PROBLEM 2.59 (Solution by Dongning Guo, June 20, 2001)

Let the autocorrelation function of $s(t)$ be

$$R_s(\tau) = \int_{-\infty}^{\infty} s(t)s(t+\tau)dt. \tag{1}$$

The matrices $\mathbf{R}[0]$ and $\mathbf{R}[1]$ are defined by

$$R_{jk}[0] = \begin{cases} 1, & \text{if } j = k; \\ \rho_{jk}, & \text{if } j < k; \\ \rho_{kj}, & \text{if } j > k; \end{cases} \tag{2}$$

$$R_{jk}[0] = \begin{cases} 0, & \text{if } j \geq k; \\ \rho_{kj}, & \text{if } j < k, \end{cases} \tag{3}$$

where for $j < k$,

$$\begin{aligned} \rho_{jk} &= \int_{-\infty}^{\infty} s_j(t)s_k(t)dt && (4) \\ &= \int_{\frac{k-1}{K}T}^{T+\frac{j-1}{K}T} s(t - \frac{j-1}{K}T)s(t - \frac{k-1}{K}T)dt && (5) \\ &= R_s(\frac{j-k}{K}T) && (6) \end{aligned}$$

and similarly

$$\begin{aligned} \rho_{kj} &= \int_{-\infty}^{\infty} s_j(t-T)s_k(t)dt && (7) \\ &= \int_{T+\frac{j-1}{K}T}^{T+\frac{k-1}{K}T} s(t - T - \frac{j-1}{K}T)s(t - \frac{k-1}{K}T)dt && (8) \\ &= R_s(T - \frac{k-j}{K}T). && (9) \end{aligned}$$