$$\left(1 - \left(\frac{\lambda_S}{\lambda_W}\right)^{-\frac{\lambda_S}{\lambda_S - \lambda_W}}, \left(\frac{\lambda_S}{\lambda_W}\right)^{-\frac{\lambda_S}{\lambda_S - \lambda_W}}\right).$$

Since $\left(\frac{\lambda_S}{\lambda_W}\right)^{-\frac{\lambda_S}{\lambda_S - \lambda_W}}$ is decreasing in $\frac{\lambda_S}{\lambda_W}$, $p^*$ is decreasing in $\frac{\lambda_S}{\lambda_W}$ and $S$'s payoff is increasing in $\frac{\lambda_S}{\lambda_W}$. Thus, having relatively a higher arrival rate makes the candidate better off. This is intuitive: If $S$ has a higher arrival rate, then (fixing $W$'s strategy,) she has a greater chance to match the $W$'s position. If $W$ has a higher arrival rate, then she can wait more at the policy profile $\{0, 1\}$ to reduce the probability of being matched afterwards.

Note that Calcagno and Lovo (2010) and Kamada and Sugaya (2010) have shown that having a higher arrival rate makes the player worse off, as it would decrease the commitment power. The difference is in the nature of the stage game being analyzed. These papers analyze coordination games, so committing to an action can help inducing the other player to accord with the player, while in this paper the game is a constant sum gam, so being unable to change an action for a longer time just means that the player suffers a low payoff with a larger probability.

### 5.3 A Generalized Model

The simple model presented in Section 2 was intended to provide a basic intuition for the dynamic incentive problems faced by candidates. This section extends this base model to more general cases. In particular, we assume that everything is the same as in the base model, while the payoff functions are different: ($S$'s payoff, $W$'s payoff)=

$$(\alpha, 1-\alpha) \quad \text{if only } W \text{ enters;}$$

$$(1-\beta, \beta) \quad \text{if only } S \text{ enters;}$$

$$(1-\gamma, \gamma) \quad \text{if } S \text{ enters and then } W \text{ enters.}$$

Note that the assumptions that we are making is that the position in the policy space that each candidate enters does not depend on the timing of entry (and this is the only restriction that we are imposing). This assumption was satisfied in our base model. Moreover, the specification fits to other cases as well. For example, this general model can be applied to the case of continuous policy space, the model that the literature on elections often considers. Specifically, the set of possible policy announcements is $\{x\}_{x\in[0,1]} \cup [0,1]$. That is, we allow the candidates to announce a specific policy $x \in [0,1]$ or an ambiguous policy $[0,1]$. Analogously to the base model, the policy set at time $-T$ is $[0,1]$. If a candidate $w$ wins the election and implements policy $x \in \{0,1\}$, the voter's utility with position $y \in [0,1]$ is defined as $u(x,y) + \delta \cdot \mathbb{I}_{w=S}$. If a candidate with the ambiguous policy $[0,1]$ wins, the voters believe the candidate implements the policies in $[0,1]$ according to the uniform distribution. Hence, the expected payoff is $\int_0^1 u(x,y)dx + \delta \cdot \mathbb{I}_{w=S}$. $u$ is strictly concave with respect to $x$ (the voters are risk averse). Again, we assume that the valence term is $\delta > 0$, but is very small so that $W$ at $\frac{1}{2}$ beats $S$ with the ambiguous policy.[14, 15]

In this class of model, we obtain the following:

**Proposition 6** *If $\beta < \gamma$, then for any $p$, there exists $t_W$, $t_S^1$, and $t_S^2$ such that $t_S^2 \geq t_S^1 \geq 0$, and when the current policy set profile is $(\{0,1\},\{0,1\})$,*

1. *$W$ does not enter if $-t < -t_W$ and he enters if $-t_W < -t$.*

[14]Specifically, $\int_0^1 u(x,y)dx + \delta < u(\frac{1}{2}, y)$ for all $y$. Note that such $\delta > 0$ exists by the strict concavity of $u$.

[15]As we mentioned in Introduction, if we assume strict convexity, ambiguity does not need valence: If candidates are symmetric, it is optimal for a candidate to announce $[0,1]$ when the opponent is announcing $\{\frac{1}{2}\}$.

*2. $S$ does not enter if $-t < -t_S^2$, she enters if $-t_S^2 < -t < -t_S^1$, and she does not enter if $-t_S^1 < -t$.*[16]

That is, for a sufficiently long election campaign phase, candidates use ambiguous language for most of the time. Notice that in the base model, $\beta = \gamma$, and if $p$ is very small then $S$ enters as soon as possible. Thus what the proposition claims is that if $S$ expects even a slightest cost of being entered by $W$ after her own entry, then she does not wish to enter as soon as possible. Remember that the model includes the case with continuous policy space with concave payoff function. Thus the proposition implies that the essence of our result is orthogonal to the convexity of payoff function, which is analyzed in Shepsle (1972) and Aragonès and Postlewaite (2002).

### 5.4 Synchronous Policy Announcement

So far we have assumed that candidates' policy announcements are asynchronous. In practice, not all the announcements are asynchronous: For example, the televised debates would be better modeled as synchronous policy announcements. To understand the role of the move structure on our ambiguity result, in this section we consider the case where all the opportunities are synchronous. That is, time flows from $-T$ to 0 and according to the Poisson process with arrival rate $\lambda$, both of the candidates receive the opportunity to announce their policy platforms simultaneously.[17] We will show that the ambiguous policy announcements are robust to this setting. The vary basic intuition— $S$ wants to wait for $W$ who does not want to be copied— is the same as in the base model, but the detailed equilibrium structure is completely different. In particular, candidates use mixed strategies at any time point close to the election date.

We assume the same voter's utility and the distribution of the median voter as in the original model explained in Section 2. For sufficiently small valence, the payoffs at the deadline 0 is given by the payoff matrix in Figure 2.

In this model, it is straightforward to see that parts 1 and 2 of Lemma 3 continue to hold. Therefore, only the relevant state is that no player has entered so far. Let $V_t^i$ be the value of candidate $i$ when no one has yet entered at $-t$ and an opportunity to enter arrives at $-t$. Suppose both of them do not enter at $-t$. Then, if they have an opportunity at $-\tau > -t$, then they will

[16] A proof of this proposition is somewhat complicated, and is available upon request from the authors.

[17] See Section 6 for the discussion of the other choices of modeling.

get $(V_\tau^S, V_\tau^W)$. Otherwise, $\{0,1\},\{0,1\}$ is realized at 0 and they get $(1,0)$. Hence, the values of choosing $\{0,1\},\{0,1\}$ at $-t$ is

$$\left(\int_0^t \lambda e^{-\lambda\tau} V_{t-\tau}^S d\tau + e^{-\lambda t}, \int_0^t \lambda e^{-\lambda\tau} V_{t-\tau}^W d\tau\right).$$

For other action profiles, parts 1 and 2 of Lemma 3 pin down the values. As in the base model, the game has a constant sum as the winning probabilities must sum up to 1, so it suffices to keep track of $S$'s payoffs. Specifically, when the candidates have an opportunity at $-t$, $S$'s payoffs for the choices of policy platforms are given by

| $S \setminus W$ | $\{0\}$ | $\{1\}$ | $\{0,1\}$ |
|---|---|---|---|
| $\{0\}$ | $1$ | $p$ | $p$ |
| $\{1\}$ | $1-p$ | $1$ | $1-p$ |
| $\{0,1\}$ | $1-pe^{-\lambda t}$ | $1-(1-p)\,e^{-\lambda t}$ | $\int_0^t e^{-\lambda\tau}\lambda V_{t-\tau}^S d\tau + e^{-\lambda t}$ |

and $V_t^S$ is the unique minmax value of this constant-sum game.

Unfortunately, a complete characterization of the equilibria for all parameter values is hard to obtain. However, the next proposition provides two analytical results.

**Proposition 7** 1. *There exists $t^* > 0$ such that for all time $-t \in (-t^*, 0]$, both candidates use completely mixed strategies conditional on the event that the opponent has not entered.*

2. *There exists $t^{**}$ such that for all $-t < -t^*$, the probability that a candidate enters at $\{0\}$ or $\{1\}$ conditional on the event that the opponent has not entered is 0.*

Part 1 of the proposition claims that if the election date is close, both candidates have to mix. This is in a stark difference from the asynchronous case, but is a natural consequence of the game representation as in the game above. The expected payoffs approach the ones in the original payoff matrix, and by the upper hemi-continuity of Nash equilibria, the result holds.

Part 2 shows the robustness of our ambiguity result with respect to the move structure. The intuition is the same as before. If $W$ enters for $-t$ sufficiently far from the election date with positive probability, it is optimal for $S$ to wait and try to copy $W$'s policy later. Given this, $W$ does not enter. $S$ gains a lot by copying $W$'s policy, so she experiences an option value of waiting

As part 1 shows, the equilibrium involves mixing when the election date is close. The mixing probabilities have to change over time, as the Nash equilibrium of the game matrix above changes as $t$ changes. The transition of mixing probabilities are complicated. We illustrate its complexity in the numerical results for $p = 0.45$ and $\lambda = 1$. This example illuminates the subtle incentive problems faced by the two candidates.

The value when $S$ takes $\{0,1\}$ at $-t$ is as depicted in Figure 5:

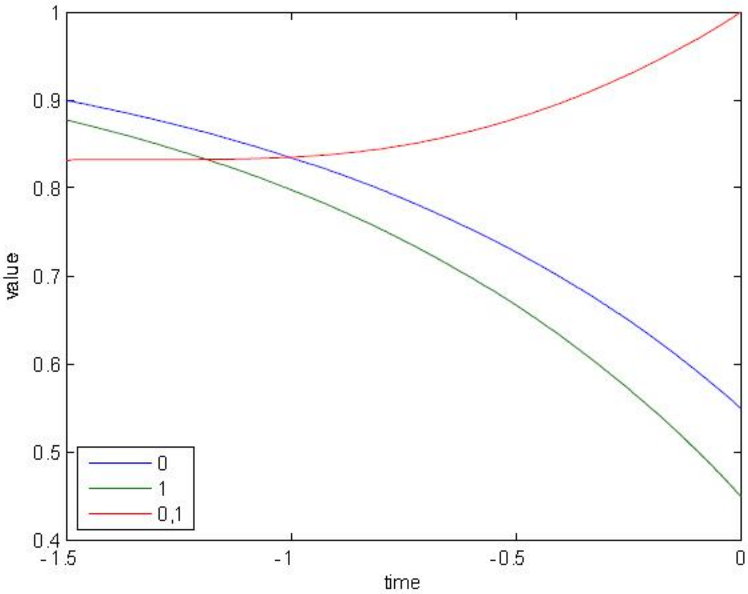
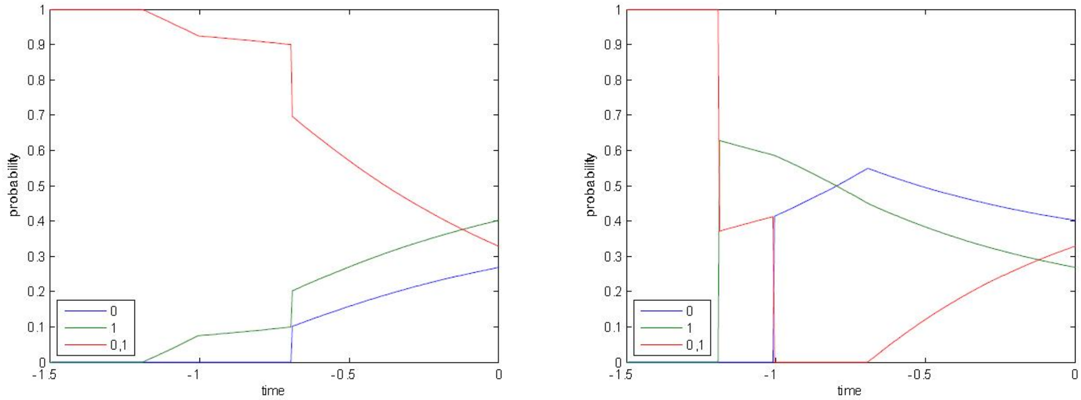



Figure 5: The value $V_t^S$ in the synchronous model

Note that the payoffs for $S$ at $(\{0,1\},\{0\})$ and $(\{0,1\},\{1\})$ at $-t$ increase as $t$ increases since the probability that $S$ can enter afterward and copy $W$'s policy increases. On the other hand, the payoff for $S$ at $(\{0,1\},\{0,1\})$ at $-t$ decreases since the weight for the highest payoff 1 decreases.

Figure 6 depicts $S$'s and $W$'s strategies respectively at $-t$. When $-t$ is sufficiently close to 0, each candidate mixes all the announcements, as we stated in part 1 of Proposition 7. $W$'s value at $(\{0,1\},\{0,1\})$ increases as the election date becomes far away, hence $\{0,1\}$ becomes more attractive to $W$. To reduce the incentive for $W$ to take $\{0,1\}$, $S$ needs to take $\{0,1\}$ with a higher probability as the deadline becomes further. On the other hand, $W$'s probability of $\{0\}$ and $\{1\}$ increases as $t$ becomes larger. This is because $S$'s value of taking $\{0,1\}$ increases as $t$ gets larger, so to incentivize




$S$'s strategy      $W$'s strategy

Figure 6: Strategies in the synchronous model

$S$ to take $\{0\}$ or $\{1\}$, $W$ needs to increase the probability of taking $\{0\}$ and $\{1\}$ (remember that $S$'s payoff is highest at $(\{0\},\{0\})$ and $(\{1\},\{1\})$).

However, around $-t = -0.7$, $\{0,1\}$ becomes too attractive for $S$ and $W$ cannot make $S$ indifferent for all the actions. Specifically, $\{0\}$ becomes less attractive than $\{0,1\}$ for $S$ in equilibrium. Note that it is less likely for the median voter to be at $\{0\}$. Conditional on that $S$ does not take $\{0\}$, $\{0,1\}$ is weakly dominated by $\{0\}$ for $W$. Hence, $W$ stops taking $\{0,1\}$.

At $-t = -1$, $S$'s value at $(\{0,1\},\{0,1\})$ becomes less than her value at $(\{0\},\{0,1\})$, which means $W$'s value at $(\{0,1\},\{0,1\})$ becomes more than his value at $(\{0\},\{0,1\})$. Then, $\{0\}$ is weakly dominated by $\{0,1\}$ for $W$ and $W$ stops taking $\{0\}$. Then, for $S$, $\{0\}$ is strictly dominated by $\{1\}$.

Finally, $S$'s value at $(\{0,1\},\{0,1\})$ becomes less than her value at $(\{1\},\{0,1\})$, which means $W$'s value at $(\{0,1\},\{0,1\})$ becomes more than his value at $(\{1\},\{0,1\})$. Then, $\{1\}$ is also weakly dominated by $\{0,1\}$ for $W$ and $W$ stops taking either $\{0\}$ or $\{1\}$. Then, for $S$, $\{0,1\}$ becomes the optimal policy. Hence, when the deadline is sufficiently far away, both candidates take the ambiguous policy $\{0,1\}$.

# 6 Conclusion

We proposed a model of a "policy announcement game" in which candidates stochastically obtain opportunities to announce their policies. We showed that, if two candidates are perfectly symmetric, they specify their policy positions as soon as possible. On the other hand, if one candidate is slightly stronger than the other, both candidates may have incentives to defer a clear announcement of their policies, depending on the opponent's current announcement and the time left until the election.

We have introduced the first model of dynamic campaigns into the literature on election by analyzing one particular simple setting and demonstrated that candidates face nontrivial dynamic incentive problems. Our work raises a wide range of new theoretical questions. Here we mention a few of them. First, we restricted ourselves to the case where policies are either perfectly ambiguous or perfectly precise. One could allow for "intermediate language" (e.g. let the candidates choose any subintervals of $[0, 1]$) and analyze how gradually candidates resolve their ambiguity over the course of the campaigns. Second, it would be more realistic if one assumes that policy announcements are sometimes synchronous and sometimes asynchronous. We conjecture that the analysis in such a model would not be just a "convex combination" of the two cases, as Ishii and Kamada (2011) show in their analysis of revision games with synchronous and asynchronous revisions. Third, we restricted ourselves to the case where once a candidate commits to a particular policy, he/she cannot change it later. Although we believe this is a reasonable starting point for analysis, one could assume that candidates can change their policies while they must incur "reputational cost" for announcing "inconsistent" policies. The idea is that if a candidate changes his opinion frequently, voters would infer that it is likely that the candidate would change his policy even after the election. Fourth, it would be interesting to enrich the model by assuming that the median voter's position becomes more and more precise over the course of campaigns, so that candidates have another reason to wait.

Finally, our work raises empirical questions as well. First, our model predicts different patterns of the timing of policy clarification for different parameter values such as $p$, which measures how much uncertainty candidates face with respect to the position of the median voters. One may want to test if this prediction is valid in the data.[18] Second, in our analysis we have essentially assumed

[18] As mentioned in the introduction, this pattern is consistent with the data in Campbell (1983), but it is of interest to test our model using more recent data.

that $\lambda T$ is large so that candidates have sufficiently many chances to announce their policies. It is desirable to examine if this assumption is correct in real election campaigns.

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# 7 Appendix

## 7.1 Proof of Lemma 2

**Proof.** Suppose that the premise of the lemma holds. Let $-t^*$ be the supremum of $-t$ such that $A_t$ is false. If $t^* = \infty$, we are done. So suppose that $t^* > \infty$. Then it must be the case that for any $\varepsilon > 0$ there exists $-\tau \in (-t^* - \varepsilon, -t^*]$ such that $A_\tau$ is false. But by the definition of $t^*$, there exists $\tilde{\varepsilon} > 0$ such that statement $A_\tau$ is true for all $-\tau \in (-t^* - \tilde{\varepsilon}, -t^*]$ because the premise of the lemma is true. Contradiction. ■

### 7.2 Proof of Proposition 4

For notational convenience, let $\theta_t = (x_S, x_W)$ with $x_S, x_W \subset \{0,1\}$ be the "state" at time $-t$ where $x_i$ is the policy announced by player $i$ most recently at $-t$.

From Lemma 3, it is always true that

- if $\theta_t = (x, \{0,1\})$ with $x \in \{0,1\}$ and $W$ can move, then $W$ is indifferent between entering at $x' \in \{0,1\}$ with $x' \neq x$ and announcing $\{0,1\}$. $S$ wins if and only if the median is located at $x$.

- if $\theta_t = (\{0,1\}, x)$ with $x \in \{0,1\}$ and $S$ can move, then $S$ should enter at $x$ as soon as possible. $S$ will win for sure.

When $-t$ is sufficiently close to the deadline 0, the following is true: at $\theta_t = (\{0,1\}, \{0,1\})$,

- if $W$ can move, then $W$ should enter at 1. Note that, since $-t$ is sufficiently close to 0, with high probability, there is no more opportunity to announce the policy. Hence, $\{1\}$ gives $W$ the payoff close to $p$, $\{0\}$ gives $W$ the payoff close to $1-p$, and $\{0,1\}$ gives $W$ the payoff close to 0. $S$ wins if and only if the median voter is located at 0.

- if $S$ can move, then $S$ should not enter. Note that, since $-t$ is sufficiently close to 0, with high probability, there is no more opportunity to announce the policy. Hence, $\{1\}$ gives $S$ the payoff close to $p$, $\{0\}$ gives $S$ the payoff close to $1-p$, and $\{0,1\}$ gives $S$ the payoff close to 1.

Let $V_i(t, \theta)$ be candidate $i$'s value when the state is $\theta$ at time $-t$. Note that it is always true that $V_W(t,\theta) = 1 - V_S(t,\theta)$. Hence, it suffices to consider $V_S(t,\theta)$. From above, for $-t$ sufficiently close to 0,

$$\begin{aligned}
\dot{V}_S(t, \{0,1\}, \{0,1\}) &= \lambda(V_S(t, \{0,1\}, \{1\}) - V_S(t, \{0,1\}, \{0,1\})),\\
\dot{V}_S(t, \{0,1\}, \{1\}) &= \lambda(1 - V_S(t, \{0,1\}, \{1\})),
\end{aligned}$$

which implies

$$\begin{aligned} V_S(t,\{0,1\},\{0,1\}) &= 1-(1-p)\,\lambda t\exp(-\lambda t), \\ V_S(t,\{0,1\},\{1\}) &= 1-(1-p)\exp(-\lambda t). \end{aligned}$$

Subtracting these from 1 yields

$$\begin{aligned} V_W(t,\{0,1\},\{0,1\}) &= (1-p)\,\lambda t\exp(-\lambda t), \\ V_W(t,\{0,1\},\{1\}) &= (1-p)\exp(-\lambda t). \end{aligned}$$

Given the above value function, at $\theta_t = (\{0,1\},\{0,1\})$, assuming that the candidate strategy is as we stated above for $-\hat{t} > -t$,

- if $W$ can move, then $W$ should enter at 1 as long as

$$V_W(t,\{0,1\},\{0,1\}) < V_W(t,\{0,1\},\{1\}) \Leftrightarrow t < \frac{1}{\lambda}.$$

- if $\theta_t = (\{0,1\},\{0,1\})$ and $S$ can move, then $S$ should not enter as long as

$$\begin{aligned} V_S(t,\{0,1\},\{0,1\}) &> \text{the value of entering at } \{1\} \\ &\Leftrightarrow 1-(1-p)\,\lambda t\exp(-\lambda t) > 1-p \\ &\Leftrightarrow 1 > \frac{p}{1-p} > \lambda t\exp(-\lambda t). \end{aligned}$$

There are following three cases:

**Case (1): there exists $t_S$ such that $\frac{p}{1-p} = \lambda t_S\exp(-\lambda t_S)$ and $t_S < \frac{1}{\lambda}$** In this case, at $-t_S$, $S$ becomes indifferent between entering at 1 and announcing $\{0,1\}$. By continuity, for $-t < -t_S$ sufficiently close to $-t_S$, at $\theta_t = (\{0,1\},\{0,1\})$, if $W$ can move, then $W$ strictly prefers entering at 1.

Let us now consider the candidates' incentive for $-t < -t_S$. The payoff of $S$ entering at 1 and ensuring $1-p$ is always strictly greater than the payoff of announcing $\{0,1\}$. To see why, suppose $S$ announces $\{0,1\}$ at $-t$. There are following cases:

1. if $W$ can move next by $-t_S$, then $W$ can always announce $\{0,1\}$. There are two subcases: if $S$ enters at $\{1\}$ by $-t_S$, $W$ gets $p$. If $S$ does not enter by $-t_S$, by definition of $-t_S$, $S$ gets $1-p$ and $W$ gets $p$. In both cases, $W$ gets at least $p$. Further, if $W$ can get a revision opportunity close to $-t_S$, $W$ gets more than $p$ since $W$ strictly prefers entering at $\{1\}$ to announcing $\{0,1\}$. Overall, $W$ gets more than $p$, which means $S$ gets less than $1-p$.

2. if $S$ can move next by $-t_S$, $S$ enters and gets $1-p$.

3. if no candidate can move by $-t_S$, then by definition, $S$ gets $1-p$.

Therefore, the value of announcing $\{0,1\}$ is strictly less than $1-p$. Note that we do not put any restriction on $W$'s strategy.

On the other hand, given $S$'s strategy above, the payoff of $W$ entering at 1 monotonically decreases if $-t$ becomes smaller since $W$ can get a positive payoff if and only if $S$ cannot have a revision opportunity after $-t$ and the probability of $S$ not having an opportunity monotonically decreases. Hence, there exists $t_W$ such that, at $-t_W$, $W$ is indifferent between entering $\{1\}$ and announcing $\{0,1\}$. For $-t < -t_W$, by the same monotonicity, $W$ always prefers $\{0,1\}$.

**Case (2):** $\frac{p}{1-p} > \exp(-1)$ Let $t^*$ be $\frac{1}{\lambda}$. Note that $W$ becomes indifferent between entering at 1 and announcing $\{0,1\}$ at $-t^*$. By continuity, for $-t < -t^*$ sufficiently close to $-t^*$, at $\theta_t = (\{0,1\},\{0,1\})$,

- if $W$ can move, then $W$ should announce $\{0,1\}$.
- if $S$ can move, then $S$ should announce $\{0,1\}$.

Given the strategy above, the value $(\{0,1\},\{0,1\})$ does not change until $-t^*$. For $S$, the payoff of entering 1 is also constant at $1-p$. For $W$, the payoff of entering at 1 monotonically decreases as $-t$ decreases since $W$ can get a positive payoff if and only if $S$ cannot have a revision opportunity after $-t$ and the probability of $S$ not having an opportunity monotonically decreases. Hence, there is no candidate who changes the strategy before $-t^*$.

**Case (3):** $\frac{p}{1-p} = \exp(-1)$ At $\theta_{t^*} = (\{0,1\},\{0,1\})$ with $t^* = \frac{1}{\lambda}$, $S$ is indifferent between entering at $\{1\}$ and ensuring $1-p$ and announcing $\{0,1\}$. At the same time, $W$ is indifferent between entering at $\{1\}$ and $\{0,1\}$.

For $-t < -t^*$, when $W$ can move, the value of $W$ not entering is at least $p$ since if $S$ enters at $\{1\}$ by $-t^*$, $W$ gets $p$. If $S$ does not enter by $-t^*$, by definition of $-t^*$, $S$ gets $1-p$ and $W$ gets $p$. On the other hand, entering at $\{1\}$ gives $W$ $(1-p)$ times the probability that $S$ does not have a revision opportunity, which is equal to $(1-p)\exp(-\lambda t) < (1-p)\exp(-\lambda t^*) = p$. Therefore, $W$ strictly prefers not entering.

Given this, $S$ is always indifferent between entering at $\{1\}$ and ensuring $1-p$ and announcing $\{0,1\}$.

### 7.3 Proof of Proposition 7

In equilibrium, there are following possibilities:

1. $S$ takes a pure strategy $\{x\}$ at $-t$. Then, $W$ should take $\{x'\}$ or $\{0,1\}$ with $x' = \{0,1\} \setminus \{x\}$. So that $x$ be optimal, $x = 1$. Consider the following two cases:

   (a) if $W$ takes a pure strategy $\{x'\}$, then $S$ should take $x$, a contradiction.

   (b) if $W$ takes $\{0,1\}$ with positive probability, then the payoff of $S$ taking $\{0,1\}$ is $1-p$ if $W$ enters at $x' = 0$ and strictly greater than $1-p$ if $W$ takes $\{0,1\}$. To see this, conditional on $W$ taking $\{0,1\}$, $S$'s payoff is given by

   | $S \setminus W$ | $\{0,1\}$ |
   |---|---|
   | $\{0\}$ | $p$ |
   | $\{1\}$ | $1-p$ |
   | $\{0,1\}$ | $\int_0^t e^{-\lambda\tau}\lambda V_{t-\tau}^S d\tau + e^{-\lambda t}$ |

   .

   Since $S$ can always enter $\{1\}$ and guarantee payoff $1-p$, $V_{t-\tau}^S \geq 1-p$ for all $\tau$. Therefore, $\int_0^t e^{-\lambda\tau}\lambda V_{t-\tau}^S d\tau + e^{-\lambda t} = \left(1 - e^{-\lambda t}\right)(1-p) + e^{-\lambda t} > 1-p$, which means it is strictly optimal for $S$ to announce $\{0,1\}$. This is a contradiction.

2. $S$ takes a mixed strategy only over $\{0\}$ and $\{1\}$ at $-t$. Then, it is strictly optimal for $W$ to take $\{0,1\}$ since it makes the probability that $S$ and $W$ enter at the same platform 0.

3. $S$ takes $\{0,1\}$ with a positive probability. We want to show that it is strictly optimal for $W$ to take $\{0,1\}$. Compare $W$'s payoff of entering at $\{x\}$ at $-t$ and that of taking $\{0,1\}$ in the

following three cases:

(a) conditional on the event that $S$ enters at $\{x\}$ at $-t$, $W$ gets 0 if $W$ enters at $\{x\}$. Compared to this, announcing $\{0,1\}$ is better since it gives him at least $1-p$.

(b) conditional on the event that $S$ enters at $\{x'\}$ at $-t$, $W$ gets $p$ by entering at $\{x\}$ if $x=0$ and $1-p$ if $x=1$. Announcing $\{0,1\}$ also gives the same payoff.

(c) conditional on the event that $S$ does not enter, $W$ gets at most $p \Pr(S \text{ will not have an opportunity}) =$ $p \exp(-\lambda t)$ by entering at $\{x\}$. On the other hand, consider the strategy that announces $\{0,1\}$ until $-\bar{t} = -\frac{1}{\lambda}$. If player $S$ has entered by $-\bar{t}$, $W$ will get at least $1-p$. Otherwise, when the candidates have an opportunity to enter at $-s \geq -\bar{t}$, then the value for $S$ should be less than the minimax value of

| $S \setminus W$ | $\{0\}$ | $\{1\}$ | $\{0,1\}$ |
|---|---|---|---|
| $\{0\}$ | $1$ | $p$ | $p$ |
| $\{1\}$ | $1-p$ | $1$ | $1-p$ |
| $\{0,1\}$ | $1-pe^{-1}$ | $1-(1-p)e^{-1}$ | $1$ |

since this payoff matrix is the same as the original payoff matrix except that we replace the payoffs at $S$ taking $\{0,1\}$ with higher payoffs. The value is bounded away from 1, which means the payoff for $W$ is bounded away from 0. Let $\underline{v}$ be this lower bound. Taking the probability that the candidates have an opportunity between $-\bar{t}$ and 0 into account, the expected payoff is no less than $\left(1-e^{-1}\right)\underline{v}$. For sufficiently large $t$, $p\exp(-\lambda t) < \min\left\{1-p, \left(1-e^{-1}\right)\underline{v}\right\}$, which means taking $\{0,1\}$ is strictly better.

In summary for Case 3, since we assume that $S$ takes $\{0,1\}$ with a positive probability, $\{0,1\}$ is strictly optimal for $W$.

Let us consider $S$'s incentive given that $W$ takes $\{0,1\}$. Remember that $S$'s payoff given that $W$ takes $\{0,1\}$ for sure is

| $S \setminus W$ | $\{0,1\}$ |
|---|---|
| $\{0\}$ | $p$ |
| $\{1\}$ | $1-p$ |
| $\{0,1\}$ | $\int_0^t e^{-\lambda\tau}\lambda V^S_{t-\tau}d\tau + e^{-\lambda t}$ |

.

By the same reason as in Case 1.(b) above, $S$ should take $\{0, 1\}$ with probability 1.