Blind Adaptive Space-Time Linear Multiuser Demodulation

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Abstract

We consider blind demodulation of multiple digital signals in a cellular environment under the assumption that training data is unavailable, and that the receiver has no *a priori* knowledge of user signatures in space or time. Our approach to this problem is to exploit diversity in the time, frequency, and spatial domains through the application of a linear receiver with multiple antenna elements. To separate the users, a subspace decomposition is used to obtain an initial estimate of the users' signatures. Once estimates of the signatures are available, the anchored algorithm described in [1] can be used to obtain initial bit estimates. These bit estimates are then used as the input to a decision-directed least squares (LS) algorithm (either full-rank or reduced-rank), which provides refined bit estimates for the packet of interest. This process can be iterated until the bit estimates remain fixed. Simulation results indicate that for the Time-Division Multiple-Access (TDMA) model considered, low to moderate uncoded error rates are achievable when the number of antennas is sufficiently large (at least twice the number of users), and when users differ substantially in received power (at least 6 dB).

1 Introduction

We consider blind demodulation of multiple digital signals in a cellular environment. The receiver is assumed to be randomly located within a particular cell, and is unable to cooperate with the transmitters. The emphasis of our study is on *adaptive multiuser* demodulation with limited side information. Although some training information may be available to aid the demodulation (e.g., the midamble in GSM), here we assume that no training data is available, that the receiver has no prior knowledge of signatures (in time or space), and that near-far effects are likely due to the random location of the receiver relative to the transmitters.

The proposed algorithm assumes a linear receiver with multiple antenna elements. Our approach is to first obtain an initial estimate of the spatial signatures via a subspace decomposition. This type of signal space decomposition has been previously considered for channel estimation [2], interference suppression [3], [4], and timing estimation [5], [6].

Once estimates of the spatial signatures are available, the anchored algorithm described in [1] can be used to obtain initial bit estimates. These bit estimates are then used as the input to a decision-directed least squares (LS) algorithm proposed in [7] , which provides refined bit estimates for the packet of interest. A related blind multiuser demodulation algorithm which uses maximum-likelihood estimation is given in [8]. This process can be iterated until the bit estimates remain fixed. By increasing the number of dimensions available through diversity, we simplify the blind multiuser demodulation problem, resulting in algorithms which have good performance with moderate complexity. In addition, diversity provides robustness in the presence of low Signal-to-Noise Ratios and fading.

In the next section, we briefly describe the TDMA model under consideration. In Section 3, we present a brief description of linear multiuser demodulation, followed by a detailed discussion of the blind algorithm in Section 4. In Section 5, we discuss enhancements to the algorithm as well as its extension to multipath channels. Simulation results are then presented in Section 6. Finally, we summarize our findings in Section 7.

2 System Model

The communications scenario considered assumes TDMA, so that the multiple users being demodulated are in different cells. We remark that, in the scenario considered here, the number of users to be demodulated is quite small (i.e., less than five), and the primary source of diversity is in the space domain. The received signal corresponding to the k^{th} user is

$$s_k(t) = \sum_i b_k(i) g(t - iT - \tau_k) \tag{1}$$

where $b_k(i)$ is the i^{th} transmitted symbol for user k, $g(t)$ is the transmitted pulse shape, T is the symbol period, and τ_k is the random delay associated with the k^{th} user. To alleviate phase slips associated with Rayleigh fading, we assume that $b_k(i)$ is differently encoded. The received signal at the m^{th} antenna is then

$$r_m(t) = \sum_m \sum_k h_k^{(m)}(t) \otimes s_k(t) + n_m(t) \tag{2}$$

where $h_k(t)$ is the channel impulse response from user k to antenna m, $n_m(t)$ is the noise on antenna m, and "$\otimes$" denotes convolution. In the model considered, we assume that each antenna element fades independently. In the case of flat fading, the spatial signature for a particular user is the vector of complex fade coefficients across the antennas. The signatures are assumed to be constant throughout each packet, but are assumed to be statistically independent from packet to packet.

For each antenna at the receiver we assume a filter matched to $g(t)$ followed by a sampler. With M antennas, the vector of M output samples at time iT can be written as

$$\mathbf{r}(i) = \mathbf{H}b(i) + \mathbf{n}(i) \tag{3}$$

where the mth component corresponds to antenna m, and the (m, k)th component of the $M \times K$ matrix $\mathbf{H}$ is the fading coefficient associated with the path from user k to antenna m (K being the number of users).

3 Linear Multiuser Demodulation

Given the sequence of received vectors, $\{\mathbf{r}(i)\}$, our objective is to obtain an estimate of the source bits transmitted by all users of interest. In what follows, we assume that we know *a priori* the number of users to be demodulated. Since a linear receiver is assumed, the decision statistic for user k at time i is given by

$$z_k(i) = \mathbf{C}_k^\dagger \mathbf{r}(i) \tag{4}$$

where $\mathbf{C}_k$ represents the filter coefficients associated with user k. Assuming DPSK modulation, the estimate of the source symbol is then

$$\hat{b}_k(i) = \text{sign}\{\text{Re}(z_k^*(i-1)z_k(i))\} \tag{5}$$

The performance criterion used to optimize the receiver is Mean Squared Error (MSE), defined as

$$\epsilon_k = E(|b_k(i) - \mathbf{C}_k^\dagger \mathbf{r}(i)|^2) \tag{6}$$

The filter which minimizes the MSE (Minimum MSE (MMSE) solution) is

$$\mathbf{C} = \mathbf{R}^{-1}\mathbf{P} \tag{7}$$

where $\mathbf{C}$ is the matrix with column vectors $\mathbf{C}_k$, $k = 1, \dots, K$,

$$\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^\dagger(i)] = \mathbf{H}\mathbf{H}^\dagger + \sigma_n^2 \mathbf{I}, \tag{8}$$

σ_n^2 is the noise variance at each sampling instant, and the k^{th} column of $\mathbf{P}$ is the signature of user k given by

$$\mathbf{P}_k = E[b_k(i)\mathbf{r}(i)] = \mathbf{H}_k \tag{9}$$

4 Blind Demodulation Algorithm

In the absence of a training sequence, blind techniques such as the constant modulus algorithm can be used; however, these tend to converge slowly [9]. Our approach is to obtain an initial estimate of the signatures via a subspace decomposition, and subsequently to substitute these estimates into an LS solution for $\mathbf{C}$. This algorithm can converge quite rapidly, although the performance will depend on the accuracy of the initial signature estimates.

4.1 Initial Signature Estimates

If the signatures corresponding to the different users are orthogonal (i.e., if $\rho_{kl} = \mathbf{H}_k^\dagger \mathbf{H}_l = 0$, $k \neq l$), then the signatures are eigenvectors of the covariance matrix $\mathbf{R}$. Although this is unlikely to be true when the elements of the signatures are independent random variables, the larger the dimension of $\mathbf{H}_l$, the lower the expected cross-correlation ρ_{kl} will be. An initial estimate of the signatures can therefore be obtained by computing the eigenvectors of the estimate of $\mathbf{R}$,

$$\widehat{\mathbf{R}} = \frac{1}{B}\sum_{i=1}^{B} \mathbf{r}(i)\mathbf{r}^\dagger(i) \tag{10}$$

where B is the number of symbols per time slot.

Since $\widehat{\mathbf{R}}$ is positive definite and full rank, we can write

$$\widehat{\mathbf{R}} = \mathbf{V}\Lambda\mathbf{V}^\dagger \tag{11}$$

where $\mathbf{V}$ is the orthogonal matrix with columns given by the eigenvectors of $\mathbf{R}$, and Λ is the diagonal matrix of eigenvalues arranged in descending order

$$\lambda_1 > \lambda_2 > \ldots \lambda_K > \sigma^2, \quad \lambda_n = \sigma^2, \quad n > K \tag{12}$$

Since we assume that the number of users to be demodulated (K) is known, the signatures are initially taken to be the K eigenvectors $\mathbf{V}_1, \ldots, \mathbf{V}_K$ corresponding to the largest eigenvalues.

4.2 Reduced-Rank LS Algorithm

Given the signature estimates, we form a reduced column matrix $\widetilde{\mathbf{V}} = [\mathbf{V}_1, \ldots, \mathbf{V}_K]$ and project the received signal onto the corresponding signal subspace to obtain

$$\tilde{\mathbf{r}}(i) = \widetilde{\mathbf{V}}'\mathbf{r}(i) \tag{13}$$

(In what follows, all variables with a tilde are reduced-rank, or projected, variables.) We wish to select $\widetilde{\mathbf{C}}$ to minimize

$$\epsilon = E|\mathbf{b}(i) - \widetilde{\mathbf{C}}'\tilde{\mathbf{r}}(i)|^2 \tag{14}$$

The solution is given by

$$\widetilde{\mathbf{C}} = \widetilde{\mathbf{R}}^{-1}\widetilde{\mathbf{P}} \tag{15}$$

where

$$\widetilde{\mathbf{R}} = E[\tilde{\mathbf{r}}(i)\tilde{\mathbf{r}}^\dagger(i)] = \widetilde{\mathbf{V}}'\mathbf{R}\widetilde{\mathbf{V}} = \widetilde{\Lambda} \tag{16}$$

and the projected signature for user k is

$$\widetilde{\mathbf{P}}_k = E[b_k(i)\widetilde{\mathbf{V}}'\mathbf{r}(i)] = \widetilde{\mathbf{V}}'E[b_k(i)\mathbf{r}(i)] = \widetilde{\mathbf{V}}'\mathbf{H}_k \tag{17}$$

Hence, we can rewrite the blind reduced-rank filter as

$$\widetilde{\mathbf{C}} = \widetilde{\Lambda}^{-1}\widetilde{\mathbf{V}}'\mathbf{H}_k, \quad \widetilde{\Lambda} = \ diag[\lambda_1, \ldots, \lambda_K] \tag{18}$$

If the signatures are orthogonal, then $\widetilde{\mathbf{V}}'\mathbf{H}_k = ||\mathbf{V}_k||^2\mathbf{e}_k$, where $\mathbf{e}_k$ is the kth unit vector. An LS algorithm is obtained by replacing the expectations in (14)-(17) by time averages.

Our initial estimates of the users' signature assume that they are orthogonal, whereas they are only approximately orthogonal. Hence, subsequent to obtaining the initial bit estimates $\{\widehat{b}_k(i)\}$, we can refine the estimate of the signatures by estimating the cross-correlation in 17. With differential decoding, the bit decisions obtained from (5) can be used to obtain the signature estimate as follows [7]:

$$\widehat{\widetilde{\mathbf{P}}}_k = \frac{1}{B}\sum_{i=1}^{B}[\widehat{b}_k(i)z_k(i-1)]^*\tilde{\mathbf{r}}(i) \tag{19}$$

This in turn gives a new set of filter coefficients $\widetilde{\mathbf{C}} = \widetilde{\Lambda}^{-1}\widehat{\widetilde{\mathbf{P}}}_k$. Note that any phase offset in $z_k(i)$ is introduced into the signature estimate, but this is inconsequential with differential decoding. Using the new set of filter coefficients, we compute new estimates of the transmitted bits. This procedure is iterated until the algorithm converges. (That is, the estimated bits do not change from one iteration to the next.) The algorithm is not guaranteed to converge, and problems with convergence do occur at relatively high error rates (> 15%). However, simulations indicate that the iterative procedure does improve the error rate when it is less than 10%.

4.3 Full-Rank LS Algorithm

The full-rank LS algorithm also relies on estimates of the user signatures via the initial eigen-decomposition. However, the estimated bits are used to minimize the "full-rank" cost function

$$\epsilon_k = \sum_{i=1}^{B} w^{-(B-i)} |b_k(i) - \mathbf{C}_k^\dagger \mathbf{r}(i)|^2 \qquad (20)$$

where w is an exponential weighting constant. The $\mathbf{C}$ which minimizes the cost function, modified for differential detection, is $\mathbf{C} = \widehat{\mathbf{R}}^{-1}\widehat{\mathbf{P}}$ where

$$\widehat{\mathbf{R}} = \sum_{i=1}^{B} w^{-(B-i)} \mathbf{r}(i)\mathbf{r}^\dagger(i), \quad \widehat{\mathbf{P}}_k = \sum_{i=1}^{B} w^{-(B-i)} \hat{b}_k(i) z_k^*(i-1)\mathbf{r}(i) \qquad (21)$$

The initial bit estimates are then used to recalculate $\mathbf{C}$ according to the LS criterion, resulting in new bit estimates. The process is iterated until the algorithm converges. For the examples considered, simulation results indicate that the full-rank algorithm performs better than the reduced-rank algorithm when the initial error rate is $\leq$ 10%.

5 Algorithmic Enhancements

In this section, we present enhancements to the basic algorithm described in the preceding section. We also discuss the effect of multipath, and propose a modification of the differentially coherent algorithm which noncoherently combines estimates from adjacent symbols.

5.1 Fractional Sampling

In addition to spatial oversampling, we can also oversample the received signal in time. However, in contrast to spatial oversampling, the performance gain due to oversampling in time is limited by the signal bandwidth. For the numerical results that follow, we assume that $g(t)$ is a raised-cosine pulse with 50% excess bandwidth. In that case, the Nyquist sampling rate is less than two samples per symbol.

It is convenient to represent the received samples during a symbol interval as the matrix $\mathbf{r}(i) = [\mathbf{r}(i;1)\ \mathbf{r}(i;2)]$ where $\mathbf{r}(i;n)$ denotes the n^{th} sample of the i^{th} sampling interval. The use of multiple samples per bit provides robustness against timing offset which is a concern in asynchronous communications system. Although fractional sampling introduces correlation into the noise samples, numerical results indicate that this causes only a slight degradation in performance.

5.2 Multi-symbol Observation Window

The discussion in the preceding section assumed that the filter spanned a single symbol interval in time. Of course, it is conceptually straightforward to expand this window to span multiple symbols in time. This is especially important when ISI and multipath are present. For the numerical results in the next section, we assume that the receiver is asynchronous. Since $g(t)$ is a raised-cosine pulse shape, all users experience ISI even without multipath. Expanding the observation window allows the filter to capture the additional energy due to multipath.

5.3 Soft Decision-Directed Adaptation

From the discussion in Section 4, the signature corresponding to user k can be estimated as

$$\widehat{\mathbf{P}}_k = \frac{1}{B}\sum_{i=1}^{B} \mathcal{F}(\mathbf{C}_k^{\dagger}\mathbf{r}(i))\mathbf{r}(i) \tag{22}$$

where $\mathcal{F}(x) = \text{sgn}(x)$ in the case of hard decisions and coherent detection. To improve performance, we can weight the decisions by a measure of reliability. One approach is to select $\mathcal{F}(x)$ to

$$\min_{\mathcal{F}} E(|\mathcal{F}(\mathbf{C}_k^{\dagger}\mathbf{r}(i)) - \mathbf{C}_k^{\dagger}\mathbf{r}(i)|^2) \tag{23}$$

where it is assumed that

$$\mathbf{C}_k^{\dagger}\mathbf{r}(i) = b_k(i) + \zeta_k(i) \tag{24}$$

where $\zeta_k(i)$ is a Gaussian random variable with mean zero and variance

$$\sigma_k^2 = \text{MSE} \approx \frac{1}{B}\sum_{i=1}^{B} |\hat{b}_k(i) - \mathbf{C}_k^{\dagger}\mathbf{r}(i)|^2 \tag{25}$$

The solution to this optimization problem, presented in [10], is

$$\mathcal{F}(x) = \tanh(\frac{x}{\sigma_k^2}) \tag{26}$$

The hyperbolic tangent function discounts samples that are close to zero while providing more weight to those that are close to ± 1. The numerical results in Section 6 were generated with soft decisions (modified for differential detection), although in most cases the performance improvement due to using soft decisions, instead of hard decisions, is relatively minor.

5.4 Multipath

To illustrate the effect of multipath, consider a two-ray channel in which the rays are separated by the symbol interval T. The received signal is then described by

$$\mathbf{r}(i) = \mathbf{H}^1\mathbf{b}(i) + \mathbf{H}^2\mathbf{b}(i-1) + \mathbf{n}(i) \tag{27}$$

and the signature of user k is comprised of $\mathbf{H}_k^1$ corresponding to the main path, and $\mathbf{H}_k^2$ corresponding to the second path.

The MMSE criterion automatically combines the two paths coherently. By combining received vectors from two symbols, we are presented with a new received vector

$$\begin{bmatrix} \mathbf{r}(i) \\ \mathbf{r}(i+1) \end{bmatrix} = \begin{bmatrix} \mathbf{H}^2 & \mathbf{H}^1 & \mathbf{0} \\ \mathbf{0} & \mathbf{H}^2 & \mathbf{H}^1 \end{bmatrix} \begin{bmatrix} \mathbf{b}(i-1) \\ \mathbf{b}(i) \\ \mathbf{b}(i+1) \end{bmatrix} + \begin{bmatrix} \mathbf{n}(i) \\ \mathbf{n}(i+1) \end{bmatrix} \tag{28}$$

The signature of user k is now given by $\mathbf{P}_k = [\mathbf{H}_k^1 \ \mathbf{H}_k^2]'$ and a signal subspace decomposition can be used as before to separate the users. Specifically, compute the new autocorrelation matrix and decompose it according to

$$\widehat{\mathbf{R}} = \frac{1}{B} \sum_{i=1}^{B} \begin{bmatrix} \mathbf{r}(i) \\ \mathbf{r}(i+1) \end{bmatrix} \begin{bmatrix} \mathbf{r}(i) \\ \mathbf{r}(i+1) \end{bmatrix}^{\dagger} = \mathbf{V}\Lambda\mathbf{V}^{\dagger} \tag{29}$$

The user signatures are then given by

$$\widehat{\mathbf{H}}_k = \mathbf{V}_k, k = 1, \ldots, K \tag{30}$$

Note that the intersymbol interference due to multipath is equivalent to additional multiple access interference.

If the filter spans only a single symbol interval, then noncoherent equal gain combining (e.g., see [11]) can be used to combine the paths associated with adjacent symbol intervals. Specifically, the algorithm relies on the assumption that the different paths are received with independent fading coefficients on each receiver antenna. Thus, each interfering path can be viewed as an independent interferer. If the different paths are sufficiently strong, then a signal subspace (eigen) decomposition should be able to resolve the paths. For example, with two users, each with two paths, the eigen-decomposition will provide four distinct eigenvectors that span the signal subspace. The eigenvalues associated with a particular user are associated with the received path energies. Of course, the multipath components must be strong relative to the background noise level. Otherwise, the associated eigenvalues are too small to allow the multipath subspace to be distinguished from the noise subspace.

Subsequent to path resolution, equal gain noncoherent combining can be used to estimate the bits. With this method, the estimated signatures associated with each path for a particular user are used to compute soft differential estimates of the data, which are added together to obtain the decision statistic. This technique is analogous to a noncoherent RAKE receiver for Direct-Sequence (DS)-Code- Division Multiple Access (CDMA) with the main difference being that the delayed time-domain CDMA signatures associated with the different paths are replaced by space-domain signatures.

6 Numerical Results

The results in this section assume a fixed number of users (two or three), with the users at different (random) locations. The modulation scheme is binary DPSK with raised cosine pulses with 50% excess bandwidth. The length of the time slot is 150 bits, and the receiver is asynchronous with respect to each user, with each user delayed by an independent random variable. Therefore $\mathbf{r}(i)$ contains both intersymbol and multiple-access interference. Both frequency-nonselective and frequency-selective (two-ray) Rayleigh fading are considered. The fading coefficients associated with the paths and antennas are assumed to be independent, and are constant for the duration of the time slot. Where fractional sampling is used, the noise correlation is taken into account.

6.1 Frequency-Nonselective Fading Channels

Unless otherwise noted, the results presented here assume that the signal-to-noise ratio for user one is 10 dB. For two users, the power difference between the strong and weak user is 8 dB. The MMSE solution sometimes gave zero errors over the duration of the simulation, in which case the results are not plotted in the figures.

Figure 1 shows results for the MMSE, blind reduced-rank, and blind full-rank algorithms assuming two samples per bit. For both users the blind algorithms perform significantly worse than the ideal MMSE solution. The full-rank LS algorithm performs slightly better than the reduced-rank LS algorithm in this case. A comparison of results obtained from sampling once and twice per symbol is shown in Figure 2. The performance improvement associated with fractional sampling is due to robustness with respect to random timing offset.

In Figure 3, error rate is shown as a function of the difference in power between the users. When the powers of the two users are close, the blind algorithm is unable to separate the users since the eigenvectors of the sample covariance matrix are no longer aligned with each user's signature.

Finally, Figure 4 illustrates the performance of the algorithm when 3 users are present. The SNR for user 1 is 11 dB. The powers of users 2 and 3 are 8 dB and 11 dB below that of user 1, respectively. When fractional sampling is used, the performance of user 1 is close to that shown in the preceding figures. The performance of the weak users, however, are substantially worse. This is due to the fact that the two weak users are close together in power. Hence, the subspace decomposition is unable to produce reliable estimates of the weak signatures.

6.2 Frequency-Selective Channels

Figures 5-6 show the performance of the algorithm in the presence of frequency-selective fading. We consider a two-path model with the second path at equal strength and delayed by the symbol interval T. Figure 5 illustrates the performance based on coherent combining of multipath signals. In this case the MMSE results are better than the analogous results for frequency-nonselective fading performance shown in Figure 1, since the MMSE solution coherently combines the different paths. The performance of the blind algorithms degrades significantly when multipath is present. This is because the additional ISI from multipath degrades the initial signature estimates.

The results for noncoherent combining using a window of two symbols is presented in Figure 6. These results show that noncoherent combining performs slightly better than coherent combining.

7 Conclusions

Our numerical results indicate that for the cases considered, the blind algorithm performs well when the number of antennas is sufficiently large (at least twice the number of users), and when users differ substantially in received power (at least 6 dB). The use of fractional sampling is important mainly due to the insensitivity provided to timing offset. We also remark that the error rates shown here are for the uncoded symbols. Coded error rates would be substantially lower, so that intelligible reception of the weak user may be possible (assuming voice transmission) when the uncoded error rate is quite high (e.g., 10%).

Perhaps the main weakness of the blind algorithm proposed is that it is unable to separate the users when the received powers are close. In addition, we have assumed that the number of users to be demodulated is known *a priori*. If this is not the case, then one possibility is to demodulate users in decreasing order of received power until the weakest user demodulated is unintelligible. This, of course, assumes that the number of active users is not too large.

Finally, our preliminary results indicate that severe multipath is likely to cause substantial degradation in performance, especially for relatively weak users. Enhancements to the algorithm which may improve performance in this situation are currently being studied.

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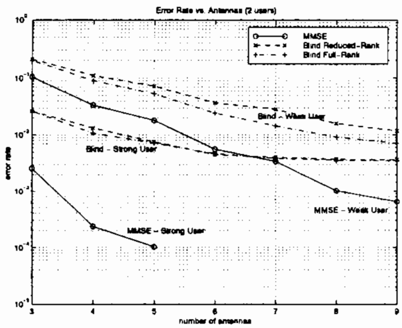

Figure 1: Error rate vs. number of antennas, 2 users, 2 samples/bit, SNR_1 = 10 dB, SNR_2 = 2 dB.

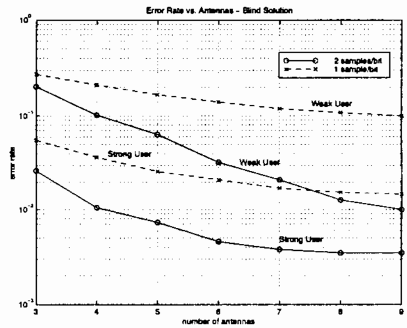

Figure 2: Fractional Sampling: Blind reduced-rank solution, 2 users, SNR_1 = 10 dB, SNR_2 = 2 dB.

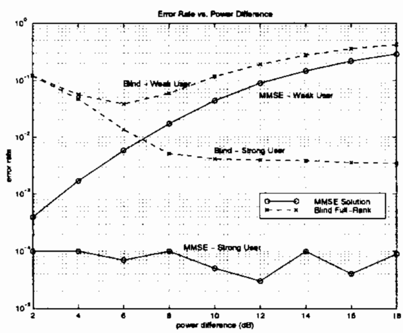

Figure 3: Error rate vs. relative power, 2 users, 5 antennas, 2 samples/bit, SNR_1 = 10 dB, SNR_2 = 2 dB.

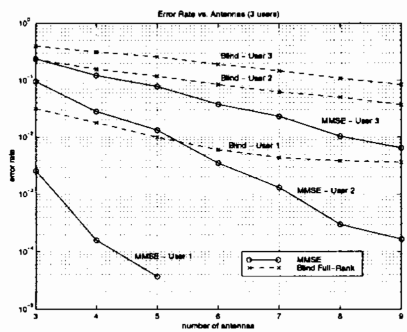

Figure 4: Error rate vs. number of antennas, 3 users, 2 samples/bit, SNR_1 = 11 dB, SNR_2 = 3 dB, SNR_3 = 0 dB.

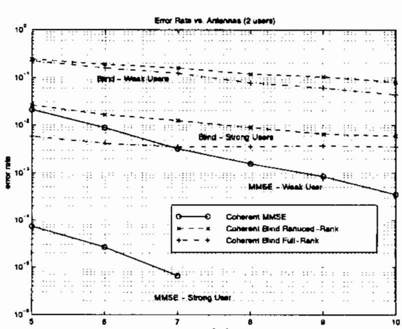

Figure 5: Coherent Combining: 2 samples/bit, SNR_1 = 10 dB, SNR_2 = 2 dB.

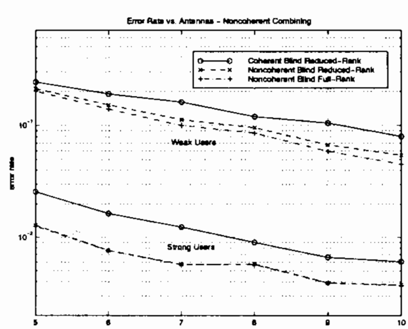

Figure 6: Noncoherent Combining: window = 2 symbols, 2 samples/bit, SNR_1 = 10 dB, SNR_2 = 2 dB.