

# BLIND ADAPTIVE INTERFERENCE SUPPRESSION FOR NEAR-FAR RESISTANT CDMA

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## ABSTRACT

Multiuser detection techniques can potentially solve the near-far problem in Code-Division Multiple-Access (CDMA) systems. However, these techniques often assume knowledge of system parameters, such as signature waveforms for all users, and associated timing, which may not be available, or may be inconvenient to obtain in practice. Recently proposed adaptive minimum mean squared error (MMSE) detectors do not require knowledge of signature waveforms; however, these techniques require a training sequence for adaptation. Here we propose a blind multiuser linear detector which requires only knowledge of the *desired* user's signature sequence (and associated timing). In particular, it does not require the use of a training sequence. Our approach is to decompose the filter impulse response into the sum of two orthogonal components: the matched filter corresponding to the desired user plus an adaptive filter. We show that if the adaptive filter is chosen to minimize the energy (i.e., variance) of output samples at each symbol interval, then a scaled version of the MMSE detector is obtained. Based on this observation, a simple adaptive gradient algorithm is derived, and numerical examples are presented which illustrate its performance in a synchronous CDMA system.

## 1. Introduction

An important problem with Code-Division Multiple Access (CDMA) systems is the *near-far* problem: a user close to the receiver can disrupt the reception of a desired user relatively far away. Multiuser detection techniques have been proposed as a solution to this problem (see [1] and the references therein). However, these techniques often require explicit knowledge of both the signature waveforms of all users, and the relative delays among the users.

Some attention has been focused recently on *adaptive* multiuser detection [2]-[5], which eliminates the need to know the signature waveforms of the interferers, as well as their delays. The adaptive multiuser detectors in [2]-[5] are based on the minimization of mean squared error (MSE) between the output of the linear receiver and the data. One potential drawback of the adaptive MMSE detectors pro-

posed thus far is the need for training sequences for each active user. Specifically, each transmitter must send a training sequence at start-up which the receiver uses for initial adaptation. After the training phase ends, adaptation during actual data transmission occurs in decision-directed mode. However, any time there is a drastic change in the interference environment (e.g., powering on of a strong interfering transmission or dynamic variations due to fading) decision-directed adaptation becomes unreliable, and data transmission (of the desired user) must be temporarily suspended and yield to a training sequence. The reliance on training sequences may therefore be cumbersome in most CDMA systems, where one of the most important advantages is the ability to have completely asynchronous and uncoordinated transmissions that switch on and off autonomously.

In this paper we present a blind adaptive detection algorithm, which is robust with respect to strong interference (i.e., it is near-far resistant in the sense defined in [6]), and requires knowledge of only the desired user's pulse shape, and associated timing. Note that this is the same knowledge required by the conventional receiver. We remark that blind single-user equalization algorithms, such as the ones discussed in [7], are not likely to be satisfactory in the multiuser environment considered here. This is because these algorithms will generally converge to the *strongest* user, which may not be the user of interest.

The approach we take is to assume a linear detector, and to decompose the filter impulse response into the sum of two orthogonal components: the (fixed) matched filter corresponding to the desired user, called the *anchor*, plus an adaptive filter. The adaptive filter is adapted to minimize the energy (i.e., variance) of the filter output at each symbol interval. We show that the optimal filter which minimizes output energy is in fact a scaled version of the minimum MSE (MMSE) filter. We therefore succeed in obtaining an adaptive MMSE multiuser detector that does not require a training sequence. This blind multiuser detector is reminiscent of the (single-user) anchored minimum energy adaptive equalizer proposed in [8]. That equalizer overcomes some of the ill-convergence problems present in conventional Godard-type blind equalizers (see [7] for a survey) by fixing one tap coefficient, and adapting the rest to minimize output

energy.

In Section 2 we derive the blind adaptation rule that minimizes MSE. In practice, exact knowledge of the signature waveform of the desired user cannot be obtained due to random and time-varying channel distortion. In Section 3, we therefore investigate the effect of using the blind algorithm with a mismatched anchor. In Section 4 we present a simulated numerical example using the algorithm proposed in Section 2. We conclude that a desirable adaptation strategy is to use the blind algorithm initially to reduce strong interference, and to switch subsequently to a conventional decision-directed algorithm.

## 2. Anchored Minimum Energy Detection

### 2.1 Channel model

Even though *synchronous* CDMA systems are more the exception than the rule, here we consider a synchronous system for simplicity and brevity. Generalization of our results to the asynchronous case presents no conceptual difficulties. It is therefore sufficient to consider the baseband received signal during a single symbol interval:

$$y(t) = \sum_{k=1}^K A_k b_k s_k(t) + \sigma n(t), \quad t \in [0, T] \quad (2.1)$$

where  $n(t)$  is white Gaussian noise with unit spectral density, the data  $b_k(t)$  are independent, equally likely to be -1 or +1,  $s_k(t)$  is the  $k$ th signature waveform, which is assumed to have unit energy ( $\|s_k\| = 1$ ), and  $A_k$  is the received amplitude of the  $k$ th user.

For convenience, we assume that the user of interest is  $k = 1$ . A linear detector for user 1 is characterized by the impulse response  $c_1 \in L_2[0, T]$ , such that the decision on  $b_1$  is

$$\hat{b}_1 = \text{sgn}(\langle y, c_1 \rangle) \quad (2.2)$$

where  $\langle x, y \rangle = \int_0^T x(t) y(t) dt$ . We define the following *canonical representation*:

$$c_1 = s_1 + x_1 \quad (2.3a)$$

where

$$\langle s_1, x_1 \rangle = 0 \quad (2.3b)$$

“Canonical” refers to the fact that any linear multiuser detector can be represented in this form. To see why this is true, note first that the set of signals that can be written as in (2.3) are those that satisfy  $\langle c_1, s_1 \rangle = \|s_1\|^2 = 1$ . There is no loss of generality in restricting attention to linear transformations that satisfy this relation, because a) we can discard linear transformations that are orthogonal to the desired signal (they result in error probability equal to 1/2), and b) the decision (2.2) is invariant to (positive) scaling.

### 2.2 MMSE linear multiuser detector

The minimum mean-square error (MMSE) linear multiuser detector for user 1 is defined as the impulse response  $c_1 \in L_2[0, T]$  that minimizes

$$MSE(c) = E[(A_1 b_1 - \langle y, c_1 \rangle)^2] \quad (2.4)$$

For each user  $k$ , we can write

$$c_k(t) = \sum_{j=1}^K c_{kj} s_j(t) \quad (2.5)$$

where  $c_{kj}$  are appropriately chosen coefficients. It is easily shown that minimizing the MSE in (2.4) with respect to these coefficients gives

$$C_{MMSE} = [R + \sigma^2 W^{-1}]^{-1} \quad (2.6)$$

where  $C_{MMSE} = \{c_{kj}\}$ ,  $W = \text{diag}\{A_1^2, \dots, A_K^2\}$ , and  $R = \{\int_0^T s_k(t) s_j(t)\}$ . Another characterization of the linear MMSE detector is given in Section 3.

### 2.3 Minimum output-energy linear detector

We now consider the linear detector in canonical form  $s_1 + x_1$  that minimizes (over all  $x_1$  orthogonal to  $s_1$ ) the mean output energy

$$MOE(x_1) = E[(\langle y, s_1 + x_1 \rangle)^2] \quad (2.7)$$

where the input  $y$  is given by (2.1). It is important to restrict the detector to be in canonical form, otherwise the output energy is trivially minimized with  $c_1 = 0$ .

The motivation for minimizing output energy, as in [8], is that the energy at the output can be written as the sum of the energy due to desired signal plus the energy due to the interference (background noise plus multiaccess interference), and the energy due to the desired signal is transparent to the choice of  $x_1$ . The canonical linear detector which minimizes output energy is, in fact, the MMSE detector:

$$\begin{aligned} MSE(x_1) &= E[(A_1 b_1 - \langle y, s_1 + x_1 \rangle)^2] \\ &= A_1^2 + MOE(x_1) - 2A_1^2 \langle s_1, s_1 + x_1 \rangle \\ &= MOE(x_1) - A_1^2. \end{aligned} \quad (2.8)$$

where the last equality follows from the fact that  $s_1$  is orthogonal to  $x_1$  and has unit energy. Note that (2.8) does not make use of the structure of the interference in (2.1). It is sufficient to assume that it is uncorrelated with the desired signal. We therefore conclude that the  $x_1$  that minimizes MOE also minimizes the MSE.

It is easily shown that the MOE cost function is strictly convex over the set of signals orthogonal to  $s_1$  (a convex set). Consequently, the output energy has no local minima other than the unique global minimum--a most desirable property for gradient adaptation. Finally, we add that the minimum output energy solution exists even in the case where  $s_1$  is spanned by the interferers, because the MMSE

solution (2.6) always exists.

#### 2.4 Blind adaptation rule

The output energy function  $MOE$  lends itself to a simple gradient-descent adaptation rule which we now present. Note that other, potentially faster, estimation techniques could be used in lieu of gradient descent; for example, least squares.

We need to find the projection of the gradient of the output energy  $MOE(x_1)$  on the linear subspace orthogonal to  $s_1$ , so that the orthogonality condition (2.3b) is satisfied at each step of the algorithm. Note that the steepest descent line along the subspace orthogonal to  $s_1$  is the projection of the gradient on that subspace. (The unconstrained gradient can be decomposed as the sum of its projections along  $s_1$  and its orthogonal subspace; steepest descent requires steepest descent in each of those directions.)

Denote the observed waveform in the  $i$ th interval  $[iT, iT + T]$  by  $y[i] \in L_2[0, T]$ . Let the  $i$ th output of the conventional single-user matched filter be the random variable

$$Z_{MF}[i] = \langle y[i], s_1 \rangle, \quad (2.9)$$

Analogously, let the  $i$ th output of the proposed linear transformation be

$$Z[i] = \langle y[i], s_1 + x_1[i-1] \rangle, \quad (2.10)$$

The unconstrained gradient of  $MOE(x_1) = E[\langle y, s_1 + x_1 \rangle^2]$  is equal to a scaled version of the observations,  $2 \langle y, s_1 + x_1 \rangle y$ , and the component of  $y$  orthogonal to  $s_1$  is equal to  $y - \langle y, s_1 \rangle s_1$ . Therefore, the adaptation rule is

$$x_1[i] = x_1[i-1] - \mu Z[i] (y[i] - Z_{MF}[i] s_1) \quad (2.11)$$

where  $\mu$  is a small step-size.

The foregoing derivation has been general enough to apply to any Hilbert space, not necessarily  $L_2[0, T]$ . In particular, in applications where there is a chip matched filter at the front end (or some other sampling mechanism) the signals  $s_1$  and  $x_1$  become vectors in  $\mathbb{R}^N$ , where  $N$  is the number of samples per symbol.

### 3. Effect of Mismatched Nominal

We assumed in Section 2 that the receiver has perfect knowledge of the waveform  $s_1$  modulating the bits of the first user. This may not be true in practice, e.g., the receiver may assume the original spreading waveform as its nominal, whereas the actual received waveform  $s_1$  may include additional multipath components. In this section, we evaluate the performance of the minimum energy detector under such a *mismatch*. Specifically, it is assumed that the linear detector for the desired user is  $c_1 = \hat{s}_1 + x_1$ , where  $\hat{s}_1$  is the assumed *nominal* anchor, and where

$\langle \hat{s}_1, x_1 \rangle = 0$ . We assume that  $\|\hat{s}_1\| = 1$  without loss of generality. When the nominal  $s_1$  is not equal to the desired waveform  $s_1$ , minimizing the energy  $E[\langle y, s_1 + x_1 \rangle^2]$  without additional constraints can cause cancellation of the desired signal, since  $x_1$  is no longer constrained to be orthogonal to the desired waveform. As we shall show, it will become necessary to impose additional constraints on  $x_1$  to prevent signal degradation while preserving the ability to suppress interference.

Henceforth, we represent signals as finite-dimensional vectors and use bold notation to denote them. This is consistent with most implementations, in which the received signal is converted to discrete time prior to processing. For  $1 \leq k \leq K$ , let  $\hat{\rho}_k = \langle s_k, \hat{s}_1 \rangle$  denote the crosscorrelation of the  $k$ th signal vector with the nominal.

Let  $\hat{\mathbf{p}}_S$  denote the projection of  $\hat{s}_1$  orthogonal to the space spanned by  $s_1$ , i.e.,  $\hat{\mathbf{p}}_S = \hat{s}_1 - \langle s_1, \hat{s}_1 \rangle s_1$ . The contribution of the desired signal to the output can be cancelled completely by choosing  $x_1$  such that  $\hat{s}_1 + x_1$  is a scaled version of  $\hat{\mathbf{p}}_S$ . The choice of  $x_1$  shown in Figure 1a is the vector with smallest energy  $\|x_1\|^2 = \chi_S$  which achieves complete signal cancellation. This is also the choice of  $x_1$  dictated by the minimum output energy criterion when there is no noise or interference. Simple geometric observations yield

$$\chi_S = \|x_1\|^2 = \|\hat{s}_1\|^2 \tan^2 \theta_S = \hat{\rho}_1^2 / (1 - \hat{\rho}_1^2), \quad (3.1)$$

where  $\theta_S$  is the angle between  $\hat{s}_1$  and  $\hat{\mathbf{p}}_S$ . In order to prevent serious signal degradation (especially for low noise and interference levels), therefore, we must constrain  $x_1$  such that  $\|x_1\|^2 < \chi_S$ .

Considering now a situation with very strong interference ( $A_2, \dots, A_K \rightarrow \infty$ ), the minimum output energy criterion implies that  $x_1$  must be chosen to completely cancel all the interference. The minimum-norm  $x_1$  satisfying this condition corresponds to  $\hat{s}_1 + x_1$  being a multiple of the projection  $\hat{\mathbf{p}}_I$  of  $\hat{s}_1$  orthogonal to the interference space  $\mathbf{S}_I$  spanned by  $s_2, \dots, s_K$ , as shown in Figure 1b. Reasoning as before, the minimum energy of  $x_1$  for complete interference cancellation, denoted by  $\chi_I$  can be computed as

$$\chi_I = \|\hat{\mathbf{p}}_I\|^{-2} - 1 = \hat{\eta}_I^{-1} - 1, \quad (3.2)$$

where  $\hat{\eta}_I = \|\hat{\mathbf{p}}_I\|^2 / \|\hat{s}_1\|^2 = \|\hat{\mathbf{p}}_I\|^2$  is the near-far resistance of  $\hat{s}_1$  with respect to the interference space, and is given by [6],[3]

$$\hat{\eta}_I = 1 - \hat{\rho}_I^T \mathbf{R}_I^{-1} \hat{\rho}_I, \quad (3.3)$$

where  $\hat{\rho}_1^T = (\hat{\rho}_2, \dots, \hat{\rho}_K)$  is the crosscorrelation vector of  $\hat{s}_1$  with the interfering signals, and  $\mathbf{R}_I = (R_{kl})_{2 \leq k, l \leq K}$  is the  $(K-1) \times (K-1)$  matrix of crosscorrelations for the interfering signals. In order to be able to suppress strong interference, therefore, we must allow  $\|\mathbf{x}_1\|^2$  to be at least  $\chi_I$ .

The preceding arguments show that the presence of mismatch forces us to constrain  $\|\mathbf{x}_1\|^2$ , which is henceforth termed the *surplus energy*  $\chi$  available to the detector. The term arises from the fact that the detector energy  $\|\mathbf{c}_1\|^2 = \|\hat{s}_1\|^2 + \|\mathbf{x}_1\|^2 = 1 + \chi$ , so that  $\chi$  is a measure of the extent to which  $\mathbf{c}_1$  can be shaped to reduce the output energy ( $\chi = 0$  corresponds to the conventional detector). In order to prevent signal degradation and noise enhancement while permitting interference suppression, the best choice of surplus energy appears to be approximately  $\chi_I$ . However, this choice avoids signal degradation only if  $\chi_I < \chi_S$  (preferably  $\chi_I \ll \chi_S$ ). The latter is therefore a *necessary* condition for obtaining near-far resistant performance without excessive signal degradation. It is easy to see from Figure 1 that this condition is equivalent to the intuitively pleasing criterion that the nominal  $s_1$  is closer in Euclidean distance to the space spanned by the desired signal than to the interference subspace  $S_I$ .

### 3.1 Minimum output energy detector with constrained surplus energy

The *constrained* minimum output energy detector minimizes (over  $\mathbf{x}_1$ ) the cost function

$$MOE(\mathbf{x}_1) = E[\langle \mathbf{y}, \hat{s}_1 + \mathbf{x}_1 \rangle^2] = E[\langle \mathbf{y}, \mathbf{c}_1 \rangle^2],$$

subject to  $\|\mathbf{x}_1\|^2 = \chi$  and  $\langle \hat{s}_1, \mathbf{x}_1 \rangle = 0$ . It is straightforward to show that the solution is given by

$$\mathbf{c}_1 = \frac{(\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \hat{s}_1}{\hat{s}_1^T (\mathbf{A} + \gamma \mathbf{I}_N)^{-1} \hat{s}_1}, \quad (3.4)$$

where  $\mathbf{A} = \sum_{k=2}^K A_k^2 \mathbf{s}_k \mathbf{s}_k^T$ ,  $\mathbf{I}_N$  is the  $N \times N$  identity matrix,  $\gamma = \nu + \sigma^2$ , and  $\nu$  is a Lagrange multiplier selected to enforce the constraint  $\|\mathbf{c}_1\|^2 = \chi + 1$ . Since  $\gamma = \nu + \sigma^2$  completely determines the minimum output energy solution, it follows that  $\nu$  plays precisely the same role as the noise variance  $\sigma^2$ , i.e., imposing an *explicit* constraint on the surplus energy is equivalent to adding more background noise. However, for a given value of  $\gamma$ , the performance will clearly be worse for a larger value of  $\sigma^2$ , since the noise contribution to the output is greater while the signal and interference contributions remain the same.

### 3.2 Numerical Results

We consider a system with  $K = 7$  and processing gain  $N = 10$ . In order to avoid averaging over the relative delays of the interferers, we assume that the interferers are synchronous. However, we compensate for this by choosing the signature sequences *randomly* rather than optimizing over deterministic sequences. The mismatch is generated by assuming that, while the nominal corresponds to the desired signature sequence, the received signal due to the desired user has two components, one main component and a multipath component with relative delay equal to half of a bit interval and relative amplitude  $a$ . We assume  $a = 1$  for our numerical results. This is a fairly large mismatch, and the corresponding minimum surplus energy for signal cancellation is  $\chi_S = 2.47$ . For convenience, we assume that all interferers have the same amplitude  $A$  relative to the desired signal, i.e., that  $A_k = A$  for  $2 \leq k \leq K$ . In our numerical results, we take  $A = \sqrt{10}$ , so that each interferer is 10 dB stronger than the desired signal. For the particular choice of signature sequences we consider, the minimum surplus energy for complete interference cancellation is  $\chi_I = 0.60$ . Since  $\chi_I < \chi_S$ , a well-designed constrained minimum energy detector is expected to perform well.

The performance criterion used is signal-to-interference ratio (SIR), defined as

$$\text{SIR} = \frac{A_1^2 (\mathbf{c}_1' \hat{s}_1)^2}{\sum_{k=2}^K A_k^2 (\mathbf{c}_1' \hat{s}_k)^2 + \sigma^2 \|\mathbf{c}_1\|^2}. \quad (3.5)$$

We compare SIR for the minimum output energy detector with mismatch (with and without explicit constraints on surplus energy) with that of the minimum output energy detector without mismatch and without an explicit constraint on surplus energy. As shown in Section 2, the latter detector is the MMSE detector.

In Figure 2, we plot the SIR versus the SNR of the desired signal, given by  $\|\hat{s}_1\|^2 / \sigma^2 = \sigma^{-2}$ . In the absence of mismatch (i.e., if the anchor takes into account the multipath component), the SIR increases almost linearly with SNR. However, with mismatch, the SIR of the minimum energy detector degrades for SNRs beyond a certain range unless there is an explicit constraint on the surplus energy. This is because for high SNR, the low noise level allows a high value of surplus energy, which leads to signal degradation and noise enhancement. The performance improves substantially when we impose an explicit constraint on surplus energy by taking  $\nu = 0.1$ . Figure 3 shows the

values of surplus energy  $\chi$  as a function of SNR. In the absence of an explicit constraint, the surplus energy grows with the SNR, leveling off at a value that permits almost complete cancellation of both desired and interfering signals. When an explicit constraint is imposed, however, the surplus energy levels off much faster, at a value which permits interference suppression without excessive signal degradation and noise enhancement.

#### 4. Performance of Adaptive Algorithm

The convergence properties of the gradient algorithm (2.11) are analyzed in the full version of this paper. Because the true gradient of the energy is approximated by its instantaneous value, algorithm "noise" contributes "excess" MSE beyond that achievable with a fixed optimal (minimum energy) tap vector  $\mathbf{c}_1$ . This excess MSE is significantly greater for the blind algorithm (2.11) than for the conventional stochastic gradient (i.e., LMS) algorithm with a training sequence. Consequently, even without mismatch it is best to switch to decision directed mode once the signal-to-interference ratio (SIR) becomes sufficiently large.

Figure 4 shows a plot of averaged SIR vs. time assuming the algorithm (2.11) is used in a synchronous CDMA system with processing gain  $N = 10$  and number of users  $K = 7$ . Averaged SIR is given by

$$\text{SIR}_{av}[i] = \frac{\sum_{r=1}^M (\mathbf{c}_r'[i] \mathbf{s}_1)^2}{\sum_{r=1}^M [\mathbf{c}_r'[i] (\mathbf{y}_r[i] - b_{1,r}[i] \mathbf{s}_1)]^2}$$

where the vector notation in Section 3 is used here, the subscript "r" denotes the particular run of the algorithm, and the number of runs  $M = 200$ . The signature sequences are also the same randomly picked sequences used to generate the numerical results in Section 3. The signal power to background noise power is 30 dB.

The interfering amplitudes  $A_2, \dots, A_7$  are each 20 times  $A_1 = 1$ , representing an extreme near-far situation. Because of the strong interference, conventional single-user blind equalization algorithms (i.e., those discussed in [7]) do not generally succeed in isolating user 1. Two plots are shown in Figure 4: one assumes no mismatch, and the other assumes a mismatched anchor obtained by adding independent Gaussian noise with variance 0.01 to each component of  $\mathbf{s}_1$ . The blind algorithm (2.11) is used for the first

800 iterations, and the conventional LMS algorithm in decision directed mode is used thereafter.

In both cases shown in Figure 4 the blind algorithm succeeds in suppressing the strong interferers, and drives the SIR above 0 dB. What is interesting is that the mismatch creates an initial condition for the conventional LMS algorithm which leads to a much lower asymptotic SIR than the case without mismatch. Additional simulation results show that different (random) mismatches lead to different asymptotic SIRs. The explanation for this is that the mismatch causes the tap vector to wander outside the space spanned by the actual signal vectors, and thereby creates an orthogonal component to the signal space which takes a long time to suppress. Possible solutions to this problem are to project the adapted vector  $\mathbf{c}$  onto the signal space (spanned by successive channel outputs  $\mathbf{y}[i]$ ), or to constrain the norm of the tap vector (say, by tap-leakage).

#### 5. Conclusions

We have presented a detection technique for CDMA systems which requires knowledge of only the desired user's signature waveform and associated timing. This algorithm achieves the MMSE solution, and is therefore near-far resistant, in the absence of mismatch. By constraining the energy, or norm, of the filter impulse response, adequate performance can also be achieved in the presence of mismatch. Numerical results indicate that the performance of the estimator appears to degrade gracefully with the amount of mismatch. A simple gradient descent algorithm has been derived which can be used to estimate the filter coefficients. More complicated estimation schemes with improved convergence properties could also be used in this context.

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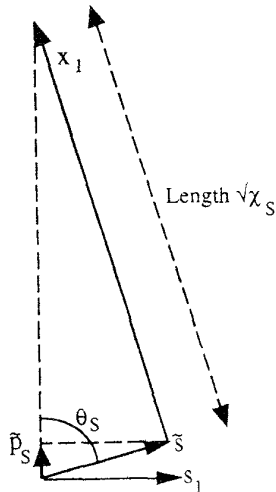


Figure 1a: Computation of surplus energy for complete signal cancellation

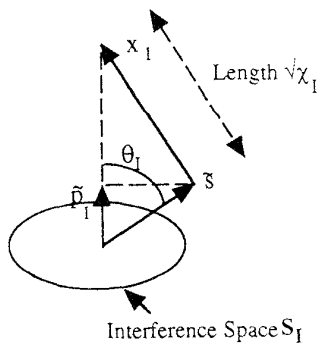


Figure 1b: Computation of surplus energy for complete interference cancellation

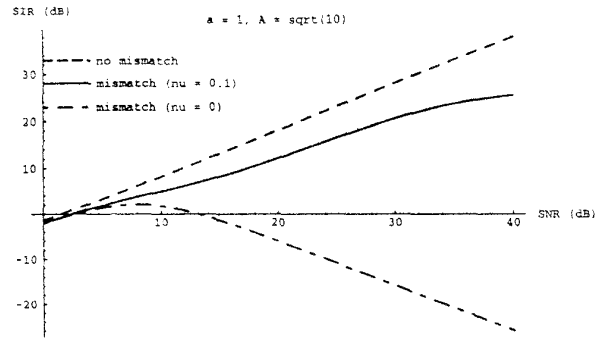


Figure 2. SIR versus SNR with and without mismatch.

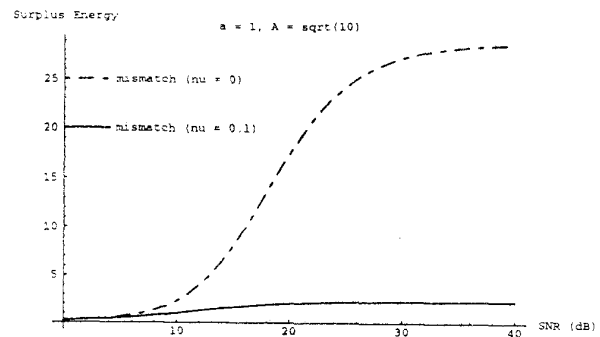


Figure 3. Surplus energy versus SNR with and without mismatch.

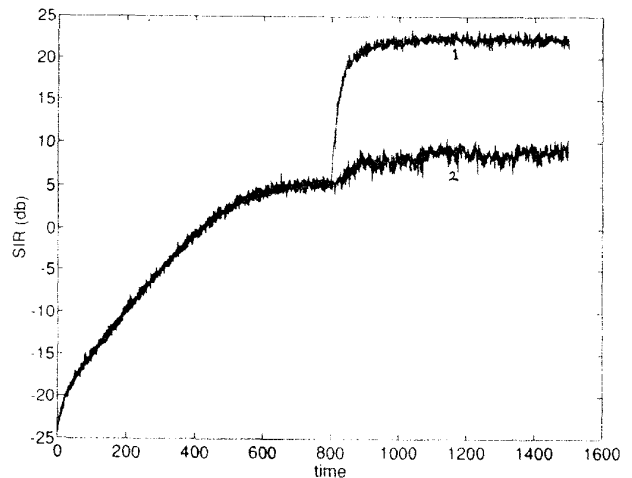


Figure 4. Averaged SIR vs. time for the algorithm (2.11). Case 1: no mismatch, Case 2: mismatch.