For contradiction’s sake assume that there exists a point in $C$ but not in $D$. Without loss of generality let the point be expressed as $z = s + v$ where

$$v = (1 - \alpha)d + w \in D,$$

where $\alpha > 0$ and

$$\langle w, d \rangle = 0.$$

Note that

$$\langle v, d \rangle = (1 - \alpha)\|d\|^2 < \|d\|^2.$$

$z \notin C$ is quite obvious. For $z \in D$ to be true, it is required that $\|v\|^2 \geq \|d\|^2$, i.e.,

$$\|w\|^2 \geq [1 - (1 - \alpha)^2]\|d\|^2.$$

By convexity of $D$, for every $0 \leq \beta \leq 1$,

$$x = \beta v + (1 - \beta)d$$

satisfies that $s + x \in D$. Let

$$\beta = \frac{\alpha}{\|w\|^2 + \alpha^2}.$$

It can be shown that $\|x\|^2 < \|d\|^2$ which violates the definition of $D$. Due to the constraint of Eqn. (4), $0 < \beta \leq \frac{1}{2}$, so the choice of $\beta$ is valid. Note that Eqn. (6) leads to that

$$\|w\|^2 = \frac{(1 - \alpha \beta)\|d\|^2}{\beta}.$$

Consider the norm of $z$,

$$\|z\|^2 = \|\beta v + (1 - \beta)d\|^2$$

$$= \| (1 - \alpha \beta)d + \beta w \|^2$$

$$= (1 - \alpha \beta)^2\|d\|^2 + \beta^2\|w\|^2$$

$$= (1 - \alpha \beta)^2\|d\|^2 + (1 - \alpha \beta)\alpha \beta\|d\|^2$$

$$< \|d\|^2.$$

This contradicts the fact that $d$ is such that

$$\min_{v \in D} \|v - s\| = \|d\|.$$

Therefore, there is no such $z \in D$ that $z \in C$. $D \subset C$ is proved.