Problem **3.49** (Solution by Dongning Guo, June 20, 2001)

I assume binary orthogonal modulation.

The conditional probility of error is

$$\begin{aligned}
& \mathsf{P}\left[|y_{1+}|^2 > |y_{1-}|^2 \,|\, b_1 = -1, b_2, \cdots, b_K\right] \\
= \; & \frac{1}{2} - \frac{\mathsf{E}\left[|y_{1-}|^2\right] - \mathsf{E}\left[|y_{1+}|^2\right]}{2\sqrt{(\mathsf{E}\left[|y_{1-}|^2 + |y_{1+}|^2\right])^2 - 4|\mathsf{E}\left[y_{1-}y_{1+}^*\right]|^2}} \qquad (1)
\end{aligned}$$

where

$$\mathsf{E}\left[|y_{1+}|^2\right] = \sigma^2 + \sum_{k=2}^{K} A_k^2 |\rho_{1k}[+, b_k]|^2, \qquad (2)$$

$$\mathsf{E}\left[|y_{1-}|^2\right] = \sigma^2 + A_1^2 + \sum_{k=2}^{K} A_k^2 |\rho_{1k}[-, b_k]|^2, \qquad (3)$$

$$\mathsf{E}\left[y_{1+}y_{1-}^*\right] = \sum_{k=2}^{K} A_k^2 \rho_{1k}[+, b_k] \rho_{1k}[-, b_k]. \qquad (4)$$

Obviously,

$$\begin{aligned}
& \lim_{\sigma\to\infty} \left[\frac{1}{2} - \mathsf{P}\left[|y_{1+}|^2 > |y_{1-}|^2 \,|\, b_1 = -1, b_2, \cdots, b_K\right]\right] \sigma^2 \\
= \; & \frac{1}{4}\left[A_1^2 + \sum_{k=2}^{K} A_k^2 (|\rho_{1k}[-, b_k]|^2 - |\rho_{1k}[+, b_k]|^2)\right]. \qquad (5)
\end{aligned}$$

By averaging the above limit over the interfering users' bits, we have

$$\begin{aligned}
& \lim_{\sigma\to\infty} \left[\frac{1}{2} - \mathsf{P}_1^{\text{noncoh}}(\sigma)\right] \sigma^2 \\
= \; & \frac{1}{4} A_1^2 + \frac{1}{8} \sum_{k=2}^{K} A_k^2 \cdot \\
& \left[|\rho_{1k}[-,-]|^2 - |\rho_{1k}[+,-]|^2 + |\rho_{1k}[-,+]|^2 - |\rho_{1k}[+,+]|^2\right]. \qquad (6)
\end{aligned}$$