(a)

First do matched filtering using the set of unnormalized spreading codes $\mathbf{SA}$. We obtain

$$y[i] = (\mathbf{A} \mathbf{S}^\top) \cdot \mathbf{S} \mathbf{A} b[i] = \mathbf{R} b[i]$$  \hspace{1cm} (1)

where $\mathbf{R}$ is the unnormalized crosscorrelation matrix. $\mathbf{R}$ is positive definite since $\mathbf{S}$ has linearly independent columns. We can find a unique lower triangular matrix $\mathbf{F}$ with positive diagonal elements such that

$$\mathbf{R} = \mathbf{F}^\top \mathbf{F}.$$  \hspace{1cm} (2)

Use matrix $\mathbf{F}^{-\top}$ to process the matched filter output. We obtain

$$\tilde{y}[i] = \mathbf{F} b[i].$$  \hspace{1cm} (3)

Notice that $\mathbf{F}$ is lower triangular. $b[i]$ can be solved efficiently.

(b)

Since the set of received signal vectors $\{\mathbf{r}[i_1], \ldots, \mathbf{r}[i_{2K}]\}$ are all different, $\{b[i_1], \ldots, b[i_{2K}]\}$ must be a permutation of the set of all $2^K$ possible antipodal vectors and $\mathbf{S}$ must have linearly independent columns. Note that

$$\mathbf{r}[i_n] - \mathbf{r}[i_m] = \mathbf{S} \mathbf{A} (b[i_n] - b[i_m])$$  \hspace{1cm} (4)

where each entry of the vector $(b[i_n] - b[i_m])$ may take its value from $\{-2, 0, 2\}$ only. Note that out of the 3 possible values, 0 occurs in $\frac{1}{2}$ of the cases, and $-2$ and 2 each occurs in $\frac{1}{4}$ of the cases. Since $b[i_n]$ and $b[i_m]$ are not equal, their difference may take any non-zero vector in $\{-2, 0, 2\}^K$. The most frequent ones have 0’s at all but one entry. In such cases, the resulting difference must be 2 times a column of $\mathbf{SA}$ or its antipodal. Therefore by taking the $K$ mostly repeated vectors in the set of $(\mathbf{r}[i_n] - \mathbf{r}[i_m])$’s and divide them by 2, we obtain the $K$ columns of $\mathbf{SA}$ subject to sign uncertainty in each column as well as uncertainty in their order.
**Problem 2.42** (Solution by Dongning Guo, June 20, 2001)

| Correction: The question in (b) shall be fixed by replacing up to a sign uncertainty in each row by up to a sign uncertainty in each column. |

(a) The data can be recovered by using $A^{-1}(S^T S)^{-1}S^T$ to filter $r[i]$. We obtain

$$A^{-1}(S^T S)^{-1}S^T \cdot SAb[i] = b[i].$$

(1)

$(S^T S)$ is invertible since $S$ has linearly independent columns.

(b) Since the set of received signal vectors $\{r[i_1], \cdots, r[i_{2^K}]\}$ are all different, $\{b[i_1], \cdots, b[i_{2^K}]\}$ must be a permutation of the set of all $2^K$ possible antipodal vectors and $S$ must have linearly independent columns. Note that

$$r[i_n] - r[i_m] = SA(b[i_n] - b[i_m])$$

(2)

where each entry of the vector $(b[i_n] - b[i_m])$ may take its value from $\{-2, 0, 2\}$ only. Note that out of the 3 possible values, 0 occurs in $\frac{1}{2}$ of the cases, and $-2$ and 2 each occurs in $\frac{1}{4}$ of the cases. Since $b[i_n]$ and $b[i_m]$ are not equal, their difference may take any non-zero vector in $\{-2, 0, 2\}^K$. The most frequent ones have 0’s at all but one entry. In such cases, the resulting difference must be 2 times a column of $SA$ or its antipodal. Therefore by taking the $K$ mostly repeated vectors in the set of $(r[i_n] - r[i_m])$’s and divide them by 2, we obtain the $K$ columns of $SA$ subject to sign uncertainty in each column as well as uncertainty in their order.