ADAPTIVE MULTIUSER DETECTION

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1. Introduction

Spurred by its applications in Code Division Multiple Access, multiuser detection has grown from its origins more than ten years ago to a vibrant research and development activity in industry and academia. As the needs to increase capacity in multiuser radio channels become more pressing, it is safe to expect that the interest in the subject will grow in the near future.

The development of multiuser detection has proceeded along a path which is typical of other areas in communications. Initially, optimum solutions were obtained along with the best possible performance achievable in Gaussian noise channels [1]. Those results showed a huge gap between the optimum performance and the performance of the conventional single-user detector (which neglects the presence of multiaccess interference). In particular, they showed that the near-far problem is not a flaw of CDMA, as widely believed, but of the inability of the conventional receiver to exploit the structure of the multiaccess interference. This feature of multiuser detection sidesteps the need for sophisticated high-precision power control in mobile communication systems. Thus, an increase in the complexity of the base station enables a considerable reduction in the complexity of the mobile transmitters. Equally important to the near-far resistant property of optimum multiuser detection, is the performance gain that it promises even in situations of exact power control (equal-power reception). This performance gain results in lower power consumption and processing gain requirements, which translate into increased battery lifes and lower bandwidth in order to support the same information rates.

The second stage in the development of multiuser detection was devoted to the analysis and design of multiuser detectors that could achieve significant performance gains over the conventional receiver without incurring in the exponential (in the number of users) complexity of the optimum multiuser detector. Notable among those efforts were the decorrelating detector of Lupas and Verdú [2], [3]; the multistage detector of Varanasi and Aazhang [4], [5]; the decision-feedback multiuser detectors of Duel-Hallen [6], [7]; and the suboptimum detectors of Xie, Rushforth and Short [8], [9].

Motivated by the channel environments encountered in many CDMA applications, the design of multiuser detectors for channels with fading, multipath, or noncoherent modulation has attracted considerable attention, as exemplified by the works of Varanasi [10]; Vasudevan and Varanasi [11], [12]; Zvonar and Brady [13], [14], [15]; Fawer and Aazhang [16].

The foregoing multiuser detectors depend on various parameters such as received amplitudes and crosscorrelations which are usually not fixed and known beforehand. Therefore, another important thrust in research in multiuser detection is the design of *adaptive* detectors, which self-tune the detector parameters from the observation of the received waveform. The very recent, and already considerable, literature on this subject is surveyed in the present tutorial paper.

Sections 2 and 3 contain background material used throughout the paper on the multiaccess channel model, optimum multiuser detection and the decorrelating detector. A comprehensive tutorial exposition of these and other topics can be found in [17]. Section 4 deals with the MMSE linear multiuser detector and its adaptive implementations. Section 5 gives an overview of adaptive tentative-decision based detectors such as those that use successive cancellation and decision-feedback. Section 6 deals with blind multiuser detection, and in particular, it presents a multiuser detector which is optimally near-far resistant and requires no more knowledge than the conventional single-user detector. Section 7 is devoted to multiuser detection using learning neural-networks

2. Optimum Detection

The asynchronous CDMA white Gaussian noise channel considered in this paper is

$$y(t) = \sum_{k=1}^{K} \sum_{i=-M}^{i=M} A_k\, b_k[i]\, s_k(t - iT - \tau_k) + \sigma\, n(t) \qquad (1)$$

where

- K is the number of users.
- A_k is the received amplitude of the kth user.
- $b_k[i] \in \{-1,+1\}$ is the data stream modulated by the kth user.
- s_k is the unit-energy signature waveform of the kth user.
- T is the inverse of the data rate (duration of s_k).
- $\tau_k \in [0,T)$ is the kth user's offset.
- $n(t)$ is normalized white Gaussian noise.

- σ^2 is the background noise power spectral density.

The model in (1) can be generalized to take into account a number of features that are relevant in practice such as quadrature and nonbinary modulation, signature waveforms spanning more than one bit epoch, intersymbol interference, etc. For conceptual clarity it is best not to generalize the model in those directions; instead, it is usually conceptually advantageous to consider the special case of (1) where the users are symbol-synchronous:

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma\, n(t) \qquad t \in [0, T] \tag{2}$$

In most cases, the analysis of multiuser detectors for the synchronous channel (2) contains all the key ingredients necessary for the analysis of the more general channel (1).

In multiuser detection, it is frequently useful to examine performance in the situation of low background noise, $\sigma \to 0$. To that end, the *asymptotic multiuser efficiency* of the kth user (whose bit error rate is denoted by $P_k(\sigma)$ is defined as

$$\eta_k = \lim_{\sigma \to 0} \frac{\sigma^2}{A_1^2} \log 1/P_k(\sigma) \tag{3}$$

which is simply the degradation (measured in SNR) suffered by a user due to the presence of other users in the channel. The worst-case asymptotic multiuser efficiency over all received interfering amplitudes is called the *near-far resistance*, denoted by $\bar{\eta}_k$. For most multiuser detectors, near-far resistance is not an overly conservative performance measure because the worst-case usually does not occur at large interfering ratios. Therefore, it is an attractive performance measure even for receivers that employ some power control.

The optimum receiver for (2) processes the received waveform with a bank of matched filters, which produce a vector of observables:

$$\mathbf{y} = \mathbf{R}\,\mathbf{A}\,\mathbf{b} + \mathbf{n} \tag{4}$$

where $\mathbf{A} = diag\{A_1, \cdots A_K\}$, $\mathbf{b} = [b_1, \cdots b_K]^T$, $\mathbf{n}$ is a zero mean Gaussian vector with covariance matrix $\mathbf{R}$ and $\mathbf{R}$ is the crosscorrelation matrix whose ij coefficient is

$$\rho_{ij} = \int_0^T s_i(t)\, s_j(t)\, dt \tag{5}$$

The optimum detector that selects the most likely data vector based on the observation of $\mathbf{y}$ must solve an NP-complete combinatorial optimization algorithm [18]. Thus, no algorithm polynomial in K is known for optimal multiuser detection. In the asynchronous case, the receiver consists of a matched filter front-end followed by a Viterbi algorithm [1]. The number of states of the Viterbi algorithm is exponential in K with metrics that are very simple to compute in terms of the matched filter outputs and crosscorrelations. The optimum kth user near-far resistance [19], [3] is equal to the minimum energy of any multiuser signal modulated by $\{-1,0,+1\}$, with fixed $A_1\, b_1[0] = 1$. In terms of the crosscorrelation matrix, the near-far resistance of the kth user is equal to the reciprocal of the kk element of the inverse of the crosscorrelation matrix:

$$\bar{\eta}_k = \frac{1}{R_{kk}^{-1}} \tag{6}$$

In practical terms, this means that there is a huge performance gap between the conventional single-user matched filter and the optimum achievable performance. For example, while the near-far resistance of the conventional receiver is zero, the expected optimum near-far resistance using direct-sequence spread-spectrum signature waveforms with N chips per symbol is lower bounded by [20]

$$E[\bar{\eta}_1] \geq 1 - \frac{K-1}{N} \tag{7}$$

These notable performance gains are obtained at the expense of:

- the signature waveforms of all users must be known.
- the received amplitudes of all users must be known.
- the timing of all users must be acquired.
- exponential complexity in the number of users.
- a centralized structure which demodulates all transmitters.

Remarkably, as we will see in Section 6, recent progress in adaptive multiuser detection has resulted in a receiver which achieves optimally near-far resistant multiuser detection with none of the above shortcomings.

3. Decorrelating Detector

The decorrelating detector outputs the signs of the matched filter outputs in (4) multiplied by the inverse crosscorrelation matrix $\mathbf{R}^{-1}$, i.e., it takes the sign of the the vector

$$\mathbf{R}^{-1}\mathbf{y} = \mathbf{A}\,\mathbf{b} + \mathbf{R}^{-1}\mathbf{n}.$$

Thus, in the hypothetical absence of background noise, the decorrelating linear transformation recovers the transmitted bits without multiuser interference. In the asynchronous case, the decorrelating detector generalizes to an infinite impulse-response filter [3]. The decorrelating detector is the maximum likelihood solution in the absence of any knowledge about the received amplitudes. A major result of Lupas and Verdú [2], [3] is that the decorrelating detector achieves optimum near-far resistance. The bit error rate of the decorrelating detector is independent of the interfering amplitudes. This is because the decorrelating linear transformation projects the received waveform on a subspace which is orthogonal to the space spanned by the interfering signature waveforms. In comparison with the above requirements of the optimum detector, the decorrelating detector has the following features:

- the signature waveforms of all users must be known.
- the received amplitudes need not be known or estimated.
- the timing of all users must be acquired.
- the matrix inverse $\mathbf{R}^{-1}$ must be computed.
- it lends itself to decentralized implementation, demodulating only the desired user.

The optimal near-far resistance property of the decorrelating detector coupled with the fact that it does not require knowledge of the received amplitudes make the decorrelating detector attractive from the standpoint of implementation. The main disadvantage is the computation required to obtain the decorrelating coefficients from the crosscorrelations. In the case of synchronous direct-sequence spread-spectrum, Chen and Roy [21] report a recursive least squares (RLS) computation of the decorrelating detector coefficients which requires knowledge of all signature sequences but sidesteps the need to perform computations with crosscorrelations. In the asynchronous case, the processing window can be truncated to the bit of interest as suggested in [22], [23]; or it can span a truncated sliding window as proposed in [24]. In the latter case, the dynamic updating of the decorrelating detector coefficients in response to variations in the crosscorrelations has been investigated in [25]. Mitra and Poor [26] advocate detecting the presence and identity of a new transmitter by processing the residual signal that results by subtracting from the received signal the multiuser signal modulated by the decorrelating detector decisions.

The optimality of the near-far resistance of the decorrelating detector with DPSK modulation has been established by Varanasi [10]. The decorrelating detector has also been used in the conjunction with DPSK and individual rake matched filters for each user (to combat multipath) in [27].

4. Linear MMSE Multiuser Detection

The decorrelating detector may have worse bit error rate than the conventional detector when all the interferers are very weak [3]. This means that it should indeed be possible to incorporate (exact or approximate) knowledge of the received amplitudes in order to obtain a linear multiuser detector that outperforms the decorrelating detector. Minimum mean-square error (MMSE) linear detection is one approach to this problem. According to this criterion, one chooses the $K \times K$ matrix $\mathbf{M}$ that achieves

$$\min_{\mathbf{M} \in R^{K \times K}} E[\|\mathbf{b} - \mathbf{M}\mathbf{y}\|^2] \tag{9}$$

where the expectation is with respect to the vector of transmitted bits $\mathbf{b}$ and the noise vector $\mathbf{n}$ which as we saw has zero mean and covariance matrix equal to $\sigma^2\mathbf{R}$. Without invoking the Gaussian nature of ■ it is possible to show that the linear MMSE detector replaces the inverse crosscorrelation matrix $\mathbf{R}^{-1}$ by the matrix

$$[\mathbf{R} + \sigma^2\mathbf{A}^{-2}]^{-1}.$$

Thus the linear MMSE has the aforementioned features of the decorrelating detector, except that it requires knowledge of the received amplitudes. If either the background noise level or the kth user received energy dominates, then the MMSE detector approaches the conventional single user matched filter; on the other hand, as the background noise level vanishes $\sigma \rightarrow 0$, the MMSE detector approaches the decorrelating detector. Therefore, the asymptotic multiuser efficiency and the near-far resistance of the MMSE detector are the same as those of the decorrelating detector. In particular, it also achieves optimal near-far resistance. The linear MMSE multiuser detector was originally proposed by Xie, Short and Rushforth [9] in the asynchronous case, and much earlier in single-user dual-polarization channels [28], which can be viewed as two-user synchronous channels.

As long as the background noise is weak, there is little point in incurring in the additional complexity over the decorrelating detector required by the need to track received amplitudes. However, the great advantage of the linear MMSE detector is the ease with which it lends itself to adaptive implementation with training sequences.

The contribution of the kth user to the penalty function in (9) is equal to

$$E[(b_k - \langle c, y \rangle)^2]. \tag{10}$$

where the linear transformation has been denoted by c. The gradient of the cost function inside the expectation in (10) is equal to

$$2(\langle c, y \rangle - b_k)\, y. \tag{11}$$

Because of the convexity of (10) in c, the gradient descent adaptive algorithm is

$$c[j] = c[j-1] - \mu(\langle c[j-1], y[j] \rangle - b_k[j])\, y[j] \tag{12}$$

will converge (with infinitesimally small step size μ) to the argument that minimizes the penalty function in (10). The update of the impulse response in (12) has the following features:

- the data stream (training sequence) of the desired user must be known.
- the received amplitudes need not be known or estimated.
- the signature waveforms of the interferers need not be known.
- the timing of the desired user must be acquired.
- the timing of interfering users need not be acquired.
- knowledge of the signature waveform of the desired user is not necessary, but it facilitates the initialization of the algorithm.
- it can be implemented in an asynchronous channel, with the only requirement that the timing of the desired user be acquired. The longer the allowed impulse response, the better the performance will be, with a judicious truncation achieving almost the same performance as a doubly infinite filter response.

The gradient descent algorithm shown in (12) is the simplest adaptation law that minimizes (10). Other more complex, but faster, algorithms can be used instead, based, for example, on recursive least-squares or in lattice structures (e.g. [29]).

In addition to the aforementioned earlier reference [28], the adaptive linear MMSE detector was proposed by Madhow and Honig [30], Rapajic and Vucetic [31] and Miller [32], [33]. The implementation of (12) is carried out with finite-dimensional vectors whose dimensionality is equal to (or twice) the number of chips per symbol. Several methods have been proposed in order to lower complexity in systems with large processing gains; for example the cyclically shifted filter bank of [30], the replacement of simple tap delays by first-order low-pass filters in [34], and the symmetric dimension reduction scheme in [35]. Lee [36] observes that the RLS algorithm is ill-conditioned in near-far environments with high SNR, and proposes a transformation of the chip matched filter outputs to overcome this problem. Significant speed-up is reported with both the gradient descent and RLS implementations of the MMSE criterion.

Joint adaptive multiuser detection and timing recovery is achieved with an RLS algorithm by Zvonar and Brady [37], [38] and with a steepest descent algorithm by Smith and Miller [39].

An interesting alternative to the minimization of mean-square error has been proposed by Mandayam and Aazhang [40]. It uses a stochastic gradient algorithm to minimize probability of error, which (for a linear detector) can be written as the sum of Q-functions. The gradient of this penalty function admits (via the chain rule) a closed-form expression. For low background noise, and assuming that at each step of the adaptation the detector can be guaranteed to have positive asymptotic efficiency (so that the adaptive law operates in the region where the cost function is convex), this detector should converge to the optimum linear multiuser detector obtained by Lupas and Verdú [2] which makes better use of the amplitudes than the MMSE detector.

5. Tentative-Decision Based Multiuser Detectors

One of the simplest ideas in multiuser detection is that of successive cancellation: detect the data of the strongest user with a conventional detector and then subtract the signal due to that user from the received waveform. The process can then be repeated with the resulting waveform which contains no trace of the signal due to the strongest user assuming no error was made in its demodulation. This technique has the disadvantage that it requires extremely accurate estimation of the received amplitudes, and unless the users can be ordered so that the received amplitudes satisfy

$$A_1 \gg A_2 \gg \cdots \gg A_K$$

its performance is actually worse than that of the decorrelating detector which requires no knowledge of the received amplitudes. A related technique is the multistage detection of Varanasi and Aazhang where the first stage consists of a bank of conventional detectors [4] or a decorrelator [5]; the second stage assumes that the previous decisions are correct and simply cancels the corresponding signals from the received waveform, thereby resulting in a clear single-user channel in the event that previous decisions are indeed correct. The decorrelating decision-feedback detector of Duel-Hallen [41] (and its adaptive version in [21]) incorporates features common to both successive cancellation and multistage detection with a decorrelating front-end. Similarly, it is possible to assume that decisions made about earlier bits in an asynchronous system are correct and therefore they can be cancelled, as in conventional single-user decision-feedback equalization (DFE). The application of this idea to multiuser detection goes back to [42].

This philosophy has been adopted in the synchronous case by Abdulrahman and Falconer [43] and in a multipath QPSK multiaccess channel by Abdulrahman, Falconer and Sheikh [44], [45] which uses a fractionally-spaced DFE detector whose feedforward and feedback coefficients are adapted to minimize mean-square error using training sequences. Another adaptive multiuser detector based on DFE is experimentally demonstrated by Stojanovic and Zvonar for a channel with severe multipath [46]. Kohno, Imai, Hatori and Pasupathy [47] consider a CDMA channel with limited bandwidth for which they design an adaptive MMSE detector that uses decision-feedback to remove intersymbol interference. The first stage in that detector (which uses knowledge of all the signature waveforms) performs preliminary decision which are then used in the adaptive stage. Rapajic and Vucetic [31], [48] find no improvement over the adaptive MMSE detector by incorporating the possibility of decision feedback. Adaptive versions of the multistage detectors of Varanasi and Aazhang have been proposed by Chen, Siveski and Bar-Ness [49] (in the case of conventional tentative decisions) and [50], [51] (in the case of a decorrelating first stage). In those detectors, the first stage is nonadaptive and requires knowledge of all the signature waveforms. However, the interference canceller is adaptive and does not require knowledge of amplitudes. The adaptation is carried out by gradient descent of the energy of the difference between y and the output of the linear adaptive canceller (or a different penalty function in [52]), and therefore, it does not require training sequences. Another adaptive two-stage multiuser detector based on soft tentative decisions is proposed by Brady and Catipovic [53], which uses knowledge of training sequences and signature waveforms in order to adapt to the channel parameters and refine a coarse initial estimate of timing and phase.

6. Blind Multiuser Detection

The requirement of training sequences in the multiuser detectors surveyed above is a cumbersome one in multiuser communications. Since transmitters start and finish their transmissions asynchronously, the "birth" (or "death") of an interferer requires the recomputation of the adaptive receiver coefficients. Often, decision-directed operation of the adaptive detector is not robust enough to take care of those sudden changes, and the desired user must be asked to interrupt its data transmission so that a training sequence is transmitted.

In this section we will review a recent adaptive multiuser detector due to Honig, Madhow and Verdú [54] which has the following features:

- it achieves optimal near-far resistance.
- (approximate) knowledge of the signature waveform of the desired user is required.
- the timing of the desired user must be acquired.
- the received amplitudes need not be known or estimated.
- the signature waveforms of the interferers need not be known.
- the timing of interfering users need not be acquired.
- Training sequences are not required for any user.

Therefore, we will see that it is possible to attain the same near-far resistance as the optimum receiver, the same asymptotic efficiency as the decorrelating detector, and the same bit error rate as the linear MMSE detector with no more than the knowledge assumed by the conventional single-user detector. Although the approach of this detector is reminiscent of that of anchored minimum energy blind equalization proposed in [55], the solution of [54] does not have a counterpart in single-user communication, in contrast to the above multiuser detectors. With few changes, it is possible to generalize the design and analysis of the blind multiuser detector below to the asynchronous case.

The blind multiuser detector of [54] adapts a linear transformation of the observations whose impulse response is c_1 (assuming that the desired user is $k = 1$), and outputs the decision

$$\hat{b}_1 = \text{sgn}\,(<y, c_1>) \tag{13}$$

Any linear multiuser detector can be written in a *canonical orthogonal decomposition:*

$$c_1 = s_1 + x_1 \tag{14a}$$

where

$$<s_1, x_1> = 0 \tag{14b}$$

The only c_1 that cannot be represented in this form are those for which

$$<c_1, s_1> \neq 1, \tag{15}$$

but the decisions are scale invariant, and if c_1 were orthogonal to s_1, the bit error rate would be 0.5. Thus, the freedom we lose in the decomposition (14), is (like in marriage) a freedom we do not need to have.

Let us focus attention on adapting x_1, while preserving orthogonality to s_1. The energy (or more precisely, the second moment) of the output of the linear transformation

$$<y, s_1 + x_1>$$

has three additive independent components: the first due to the desired user, the second due to the multiaccess interference, and the third due to the background noise. The first component is transparent to the choice of x_1. Thus, by varying x_1 can can only change the energy of the second and third components. Accordingly, a very simple and sensible strategy is to choose x_1 that minimizes the output energy:

$$MOE(x_1) = E[(<y, s_1 + x_1>)^2] \tag{16}$$

We would expect that if the background noise is comparatively small, the argument x_1 that minimizes (16) is such that it (almost) eliminates the contribution of the multiuser interferers to the output, in other words $s_1 + x_1$ would approach the decorrelating detector. For higher background noise, x_1 would try to attenuate the contribution of the multiaccess interference, but without becoming too large in norm, and thus contributing a large component due to the background noise. We need to speculate no further about the nature of the minimum output energy detector, because it is easy to check that the output energy in (16) is a translated version of the mean-square error:

$$MOE(x_1) = E[(A_1 b_1 - <y, s_1 + x_1>)^2] + A_1^2. \tag{17}$$

Therefore, the x_1 that minimizes (16) is such that $s_1 + x_1$ is the MMSE linear detector of Section 4. If the minimum output energy detector is the MMSE detector, what is the point of this alternative derivation? The adaptive minimization of mean square error requires training sequences, whereas the minimization of output energy does not. Therefore, the minimization of the convex cost function (16) lends itself to blind adaptation. The simple method of projected gradient descent is adopted in [54] to show the following blind adaptation rule, which is guaranteed to converge globally:

$$x_1[i] = x_1[i-1] - \mu Z[i]\,(y[i] - Z_{MF}[i]\,s_1) \tag{18}$$

where Z_{MF} and Z are the outputs of the conventional single-user matched filter and of the proposed linear transformation:

$$Z_{MF}[i] = <y[i], s_1>, \tag{19}$$

$$Z[i] = <y[i], s_1 + x_1[i-1]>, \tag{20}$$

The generalization of (18) to the asynchronous case is straightforward. In fact, in order to write the key equation (17) we did not invoke any structure of the multiaccess interference. In the asynchronous case, we can work with signals (or finite dimensional vectors) that span only one bit, or in order to improve performance, we can lengthen the duration of the linear transformation on both sides of the timelimited signal s_1. As usual, it should be possible to speed up convergence speed at the expense of computational complexity by adopting an RLS-based method.

The foregoing simple blind adaptive multiuser detector, which as we have seen, has no more requirements than the conventional detector, and yet, converges always to an optimally near-far resistant solution, is ideally suited to cope with transients due to initialization, powering on/off of interferers, or sudden changes in received power. The slower variations that occur due to offset drift, slow fading, etc. could be followed more closely (albeit, less robustly), by an MMSE adaptive detector operating in decision-directed mode, in lieu of training sequences.

In practice, there will always be some mismatch between the received signature waveform s_1 of the desired user and the assumed (nominal) waveform $\hat{s}_1$. So the natural question to investigate is how robust will the blind multiuser detector be to mismatch? The answer depends on the background noise level. If $\hat{s}_1$ is different from s_1 as well as from the other interfering signature waveforms, one can always choose x_1 orthogonal to $\hat{s}_1$, so that $\hat{s}_1 + x_1$ will be orthogonal to the signals of all users: $s_1, \cdots s_K$. This may require an x_1 with huge norm, but if $\sigma \rightarrow 0$, then this will indeed be the solution that minimizes output energy. This means that in high SNR channels, the foregoing detector is not robust at all against mismatch in the nominal signal. In particular, as long as $\hat{s}_1$ is different from s_1, the asymptotic multiuser efficiency is equal to zero. An increase in background noise will have a robustifying effect. In that case a desired signal suffering a small mismatch will not be cancelled because that would require an x_1 with very large energy, and thus a correspondingly large contribution to the output energy due to the background noise. Fortunately, we can achieve the same robustifying effects of background noise even in high SNR situations by simply putting a constraint on the maximum allowable energy of x_1, referred to as *surplus energy* in [54]. The modified blind algorithm with constrained surplus energy is

$$x_1[i] = \beta\, x_1[i-1] - \mu Z[i]\,(y[i] - Z_{MF}[i]\, s_1) \qquad (21)$$

where $0 < \beta < 1$. Note that the conventional single-user receiver corresponds to a blind multiuser detector with zero surplus energy, while allowing unlimited surplus energy makes the detector non-robust against desired signal mismatch in high SNR channels. A good choice for the surplus energy is the energy necessary to *eliminate the interfering signals*, which turns out to be -1 plus the reciprocal of the near-far resistance of a desired user with signal $\hat{s}_1$ and interfering signals $s_2, \ldots s_K$. In general, it is necessary that the nominal signal $\hat{s}_1$ be closer to s_1 than to the space spanned by the interfering signals. Provided this is satisfied and the blind detector has reached a stage in its convergence where the bit error rate is not too high, the assumed nominal can be refined by correlating the received waveform with the decisions of the user of interest:

$$\frac{1}{L}\sum_{i}^{L} \hat{b}_1[i]\, y[i]. \qquad (22)$$

The estimator in (22) converges, by the law of large numbers, to a scaled version of the received signature waveform of the desired user s_1.

There have been other efforts in blind multiuser detection. Oda and Sato [56] consider a multidimensional generalization of the conventional single-user blind equalization methods that attempt to minimize a nonconvex function of the output. The channel model can be specialized to synchronous CDMA; however since the equalizer in [56] does not use knowledge of any signature waveforms or data, bit error rate performance would be poor for weak users. Convergence is (as in the single-user case) not guaranteed using this method. Soon and Tong [57] develop a blind identification algorithm for a synchronous noiseless multiuser channel, which requires introducing a different amount of correlation in the data modulated by each user. The method is based on the singular value decomposition of the estimated covariance of the vector of observables (obtained by fractional sampling). Paris[58] proposes a blind seft-tuning maximum likelihood sequence estimator which, in principle, could be used for optimum asynchronous multiuser detection without prior knowledge of amplitudes and crosscorrelations.

7. Neural Network Multiuser Detectors

The first paper that considered the applicability of adaptive neural network receivers to multiuser detection is due to Aazhang, Paris and Orsak [59] where they study a multilayer perceptron. Each node in the first stage computes a *nonlinear function of a linear transformation of the* matched filter outputs. The number of neurons grows exponentially with the number of users. The signature waveforms are assumed known and training sequences are employed in order to adapt the linear transformation of the matched filter outputs. The adaptation is by gradient descent of mean square error (which in the context of neural networks is known as backpropagation), although in this problem this cost function is not convex and has local minima. Two different configurations are simulated: with training sequences for the desired users, and with training sequences for all users. Simulations show that this difference turns out to have a very important effect on the nature of the detector to which the network converges. Assuming knowledge of the desired user's spreading code, Mitra and Poor [60] give a convergence analysis of a single layer perceptron, which can be viewed as a modified version of (12) where the update term is multiplied by a nonlinear function of the adaptive linear transformation. A so-called radial-basis-function neural network is proposed in [61] for single-user equalization and investigated in [62] in synchronous multiuser detection. The number of nodes is exponential with the number of users, and the decision statistic is a linear combination of nonlinear transformations of the observables.

Miyajima, Hasegawa and Haneishi [63] propose a Hopfield neural network for synchronous multiuser detection using the likelihood function as the energy function to be minimized. The weights of the network are nonadaptive and equal to the crosscorrelations times the corresponding amplitudes, both of which are assumed known. When the true minimum of the function is found, the decisions are optimum. Although the network does not always converge to the global minimum, this approach has shown promise in the solution of other NP-complete combinatorial optimization problems. It is shown empirically in [63] that the probability of convergence to spurious local minima increases with the number of users, the background noise level, or when the interfering signals are weak. However, the achieved bit error rate is near-optimum.

Finally we mention the application of Kohonen's Self-Organizing Map to synchronous multiuser detection to be presented at this conference [64]. This algorithm works with a matched filter bank front-end, and thus, it assumes knowledge of the signature waveforms; however it does not require the use of training sequences or knowledge of amplitudes in order to adapt the decision boundaries of the detector.

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