# An Efficient Scheme for Reliable Error Correction with Limited Feedback

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***Abstract*— This paper proposes a practical scheme to transmit reliable information through a noisy symmetric DMC using limited noiseless feedback. The ratio of feedback rate to feedforward rate is a design parameter that can be selected from zero to $1 - C$, where $C$ is the capacity of the channel. The proposed scheme uses a concatenation of low-density parity-check codes, belief propagation, and a noisy version of the closed-loop iterative doping algorithm, previously proposed by the authors for data compression using linear codes. Our scheme takes advantage of the availability of a modicum of feedback to achieve very small block error rates.**

## I. Introduction

### A. Shannon Feedback

We refer to instantaneous, causal, noiseless feedback as *Shannon feedback.* According to this model, the feedback link supplies the $i - 1$ output of the channel to the encoder prior to the transmission of the $i$th symbol through the channel. The classical result that Shannon feedback, and thus any other type of realizable feedback, does not increase the capacity of discrete *memoryless* channels (DMC) goes back to Shannon's 1956 paper [1].

However the fact that feedback impacts dramatically the performance versus blocklength tradeoff, as well as the complexity of encoding and decoding is also well known by now. For DMCs, the increase in the error exponent afforded by feedback has been studied in [2], [3], [4], [5] following the block coding paradigm.

In view of the fact that not even for BSCs has the reliability function been characterized in the absence of feedback, a major result in variable-length coding is Burnashev's closed form expression for the reliability function of the DMC with Shannon feedback [6] (rediscovered in [7]):

$$E_f(R) = \overline{D}(1 - R/C)$$

where $C$ is the channel capacity and

$$\overline{D} = \max_{a,b \in \mathcal{X}} D(P_{Y|X=a} \| P_{Y|X=b})$$

is the maximal divergence between any two conditional output distributions. To achieve this behavior, [7] uses a conceptually simple two-step scheme: first, the information message is transmitted using a good near-capacity achieving code for the underlying DMC (without feedback); at the end of this step the encoder knows exactly the state of the decoder and sends an accept/deny message by using repetition coding, i.e., by repeating $L$ times the letter $a$ (accept) or $b$ (deny) where $a$ and $b$ are chosen to achieve $\overline{D}$. If "deny" is decoded, the whole block is re-transmitted. If "accept" is decoded, transmission of the current block stops. An error occurs when "accept" is decoded while "deny" was transmitted.

The schemes [6], [7], while theoretically fundamental, require the design of a good capacity-approaching coding for the DMC without feedback, not only achieving capacity, but with block error probability that decreases exponentially with the blocklength for all rates $R < C$. Therefore, from the coding construction point of view, these schemes may be as complicated as the coding problem for the forward channel alone.

An alternative approach to reliable transmission with Shannon feedback consists of the sequential schemes such as proposed and analyzed in [8], [9], [10]. In particular, Hornstein's seminal sequential scheme for the BSC [8] is very simple and achieves excellent performance versus expected decision delay tradeoff.

Although our main concern is the case of limited feedback, a special case of the scheme presented in this paper enables us to harness the power of Shannon feedback to obtain a particularly simple algorithm that exhibits an excellent performance/complexity tradeoff.

### B. Limited Feedback

While Shannon feedback is an important paradigm, particularly to prove negative results, its relevance to practice is severely limited to situations where a return channel is available with very high capacity. In most practical scenarios, the available feedback link offers much less capacity than the forward channel, let alone that required by Shannon feedback.

One of the most popular practical schemes is the automatic repeat-request (ARQ) system, where a minimal degree of feedback in terms of Ack/Nack is employed. The classical paper [13] considers block codes transmitted over a DMC in the presence of Ack/Nack feedback and established that the decoding error exponent has a slope of -1 at capacity (in contrast to a zero slope without feedback and slope $-\overline{D}/C \leq -1$ with Shannon feedback) provided that the "erasure" probability (i.e., the probability of sending Nack while decoding was correct) is allowed to be an arbitrarily small number.

Modern information theoretic analysis of ARQ based, incremental redundancy, packet protocols is provided in [14]. The importance of the incremental redundancy concept propelled intensive research into actual applications of advanced turbo and LDPC codes, operating efficiently in this regime. The performance of random binary coding as well as an efficient LDPC based approach is reported in [15], and it is concluded that LDPC designed for this purpose based on a simplified version of density evolution are quite efficient. Turbo coding operating in the HARQ (hybrid ARQ incremental redundancy) regime is also studied recently in [16]. The recently introduced fountain codes, such as the LT codes [17] and raptor codes [18] are particularly suitable for applications involving incremental rates. Feedback schemes tailored to belief propagation decoders were initially proposed in [19], but they are more complex and do not offer as good performance as the scheme proposed in this paper.

In this work, we propose a simple practical scheme that is flexible enough in order to handle feedback rates between $\approx 0$ and $\approx 1 - C$. Our rationale builds on [6], [7], in the sense that we use a two-step procedure where the first step is based on a conventional code for the forward channel. However, we do not require that its *block error rate* vanishes exponentially with the blocklength, but only that its post-decoding bit-error rate (BER) is small. Then, we also build on the analogy of the sequential schemes of [8], [9], [10] with data compression (arithmetic coding) in order to propose a second step that is reminiscent of the data compression scheme based on linear error correcting codes proposed by the authors in [11], [12].

## II. The Proposed Feedback Scheme

For the sake of clarity we limit our exposition to a BSC with known crossover probability $\delta$. The scheme can be readily extended to DMCs with additive independent noise (in a finite field) for which linear codes are optimum, and to channels such as the binary input additive Gaussian noise channel, as done in Example 2 of Section III.

Noiseless feedback is available with a normalized rate $R_f$, defined as the ratio between the number of feedback channel uses over the number of forward channel uses. Thus, Shannon feedback requires $R_f = 1$, while we only consider the case of $0 \leq R_f \leq 1 - C$.

Although we work in the regime of limited feedback rate, we are able to maintain the encoder informed of the state of the decoder at every point in time. This enables the encoder to strategically select symbols to transmit in order to stir the decoder to the correct transmitted message. In principle, for $R_f = H(Y|X) = h(\delta) = 1 - C$, the decoder can inform the encoder about its state with small error by encoding the received signal $\mathbf{y}$ using Slepian-Wolf encoding. However, Slepian-Wolf encoding is tantamount to a channel coding problem and communicating the decoder state reliably to the encoder is as difficult as communicating reliably over the forward channel alone. Nevertheless, as we will see it is possible to build on the analogy between coding with feedback and data compression in order to devise a simple and efficient practical scheme for $0 \leq R_f \leq 1 - C$.

### A. Noisy Closed Loop Iterative Doping

The noiseless closed-loop iterative doping (CLID) algorithm introduced in [11], [12] was instrumental in obtaining lossless data compression using linear codes with performance quite competitive [20] with state-of-the-art compression schemes. In the data compression setting, CLID works as follows:

The belief propagation (BP) algorithm operates on the Tanner graph of a linear code and at each iteration computes the reliabilities (magnitude of the posterior symbol-wise approximated log-likelihood ratios computed by BP) of the $n$ variable bits An effective way to drive BP to the correct codeword is to supply, every $L \geq 1$ iterations, the true value of the bit with the lowest reliability. Thereafter, the reliability of this *doped* bit remains equal to $\pm\infty$ (where the sign depends on the bit's true value). To signal the identity of the lowest reliability bit to the encoder would be costly as it requires about $\log_2 n$ bits per doping iteration. An effective alternative is to run an identical copy of BP at the encoder, which at each iteration knows which bit it needs to supply to the decoder. In the original data compression application of CLID [20], it was possible to run an identical copy of the belief propagation algorithm at the compressor since the compressor has access to all the information available to the decompressor. The same holds in the case of a noisy channel with Shannon feedback. In the latter case, though, the bit values are sent to the decoder via the noisy forward channel. Hence, the basic CLID algorithm is modified: instead of setting the doped bit messages to $\pm\infty$, we incorporate in the BP the additional messages $(-1)^y \log \frac{1-\delta}{\delta}$, where $y$ is the channel output corresponding to a doped bit. Notice that with Shannon feedback the transmitter knows $y$ for each doping round, and therefore can keep its own copy of the BP decoder perfectly synchronized with that of the decoder such that the position of the bits to be doped need not be communicated explicitly. We shall refer to this modified CLID algorithm as the "noisy-CLID" strategy.

We have experimented on a BSC with $\delta = 0.1$ and an optimized irregular LDPC code of length $n = 20000$ and rate $r = 0.5$ with degree sequences given in Table I. With Shannon feedback, we found zero block errors in $10^5$ simulated blocks with average transmission rate, defined as

$$R = \frac{rn}{n + \mathbb{E}[d]}$$

where the LDPC code has rate $r$ and blocklength $n$, and $\mathbb{E}[d]$ is the average number of doping bits. To achieve this excellent performance with noisy CLID, we used $R = 0.495$, indicating that the expected number of doping rounds per block is indeed very small.

### B. Concatenated coding for $R_f \leq 1 - C$

In this subsection, we show how to use the noisy CLID scheme of Section II-A, with limited feedback. Consider a transmitter and a receiver, denoted by Tx and Rx in the following. The scheme is a two-step process based on the serial concatenation of two codes, where the outer code is linear.

*Step 1* (Figure 1) Tx encodes an information message of $k$ bits by the outer code whose parity-check matrix is $\mathbf{H}_1$, producing a codeword $\mathbf{x}_1$ of $m$ coded bits. Then, $\mathbf{x}_1$ is input to the inner encoder producing a codeword $\mathbf{x}_2$ of $n$ coded bits transmitted through the forward channel. Upon reception of the channel output $\mathbf{y}$, Rx runs a decoder for the concatenated code. For example, it can run BP over the Tanner graph of the concatenated scheme. Upon completion of decoding (e.g., after a predetermined number of BP iterations), Rx obtains an estimate $\widehat{\mathbf{x}}_1$ of $\mathbf{x}_1$. Let $\mathbf{e}_1 = \widehat{\mathbf{x}} - \mathbf{x}_1$ denote the error pattern. Typically, the concatenated scheme is designed such that the residual BER is small, i.e., $\mathbf{e}_1$ has small Hamming weight. Then, Rx computes the syndrome

$$\mathbf{z}_1 = \mathbf{H}_1\hat{\mathbf{x}}_1 = \mathbf{H}_1\mathbf{e}_1,$$

If $\mathbf{z}_1 = 0$, Rx declares the information message corresponding to $\hat{\mathbf{x}}_1$ as the output and the algorithm finishes without the use of the feedback link.

Note that conditioned on $\mathbf{z}_1 = 0$, this decision has a certain probability of being erroneous. If $\mathbf{z}_1 \neq 0$, $\mathbf{z}_1$ is sent over the noiseless feedback link.

*Step 2* (Figure 2) Tx knows $\mathbf{x}_1$ and $\mathbf{z}_1$ and Rx knows $\hat{\mathbf{x}}_1$ and $\mathbf{z}_1$, but neither knows $\mathbf{e}_1$. In step 2, Tx and Rx maintain a "dialog" that enables them to agree on a value of $\mathbf{e}_1$. Both Tx and Rx run BP simultaneously on the Tanner graph of the outer code with syndrome (i.e., values of the parity check sums) equal to $\mathbf{z}_1$, and variable nodes associated to the elements of $\mathbf{e}_1$. The initial values of the nodes $\mathbf{e}_1$ are determined by assuming that the equivalent channel from $\mathbf{x}_1$ to $\hat{\mathbf{x}}_1$ is a BSC whose crossover probability, $\delta'$, is experimentally determined as the residual BER of the open-loop decoding step. The synchronized versions of BP at Tx and Rx make use of the noisy CLID strategy described before, modified as follows. At each doping round, both Tx and Rx identify the least reliable bit node, say $\ell$. The Tx sends $x_{1,\ell}$ to Rx through the forward channel which outputs $y_{1,\ell} = x_{1,\ell} + w_\ell$, where $w_\ell$ is the BSC noise variable. Rx computes $\widehat{x}_{1,\ell} - y_{1,\ell} = e_{1,\ell} - w_\ell$ and feeds it back to Tx via the noiseless feedback link. Hence, both Tx and Rx have the same noisy observation of $e_{1,\ell}$ and can update their BP algorithm by incorporating the additional message $(-1)^{\widehat{x}_{1,\ell} - y_{1,\ell}} \log \frac{1-\delta}{\delta}$ for the $\ell$-th variable node. Clearly, the BP algorithms at Tx and Rx remain synchronized. The decoding process stops at an iteration when its guess $\widehat{\mathbf{e}}_1$ satisfies the parity-check equations, i.e., when

$$\mathbf{z}_1 = \mathbf{H}_1\widehat{\mathbf{e}}_1.$$

Finally, Rx makes use of the estimated error pattern to compute $\widetilde{\mathbf{x}}_1 = \widehat{\mathbf{x}}_1 - \widehat{\mathbf{e}}_1$ and outputs the information message corresponding to $\widetilde{\mathbf{x}}_1$.

In order to achieve good performance, the outer code must be optimized for a BSC with probability of error $\delta'$. Hence, the outer coding rate must satisfy $r_1 < 1 - h(\delta')$. Letting $r_2$ denote the rate of the inner code and $\mathbb{E}[d]$ the expected number of doped bits, the resulting feedback rate is given by

$$R_f = \frac{nr_2(1-r_1) + \mathbb{E}[d]}{n + \mathbb{E}[d]}$$

For $n \gg \mathbb{E}[d]$, this yields $R_f \approx r_2(1-r_1) + \eta$, where $\eta = (1 - r_2(1-r_1))\mathbb{E}[d]/n$ is a small doping redundancy. We illustrate the choice of rates in the concatenated scheme with two extreme cases. Choosing a trivial inner code and an outer code optimized for the forward channel, we get $r_2 = 1$ and $r_1 = 1 - h(\delta) - \epsilon$, with some gap-to-capacity $\epsilon > 0$ which is a code design parameter. This choice yields $R_f \approx h(\delta) + \epsilon + \eta$, which is close to the rate that would by achieved with Slepian-Wolf coding in the feedback link as mentioned in the preamble of Section II. At the other extreme, choosing an inner code optimized for the forward channel yields $r_2 = 1 - h(\delta) - \epsilon$ and very small residual BER $\delta'$, such that $r_1 = 1 - h(\delta') - \epsilon$ is close to 1. This yields

$$R_f = (1 - h(\delta) - \epsilon)(h(\delta') + \epsilon) + \eta$$

which can be made as small as desired.

It is easy to check that in both the above extreme design cases jointly decoding the Tanner graph of the concatenated scheme in the open-loop decoding step does not bring any improvement. In the case of the trivial inner code this is obvious, and in the case of the inner code designed to have iterative decoding threshold larger or equal to $\delta$ this is also clear since running BP over the section of the Tanner graph corresponding to the inner code alone is already sufficient to achieve very small BER $\delta'$. This consideration is valid in the case of very large blocklength. For finite length, performing BP over the joint Tanner graph does indeed improve performance.

We list several extensions and modifications of the basic scheme which can be used to improve performance at the expense of an increase in latency.

- In order to combat the mismatch of the approximation of the modelling of the channel $\mathbf{x}_1 \to \hat{\mathbf{x}}_1$ as a BSC, it is useful to interleave consecutive blocks of the intermediate vectors $\mathbf{x}_1$.
- Better performance is achieved by doping at nonconsecutive iterations ($L > 1$). The doping scheduling strategy can be optimized as long as encoder and decoder keep synchronism.
- A more general operation of noisy-CLID is feasible, where instead of raw bit-by-bit transmission, encoder and decoder identify blocks of $K$ least reliable bits and send their value through the noisy channel by using coding. To that end, the following alternatives are possible:
  1) Open-loop coded transmission of the block of $K$ bits with a linear $K$-to-$N$ code. The new $N - K$ parity-check equations can be incorporated in the BP, or expressions for the marginal probabilities of the doped values can be precomputed if the code is sufficiently simple. This requires $N$ feedforward bits and $N$ feedback bits.

2) Closed-loop coded transmission of the block of $K$ bits with a sequential code that uses Shannon feedback such as the ones developed in [8], [9], [10].

## III. Examples

We have performed some experiments to test and validate the proposed scheme. In particular, we have considered the case of a BSC where the inner code is optimized for the forward channel and the feedback rate is very small. Then, we also show how the proposed scheme can improve an existing physical layer standard (that cannot be changed for standardization reasons) by adding a coding and feedback scheme at an upper protocol layer.

*Example 1.* We test the case of a BSC with $\delta = 0.1$, information vector length $k = 10000$, an LDPC outer code of rate $r_1 = 0.93$ designed to handle a residual BER up to $5 \cdot 10^{-3}$, specified in Table II, and as inner code the same $r_2 = 0.5$ code of Table I. We simulated $10^5$ frames without finding any error. The normalized average number of doping bits is $\mathbb{E}[d]/n = 0.0022$, illustrating that of $\mathbb{E}[d]/n \ll 1$ is indeed satisfied if the component codes are properly designed. The average forward coding rate is given by $R = 0.464$, and the resulting feedback rate is given by $R_f = 0.037$. The channel capacity is given by $C = 1 - h(0.1) = 0.53$ bit/symbol, i.e., we are 0.067 bit/symbols away from capacity. For the sake of comparison, we have simulated the LDPC code of the same blocklength $n = 21505$ and rate 0.465 bit/symbol given in Table III optimized for the BSC with $\delta = 0.1$. The resulting block error rate for the standard random ensemble is $\approx 10^{-3}$. Notice that in this example the LDPC finite-length graphs are generated randomly from the standard ensemble of finite-length random graphs without using any particular graph optimization technique and performance is averaged also over the random selection of the graphs.

*Example 2.* Consider the recently standardized satellite digital video broadcasting downlink system DVB-S2 [21] and its companion uplink system DVB-RCS [22]. The downlink (from the gateway through the satellite to the user terminals) is designed for high-definition digital video broadcasting, and achieves a very low BER $\leq 10^{-12}$. Hence, it can be regarded as practically error-free. On the contrary, the uplink is based on packet TDMA with relatively short data frame and QPSK modulation. A set of turbo codes with rates ranging from $1/3$ to $6/7$ and blocklengths ranging from 96 to 1712 information bits is specified [22]. The target block error rate is $10^{-4}$. Consider for example a channel with $E_b/N_0 = 5$ dB and blocklength $440$ information bits (1 ATM cell, according to the standard). From Fig.3 we see that a rate $6/7$ turbo code can be selected which lies 2.38dB away from QPSK capacity. If the channel SNR degrades (e.g., due to rain) to some value between 3.0 and 3.5, the system would select coding rate $2/3$ operating 1.94dB away from capacity at $E_b/N_0 = 3.0$ dB.

Alternatively, we can insist on using turbo coding rate $6/7$ provided that we add an outer coding LDPC level at some higher protocol layer. This is designed to handle the resulting inner code BER. For example, at $E_b/N_0 = 3.5$ dB, the BER is $\delta' = 5 \cdot 10^{-3}$, and we can use the same outer code developed for Example 1 achieving a point 1.17dB away from capacity. At $E_b/N_0 = 3.0$ dB, the BER is $\delta' = 2 \cdot 10^{-2}$, and we have designed an LDPC of rate $0.837$ (capacity is $1 - h(\delta') = 0.858$) which operates at 1.59dB away from capacity

This example illustrates that the noisy CLID algorithm can be used and achieves extremely small block error rate thanks to the virtually error-free forward link (acting as noiseless feedback for the return link). Hence, the proposed LDPC-coded feedback scheme can be practically used to replace classical block-based retransmission schemes such as those implemented in standard protocols such as TCP.

Note that not only our feedback scheme achieves greater power efficiency and reduced block error rate, but affords finer granularity in the choice of rates.

## IV. Conclusions

We proposed a coding scheme for channels with noiseless feedback, with feedback rate $R_f \in [\eta, 1 - C + \eta]$, where $\eta > 0$ is small. Our scheme is based on the concatenation of an outer LDPC code with an inner code. When the inner code is designed to operate at rate close to $C$ with small BER, our scheme is able to achieve very small block-error rate and yet operate close to $C$. Therefore, it provides a simple and efficient alternative to the conventional concatenated coding scheme where an inner LDPC code is concatenated with an outer expander/BCH or RS code in order to achieve small (open-loop) block-error rate. When the inner code is fixed by an existing physical layer, our scheme provides an attractive alternative to higher-layer protocol retransmission schemes, that are implemented in most data communication systems.

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| deg. | $\lambda$ | $\rho$ |
|---|---|---|
| 2 | 0.1604240 | |
| 3 | 0.1605410 | |
| 6 | 0.0610339 | |
| 7 | 0.1534340 | |
| 11 | | 1.0 |
| 13 | 0.0369041 | |
| 16 | 0.0200680 | |
| 17 | 0.0054856 | |
| 20 | 0.1281270 | |
| 25 | 0.0233812 | |
| 35 | 0.0528542 | |
| 68 | 0.0574104 | |
| 69 | 0.0898442 | |
| 86 | 0.0504923 | |

TABLE I

RATE 0.5 ENSEMBLE (LEFT ($\lambda$) AND RIGHT ($\rho$) SEQ.).

| deg. | $\lambda$ | $\rho$ |
|---|---|---|
| 5 | 0.6420 | |
| 27 | 0.2267 | |
| 28 | 0.1312 | |
| 100 | | 1.0 |

TABLE II

RATE 0.93 ENSEMBLE (LEFT ($\lambda$) AND RIGHT ($\rho$) SEQ.).

| deg. | $\lambda$ | $\rho$ |
|---|---|---|
| 3 | 0.4455 | |
| 11 | | 1.0 |
| 16 | 0.1657 | |
| 17 | 0.1455 | |
| 100 | 0.2433 | |

TABLE III

RATE 0.465 ENSEMBLE (LEFT ($\lambda$) AND RIGHT ($\rho$) SEQ.).

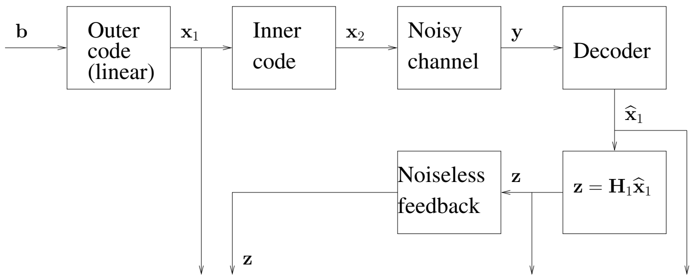


Fig. 1. Step 1: Concatenated coding and standard (open-loop) decoding. The syndrome $\mathbf{z}$ is computed and fed back via the noiseless feedback link.

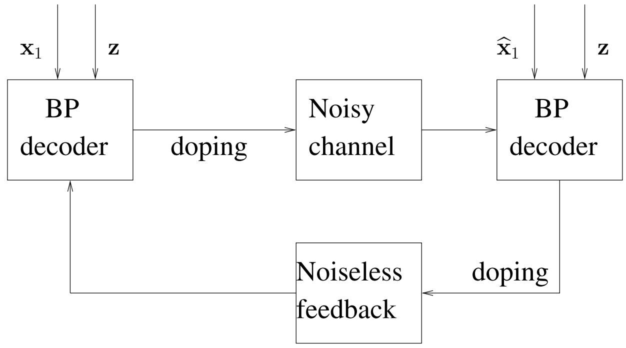


Fig. 2. Step 2: BP decoding with noisy closed-loop iterative doping.

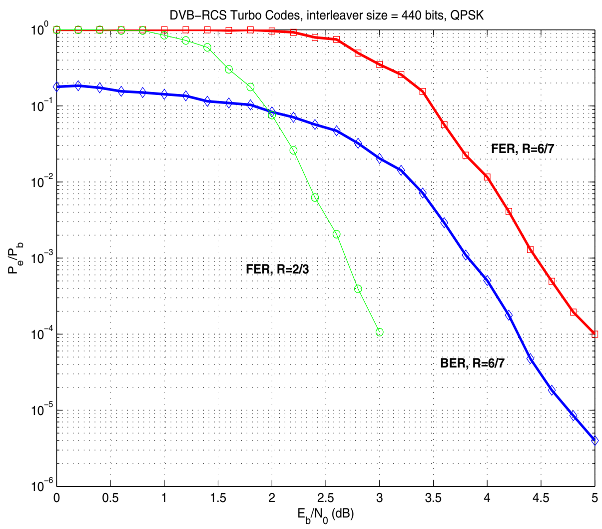


Fig. 3. Block-error rate (FER) and BER performance of some turbo-codes specified by the DVB-RCS standard.