PROBLEM **4.31** (Solution by Dongning Guo, June 20, 2001)

Let $\mathbf{s}$ be the signature waveform assigned to both users. A sufficient decision statistic is obtained by matched filtering using $\mathbf{s}$,

$$y = A_1(b_1 + 2b_2) + n. \tag{1}$$

where $n$ is a zero-mean Gaussian random variable with variance $\sigma^2$, and we have used the fact that $2A_1 = A_2$. The constellation points corresponding to the 4 hypotheses are

$$H_{++}: \quad 3A_1, \tag{2}$$
$$H_{+-}: \quad -A_1, \tag{3}$$
$$H_{-+}: \quad A_1, \tag{4}$$
$$H_{--}: \quad -3A_1. \tag{5}$$

The decision regions are

$$H_{++}: \quad y \in [2A_1, \infty), \tag{6}$$
$$H_{+-}: \quad y \in [-2A_1, 0), \tag{7}$$
$$H_{-+}: \quad y \in [0, 2A_1], \tag{8}$$
$$H_{--}: \quad y \in (-\infty, -2A_1). \tag{9}$$

The joint region correspinding to a composite hypothesis $H_{++} \cup H_{+-}$, i.e., $b_1 = +1$, is $[-2A_1, 0) \cup [2A_1, \infty)$. The bit-error-rate of user 1 is

$$\begin{aligned} \mathsf{P}_1 &= \mathsf{P}\left[y \notin [-2A_1, 0) \cup [2A_1, \infty) | H_{++} \cup H_{+-}\right] && (10)\\ &< \tfrac{1}{2}\mathsf{P}\left[y \notin [2A_1, \infty) | H_{++}\right] + \tfrac{1}{2}\mathsf{P}\left[y \notin [-2A_1, 0) | H_{+-}\right] && (11)\\ &= \frac{3}{2} Q\left(\frac{A_1}{\sigma}\right). && (12) \end{aligned}$$

Note that $\mathsf{P}_1$ is upper bounded by $\frac{3}{2}$ times the single user performance. Therefore, the asymptotic multiuser efficiency is 1.