**Problem 2.55** (Solution by Dongning Guo, June 20, 2001)

Correction: Eqn. (2.137) shall be fixed as

\[ F_{kk} = \frac{1}{\sqrt{(R[k,K])^{-1}_{11}}} \]  \hfill (1)

Let \( G \) be a \((K - k + 1)\) dimensional square matrix defined by

\[ G_{ij} = F_{k+i-1,k+j-1} \quad \text{for} \quad i, j = 1, \cdots, K - k + 1. \]

Then

\[
[G^T G]_{ij} = \sum_{l=1}^{K-k+1} G_{il} G_{lj} \quad \text{(2)}
\]

\[
= \sum_{l=1}^{K-k+1} F_{k+l-1,k+i-1} F_{k+l-1,k+j-1} \quad \text{(3)}
\]

\[
= \sum_{l=k}^{K} F_{l,k+i-1} F_{l,k+j-1} \quad \text{(4)}
\]

\[
= \sum_{l=1}^{K} F_{l,k+i-1} F_{l,k+j-1} \quad \text{(5)}
\]

\[
= R_{k+i-1,k+j-1} \quad \text{(6)}
\]

\[
= R_{ij}[k,K]. \quad \text{(7)}
\]

Therefore \( F_{ij}[k,K] = F_{k+i-1,k+j-1}. \)

Assume that \( R[k,K] \) is invertible. Then

\[
R[k,K]^{-1} = F[k,K]^{-1}(F[k,K]^{-1})^T. \quad \text{(8)}
\]

Note that the inverse of a lower-triangular matrix is still a lower-triangular one. Therefore

\[
(F[k,K])^{-1}_{11} = \frac{1}{F_{11}[k,K]} \quad \text{(9)}
\]

and

\[
(R[k,K])^{-1}_{11} = (F[k,K])^{-1}_{11} (F[k,K])^{-1}_{11}. \quad \text{(10)}
\]

Therefore,

\[ F_{kk} = F_{11}[k,K] = \frac{1}{\sqrt{(R[k,K])^{-1}_{11}}}. \quad \text{(11)}\]