Asymptotic Eigenvalue Moments for Linear Multiuser Detection

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Abstract

Using the theory of lattices of non-crossing partitions, an explicit expression for the asymptotic moments of certain infinite random matrices is obtained and extended to several cases. By using this explicit expression, we obtain a self-contained proof of the Tse-Hanly formula for the output signal-to-interference-plus-noise ratio of MMSE multiuser detector. The asymptotic moment results are used to design a low-complexity polynomial approximation of linear multiuser detector. Analytical and numerical results of their performance analysis are also given.

1 Introduction

In order to gain insight into the performance of receivers in a DS-CDMA system with large processing gain and many users, much work has been devoted to the asymptotic analysis for synchronous DS-CDMA with random spreading [8, 6].

In Section 2, using the theory of lattices of non-crossing partitions, explicit expressions for the asymptotic eigenvalue moments of several classes of infinite random matrices are obtained.

There are several motivations and applications for these results. In Section 3, the large-system limit of the optimal weights for the reduced-rank MMSE receiver and the reduced-rank Linear-Conjugate MMSE receiver (LCM) [7] are obtained in six environments from the explicit expression of the moments. The desirable feature of using the asymptotic limit of the optimal weights instead of the optimal weights themselves is that the asymptotic limit does not depend on the realizations of the spreading sequences, which is particularly useful when the CDMA system uses long sequences. The asymptotic performance of the full-rank receivers in the environments, considered in Section 3, is given and the asymptotic equivalence of them is established. In Section 4, we briefly describe how to derive the fixed point equation satisfied by the asymptotic output SINR of the MMSE receiver based on the asymptotic moments. The equation is originally derived in [6] by means of the Silverstein and Bai theorem [5]. Section 5 contains our numerical results.

2 Asymptotic Eigenvalue Moments

If $\mathbf{V}_n$ is a self-adjoint matrix, *the asymptotic eigenvalue distribution* of $\mathbf{V}_n$ is the function $F(\lambda)$ described by [9]:

$$\int \lambda^m dF(\lambda) \triangleq \lim_{n\to\infty} \frac{1}{n}\mathrm{E}[\mathrm{Trace}\{\mathbf{V}_n^m\}]$$

where $\lim_{n\to\infty} \frac{1}{n}\mathrm{E}[\mathrm{Trace}\{\mathbf{V}_n^m\}]$ is the m-th asymptotic moment of $\mathbf{V}_n$. It is easy to see that [9]:

$$F(\lambda) = \lim_{n\to\infty} E[F_n(\lambda)]$$

where $F_n(\lambda)$ is the empirical distribution of the eigenvalues of $\mathbf{V}_n$ [9].

Let us introduce here the necessary notation used in the following results: suppose a vector of k integers $(m_1, \ldots, m_k)$ is partitioned into n equivalence classes under the equivalence relation $a = b$, and the cardinalities of the equivalence classes are given by $f_1, \ldots, f_n$, then $f(m_1, \ldots, m_k) \triangleq f_1! \cdots f_n!$. For example, $f(1,1,4,2,1,2) = 3! \cdot 2! \cdot 1!$. In the following sections we give six propositions; for their proofs please refer to [3].

2.1 I.I.D. Sequences

Proposition 1: *Suppose the* $N \times K$ *matrix* $\mathbf{S}$ *consists of i.i.d. zero-mean random variables with variance* $1/N$, $\mathbf{D}$ *is a* $K \times K$ *diagonal matrix with non-negative diagonal elements with empirical eigenvalue distribution converging in distribution to a non-random limit function. Then the* m^{th} *moment* λ_m *of* $\mathbf{SDS}^H$ *as* K, N *go to infinity with* $K/N = \beta$ *is:*

$$\lambda_m = \sum_{k=1}^{m} \beta^k \sum_{m_1+\cdots+m_k=m} c(m_1, \ldots, m_k)\mathrm{E}[\Lambda^{m_1}] \cdots \mathrm{E}[\Lambda^{m_k}] \quad (1)$$

where Λ is a nonnegative random variable whose distribution is the non-random limit distribution of $\mathbf{D}$, and

$$c(m_1,\dots,m_k) = \frac{m!}{(m-k+1)!\cdot f(m_1,\dots,m_k)}, \qquad (2)$$

with $f(m_1,\dots,m_k)$ as defined above.

2.2 Repeated Sequences with Block I.I.D. Phases

Proposition 2: *Let $N{=}LN'$. Let $\mathbf{t}_k = (U_{k,1},\dots,U_{k,N'})^T$ $(1 \le k \le K)$, where $U_{k,j}$ $(1{\le}k{\le}K$ $1{\le}j{\le}N')$ are real- valued i.i.d. zero-mean random variables with unit variance. Let $\mathbf{S} = (\mathbf{s}_1,\dots,\mathbf{s}_K)$ where*

$$\mathbf{s}_k = \frac{1}{\sqrt{N}}(\mathbf{t}_k^T e^{j\phi_{k,1}},\dots,\mathbf{t}_k^T e^{j\phi_{k,L}})^T,$$

and $\phi_{k,l}$ $(1{\le}k{\le}K,\ 1{\le}l{\le}L)$ are i.i.d. random variables uniformly distributed on $[0,2\pi)$ $(\mathcal{U} \sim (0,2\pi))$. $\mathbf{D}$ is as defined in Proposition 1. Then the m^{th} moment λ_m of $\mathbf{SDS}^H$ is given by (1).

2.3 Conjugate Sequences

Proposition 3: *Let $N{=}2N'$. Let $\mathbf{t}_k = (U_{k,1},\dots,U_{k,N'})^T$ $(1 \le k \le K)$, where $U_{k,j}$ $(1{\le}k{\le}K$ $1{\le}j{\le}N')$ are real- valued i.i.d. zero-mean random variables with unit variance. Let $\mathbf{S} = (\mathbf{s}_1,\dots,\mathbf{s}_K)$ where*

$$\mathbf{s}_k = \frac{1}{\sqrt{N}}(\mathbf{t}_k^T e^{j\phi_k},\mathbf{t}_k^T e^{-j\phi_k})^T,$$

and ϕ_k $(1 \le k \le K)$ are i.i.d. random variables uniformly distributed on $[0,2\pi)$, and $1/\sqrt{N}$ is the normalizing factor. $\mathbf{D}$ is as defined in Proposition 1. Then the m^{th} moment λ_m of $\mathbf{SDS}^H$ is given by (1).

2.4 Repeated Sequences with Conjugate Block I.I.D. Phases

Proposition 4: *Let $N{=}2LN'$. Let $\mathbf{t}_k = (U_{k,1},\dots,U_{k,N'})^T$ $(1 \le k \le K)$, where $U_{k,j}$ $(1{\le}k{\le}K$ $1{\le}j{\le}N')$ are real- valued i.i.d. zero-mean random variables with unit variance. Let $\mathbf{S} = (\mathbf{s}_1,\dots,\mathbf{s}_K)$ where $\mathbf{s}_k = \frac{1}{\sqrt{N}}(\mathbf{v}_k^T,\mathbf{v}_k^H)^T$*

$$\mathbf{v}_k = (\mathbf{t}_k^T e^{j\phi_{k,1}},\dots,\mathbf{t}_k^T e^{j\phi_{k,L}})^T,$$

and $\phi_{k,l}$ $(1{\le}k{\le}K,\ 1{\le}l{\le}L)$ are i.i.d. random variables $\mathcal{U} \sim (0,2\pi)$. $\mathbf{D}$ is as defined in Proposition 1. Then the m^{th} moment λ_m of $\mathbf{SDS}^H$ is given by (1).

2.5 Repeated Sequences with Linearly Increasing Phases

Proposition 5: *Let $N{=}LN'$. Let $\mathbf{t}_k = (U_{k,1},\dots,U_{k,N'})^T$ $(1 \le k \le K)$, where $U_{k,j}$ $(1{\le}k{\le}K$ $1{\le}j{\le}N')$ are real- valued i.i.d. zero-mean random variables with unit variance. Let $\mathbf{S} = (\mathbf{s}_1,\dots,\mathbf{s}_K)$ where*

$$\mathbf{s}_k = \frac{1}{\sqrt{N}}(\mathbf{t}_k^T e^{j\phi_k},\mathbf{t}_k^T e^{2j\phi_k},\dots,\mathbf{t}_k^T e^{Lj\phi_k})^T,$$

and ϕ_k $(1 \le k \le K)$ are i.i.d. random variables $\mathcal{U} \sim (0,2\pi)$. $\mathbf{D}$ is as defined in Proposition 1. Then the m^{th} moment λ_m of $\mathbf{SDS}^H$ is given by (1).

2.6 Repeated Sequences with Conjugate Linearly Increasing Phases

Proposition 6: *Let $N{=}2LN'$. Let $\mathbf{t}_k = (U_{k,1},\dots,U_{k,N'})^T$ $(1 \le k \le K)$, where $U_{k,j}$ $(1{\le}k{\le}K$ $1{\le}j{\le}N')$ are real- valued i.i.d. zero-mean random variables with unit variance. Let $\mathbf{S} = (\mathbf{s}_1,\dots,\mathbf{s}_K)$ where $\mathbf{s}_k = \frac{1}{\sqrt{N}}(\mathbf{v}_k^T,\mathbf{v}_k^H)^T$*

$$\mathbf{v}_k = (\mathbf{t}_k^T e^{j\phi_k},\dots,\mathbf{t}_k^T e^{Lj\phi_k})^T,$$

and ϕ_k $(1 \le k \le K)$ are i.i.d. random variables $\mathcal{U} \sim (0,2\pi)$. $\mathbf{D}$ is as defined in Proposition 1. Then the m^{th} moment λ_m of $\mathbf{SDS}^H$ is given by (1).

3 Asymptotic Reduced-Rank Receiver

Expressions for the asymptotic moments can be used to implement the asymptotic reduced-rank MMSE/LCM receivers. In what follows, we consider a synchronous DS-CDMA with K active users and processing gain N, analyzed in six different environments: (1) reduced-rank single antenna MMSE receiver; (2) reduced-rank multiantenna MMSE receiver; (3) reduced-rank single antenna LCM receiver; (4) reduced-rank multiantenna LCM receiver; (5) reduced-rank multiantenna MMSE receiver in line-of-sight transmission; (6) reduced-rank multiantenna LCM receiver in line-of-sight transmission. These situations correspond to applications of Propositions 1-6, respectively. In what follows, A_k is the received amplitude of user k, $b_k \in \{\pm 1\}$ is the bit transmitted by user k, and $\mathbf{s}_k$ is the spreading sequence of user k. The phase of the complex fading coefficient of user k at the l^{th} antenna element is $e^{j\phi_k(l)}$. The phases $\phi_k(l)$ are modelled as i.i.d. random variables uniformly distributed on $[0,2\pi)$. The spreading sequences and the fading channel coefficients are assumed known at the receiver. $\mathbf{n}_l$ is the noise vector at l^{th} receiver antenna modelled as additive white Gaussian noise with covariance matrix $\sigma^2\mathbf{I}$. $\mathbf{s}_k$ is the unitary energy spreading sequences of the k^{th} user modelled as random vector with i.i.d zero-mean entries with variance $1/\sqrt{N}$.

3.1 Reduced-Rank multi-antenna MMSE Receiver

The output of the chip-matched filter at the l^{th} antenna element is

$$\mathbf{r}_l = \sum_{k=1}^{K} A_k e^{j\phi_k(l)} b_k \mathbf{s}_k + \mathbf{n}_l \qquad l = 1, \ldots, L \tag{3}$$

In the equation above, we assume that, for each user, the received amplitudes at different antenna elements are the same for simplicity. This is valid if the antenna elements are close enough to each other so that for a given user, the received powers at different antenna elements vary little. However, the received phases still vary considerably because different propagation paths have a much greater impact on the received phases due to the usually high carrier frequency and can be modelled $\mathcal{U} \sim (0, 2\pi)$. In matrix form

$$\bar{\mathbf{r}} = \overline{\mathbf{S}}\mathbf{A}\mathbf{b} + \bar{\mathbf{n}}, \tag{4}$$

where $\bar{\mathbf{r}} = (\mathbf{r}_1^T, \ldots, \mathbf{r}_L^T)^T$, $\overline{\mathbf{S}} = (\bar{\mathbf{s}}_1, \ldots, \bar{\mathbf{s}}_K)$, $\bar{\mathbf{s}}_k = (\mathbf{s}_k^T e^{j\phi_k(1)}, \ldots, \mathbf{s}_k^T e^{j\phi_k(L)})^T$, $\mathbf{A} = \mathrm{diag}\{A_1, \ldots, A_K\}$, $\mathbf{b} = (b_1, \ldots, b_K)^T$, and $\bar{\mathbf{n}} = (\mathbf{n}_1^T, \ldots, \mathbf{n}_L^T)^T$. The MMSE receiver for user 1 is

$$\mathbf{c} = (\overline{\mathbf{S}}_1 \mathbf{D}_1 \overline{\mathbf{S}}_1^H + \sigma^2 \mathbf{I})^{-1} \bar{\mathbf{s}}_1, \tag{5}$$

where $\overline{\mathbf{S}}_1 = (\bar{\mathbf{s}}_2, \ldots, \bar{\mathbf{s}}_K)$, and $\mathbf{D}_1 = \mathrm{diag}\{A_2^2, \ldots, A_K^2\}$. The rank D ($D \leq LN$) reduced-rank MMSE receiver for user 1 is

$$\mathbf{c}_D = \sum_{m=0}^{D-1} w_m \left(\overline{\mathbf{S}}_1 \mathbf{D}_1 \overline{\mathbf{S}}_1^H + \sigma^2 \mathbf{I}\right)^m \bar{\mathbf{s}}_1. \tag{6}$$

where the weight vector $\mathbf{w} = (w_0, \ldots, w_{D-1})^T$ is chosen to maximize the output SINR and is given by [4, 7]

$$\mathbf{w}^T = [\mathcal{H}_0 \ldots \mathcal{H}_{D-1}]^T (\mathbf{M}^{-1})^T, \tag{7}$$

where the (i, j)-entry of the matrix $\mathbf{M}$ is $(\mathbf{M})_{i,j} = \mathcal{H}_{i+j-1} + A_1^2 \mathcal{H}_{i-1} \mathcal{H}_{j-1}$ and

$$\mathcal{H}_m = \bar{\mathbf{s}}_1^H \left(\overline{\mathbf{S}}_1 \mathbf{D}_1 \overline{\mathbf{S}}_1^H + \sigma^2 \mathbf{I}\right)^m \bar{\mathbf{s}}_1. \tag{8}$$

We can see that $\mathbf{w}$ depends on the realizations of the spreading sequences. Therefore, in a system using long sequences, they need to be computed and updated symbol to symbol, which hampers real-time implementation. The large system limit of the weights seems to be a promising solution to this problem. The large system limit is taken as N and K go to infinity with $K/N = \beta$. By using the theory of non crossing partitions [3], it can be shown that $\mathcal{H}_m^\infty$ converges in probability to:

$$\begin{aligned}
\mathcal{H}_m^\infty &\triangleq \lim_{\substack{N,K \to \infty \\ \frac{K}{N} = \beta}} \bar{\mathbf{s}}_1^H \left(\overline{\mathbf{S}}_1 \mathbf{D}_1 \overline{\mathbf{S}}_1^H + \sigma^2 \mathbf{I}\right)^m \bar{\mathbf{s}}_1 \\
&\to \lim_{\substack{N,K \to \infty \\ \frac{K}{N} = \beta}} \frac{1}{N} \mathrm{E}[\mathrm{Trace}\{\left(\overline{\mathbf{S}}_1 \mathbf{D}_1 \overline{\mathbf{S}}_1^H + \sigma^2 \mathbf{I}\right)^m\}] \\
&= L^{m+1} \sum_{n=0}^{m} \sigma^{2m-2n} \lim_{\substack{N,K \to \infty \\ \frac{K}{N} = \beta}} \frac{1}{LN} \mathrm{E}\left[\mathrm{Tr}\left\{\left(\widetilde{\mathbf{S}}_1 \mathbf{D}_1 \widetilde{\mathbf{S}}_1^H\right)^n\right\}\right] \\
&= \sum_{n=0}^{m} L^{n+1} \binom{m}{n} \mu_n^\infty \sigma^{2m-2n}
\end{aligned} \tag{9}$$

where μ_n is the asymptotic moments of the $LN \times K$ dimensional matrix $\tilde{\overline{\mathbf{S}}}_1 \mathbf{D}_1 \tilde{\overline{\mathbf{S}}}_1^H$ with $\tilde{\overline{\mathbf{S}}}_1 = \overline{\mathbf{S}}_1/\sqrt{L}$. Proposition 2 can be applied to find $\mathcal{H}_m^\infty$ and therefore the asymptotic weights.

3.2 Reduced-Rank multi-antenna LCM Receiver

Similar to the single antenna case, the multi-antenna LCM receiver for user 1 is

$$\mathbf{c} = \mathbf{F}\left(\overline{\mathbf{S}}_{1a} \mathbf{D}_1 \overline{\mathbf{S}}_{1a}^H + \sigma^2 \mathbf{I}\right)^{-1} \bar{\mathbf{s}}_{1a}, \tag{10}$$

where $\mathbf{F} = [\mathbf{I}_{LN} \ \ \mathbf{0}]$, and

$$\bar{\mathbf{s}}_{1a} = \begin{bmatrix} \bar{\mathbf{s}}_1 \\ \bar{\mathbf{s}}_1^* \end{bmatrix}, \overline{\mathbf{S}}_{1a} = \begin{bmatrix} \bar{\mathbf{s}}_2 & \cdots & \bar{\mathbf{s}}_K \\ \bar{\mathbf{s}}_2^* & \cdots & \bar{\mathbf{s}}_K^* \end{bmatrix}. \tag{11}$$

The rank D ($D \leq 2LN$) reduced-rank LCM receiver for user 1 is

$$\mathbf{c}_D = \sum_{m=0}^{D-1} w_m \mathbf{F}\left(\overline{\mathbf{S}}_{1a} \mathbf{D}_1 \overline{\mathbf{S}}_{1a}^H + \sigma^2 \mathbf{I}\right)^m \bar{\mathbf{s}}_{1a}. \tag{12}$$

The weights are again given by (7) where $(\mathbf{M})_{i,j} = \mathcal{H}_{i+j-1}$ with

$$\mathcal{H}_m = \bar{\mathbf{s}}_{1a}^H \left(\overline{\mathbf{S}}_{1a} \mathbf{D}_1 \overline{\mathbf{S}}_{1a}^H + \sigma^2 \mathbf{I}\right)^m \bar{\mathbf{s}}_{1a}. \tag{13}$$

To calculate $\mathcal{H}_m^\infty$, we notice that the energy (defined by $\mathrm{E}\{\| \cdot \|^2\}$) of $\bar{\mathbf{s}}_{1a}$ and every column of $\overline{\mathbf{S}}_{1a}$ is $2L$. By using again the theory of non-crossing partition and following similar arguments in Section (3.1), it is easily shown that $\mathcal{H}_m^\infty$ converges in probability to::

$$\begin{aligned}
\mathcal{H}_m^\infty &\to (2L)^{m+1} \lim_{\substack{N,K \to \infty \\ \frac{K}{N} = \beta}} \frac{1}{2NL} \mathrm{E}\left[\mathrm{Tr}\left\{\left(\widetilde{\mathbf{S}}_{1a} \mathbf{D}_1 \widetilde{\mathbf{S}}_{1a}^H + \frac{\sigma^2}{2L}\mathbf{I}\right)^m\right\}\right] \\
&= \sum_{n=0}^{m} 2^{n+1} L^{n+1} \binom{m}{n} \tau_n^\infty \sigma^{2m-2n}
\end{aligned} \tag{14}$$

where τ_n is the asymptotic n^{th} moment of the $2LN \times K$ matrix $\tilde{\bar{\mathbf{S}}}_{1a}\mathbf{D}_1\tilde{\bar{\mathbf{S}}}_{1a}^H$ with $\tilde{\bar{\mathbf{S}}}_{1a} = \bar{\mathbf{S}}_{1a}/\sqrt{2L}$. Proposition 4 can be applied to find $\mathcal{H}_m^\infty$ and the asymptotic weight vector $\mathbf{w}^\infty$.

3.3 All other cases

a) **Reduced-rank single-antenna MMSE and LCM receivers.** Both receivers can be obtained as a special case of reduced-rank multi-antenna MMSE receiver and reduced-rank multi-antenna LCM receiver respectively when L=1. The asymptotic weight of Reduced-rank single-antenna MMSE and LCM receivers are given by (9) and (14) respectively where μ_n^∞ is the n^{th} moments of $\mathbf{S}_1\mathbf{D}_1\mathbf{S}_1^H$ whose moments explicit expression can be computed by Proposition 1, and τ_n^∞ is the n^{th} moments of $N \times K$ dimensional matrix $\tilde{\bar{\mathbf{S}}}_{1a}\mathbf{D}_1\tilde{\bar{\mathbf{S}}}_{1a}^H$ with $L = 1$ whose moments explicit expression can be computes by Proposition 3.

b) **Reduced-rank multi-antenna MMSE Receiver in Line-of-Sight Transmission.** In a line-of-sight transmission DS-CDMA system with multiple receive antennas, the received vector is given by (3) where $\phi_k(l) = 2\pi d \cos\alpha_k l/\lambda = \phi_k l$ with d the distance between two adjacent antennas, α_k the incident angle of user k, and λ the wavelength. Due to the randomness in the geographic distribution of users, it is reasonable to assume that ϕ_k are i.i.d. random variables uniformly distributed in $[0, 2\pi)$. As a consequence the asymptotic weight of the reduced-rank multi-antenna MMSE receivers in LoST are again given by (9). In this case μ_n is the n^{th} moment of $\tilde{\mathbf{S}}_1\mathbf{D}_1\tilde{\mathbf{S}}_1^H$ with $\phi_k(l) = \phi_k l$. Its moments explicit expression can be computed using Proposition 5.

c) **Reduced-rank multi-antenna LCM Receiver in Line-of-Sight Transmission.** Again in this case the received vector is given by (3) where $\phi_k(l) = \phi_k l$. The asymptotic weight of the reduced-rank multi-antenna LCM receivers in LoST are again given by (14) where, in this case τ_n is the n^{th} moment of $\tilde{\bar{\mathbf{S}}}_{1a}\mathbf{D}_1\tilde{\bar{\mathbf{S}}}_{1a}^H$ with $\phi_k(l) = \phi_k l$ whose moments explicit expression can be computed using Proposition 6.

Notice that from the equality of the asymptotic moments in Propositions 1 and 4, it is easy to see that the asymptotic output SINR of the single antenna MMSE receiver with $(K, 2LN, \sigma^2/2)$ is the same as that of the multi-antenna LCM receiver with (K, N, σ^2). Similar results hold for the multi-antenna MMSE receiver, the single antenna LCM receiver, and the MMSE/LCM receivers in line-of-sight transmission. We note that the results on the multi-antenna MMSE receiver and the single-antenna LCM receiver were originally obtained in [2] and [7], respectively, by using free probability theory.

4 Deriving the Tse-Hanly Formula by Means of the Moments

By using the Silverstein-Bai theorem [5], it is shown in [6] that the asymptotic output SINR γ of the MMSE receiver in a DS-CDMA system with random spreading satisfies the following fixed point equation (without loss of generality, the desired user is received with unit power)

$$\gamma = \frac{1}{\sigma^2 + \int \frac{\beta P}{1+P\gamma} dF(P)}, \tag{15}$$

where $F(P)$ is the limit of the empirical distribution of $P_2, \ldots, P_K$ as K goes to infinity. We point out that the fixed-point equation can also be proved by means of the asymptotic moments. Specifically, we reduce (15) to a combinatorial equality involving the asymptotic moments. Then we prove the equality by using the combinatorial convolution defined on the incidence algebra of non-crossing partitions. For details, please refer to [3].

5 Numerical Results

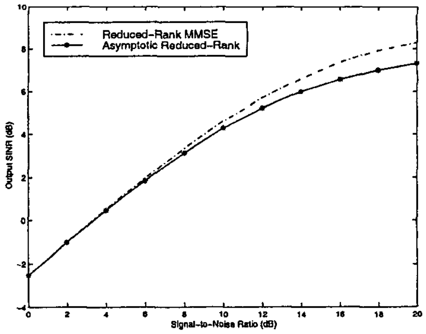

Figure 1: Output SINR vs. input signal-to-noise ratio for two reduced-rank single-antenna receivers.

In all numerical results, $N = 32$, $K = 25$, $L = 2$ (number of receive antenna elements), $D = 5$ (number of stages of the reduced-rank receiver) are used. The received powers of the interfering users are equal and 5 dB above the desired user (power-controlled interferers). The horizontal axis represents the input signal-to-noise ratio P_1/σ^2 in dB. For brevity, we do not analize all cases (for more detatils please refer to [3]).

Fig. 1 shows the performance of two reduced-rank single-antenna MMSE receivers. The upper curve is the output SINR of the reduced-rank MMSE receiver that uses exact optimal weights, and the lower curve is for the asymptotic reduced-rank receiver that uses asymptotic weights. From the figure we can see that the latter lies within 1 dB of the former. We found (not shown in the figure) that if $N > 80$, the performance loss due to using asymptotic weights will be less than 0.5 dB.

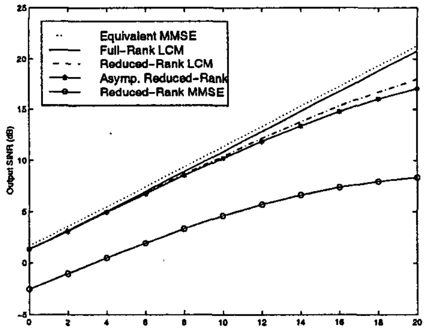

Figure 2: Output SINR vs. input signal-to-noise ratio for single-antenna LCM receivers.

Fig. 2 illustrates the performance of the single-antenna LCM receivers. We observe that the penalty due to using asymptotic weights is within 1.25 dB. As predicted, performance of the full-rank LCM receiver is very close to that of the asymptotically equivalent MMSE receiver (i.e. single-antenna case (L=1) with processing gain $2N$ and noise level $\sigma^2/2$). We found (not shown in the figure) that if $N > 60$, the performance loss due to using asymptotic weights will be less than 0.5 dB, and if $D \geq 8$, the performance gap between the reduced- rank receiver and the full-rank one will be less than 0.5 dB. For comparison purposes, the output SINR of the reduced-rank MMSE receiver in Fig. 1 is also shown here. We can see the significant performance gain obtained by LCM processing.

6 Conclusion

In this paper, we have obtained explicit expressions for the asymptotic eigenvalue moments of several classes of infinite random matrices that arise in multiuser detection in DS-CDMA system with random spreading. The results are useful in the design of the asymptotic reduced-rank MMSE/LCM receivers that use the asymptotic values of the optimal weights. Numerical results show that the penalty in the output SINR due to using the asymptotic weights in place of the exact optimal weights is acceptable for reasonably large processing gain. Finally a self-contained proof of the Tse-Hanly formula has been obtained.

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