Solution

From (5.9), we know \( \langle \tilde{s}_k, s_k \rangle = 1 \). Because \( \| s_k \| = 1 \), by Cauchy-Schwarz inequality we have \( \| \tilde{s}_k \| \geq 1 \) with equality if and only if \( \tilde{s}_k = \alpha s_k \) where \( \alpha \) is a real number.

Hence \( \| \tilde{s}_k \| = 1 \) if and only if

\[
\tilde{s}_k = s_k, \tag{1}
\]

which is equivalent to that \( s_k \) is orthogonal to the other signature vectors because \( \tilde{s}_k \) is a decorrelating transformation for \( k \)th user.

So, \( \| \tilde{s}_k \| = 1 \) if and only if \( s_k \) is orthogonal to the other signature vectors.

\( (\| \tilde{s}_k \| = 1 \text{ for all } k \text{ if and only if } s_1, \ldots, s_K \text{ are orthogonal.}) \)