Blind Equalization using Fractional Sampling and an Energy Cost Function*

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Abstract

In this paper we present a simple and intuitively appealing strategy to achieve convergent blind equalization. This method is based on sampling the received signal at faster than the symbol rate and it uses energy cost function which enables simple gradient based adaptation. Necessary and sufficient conditions The necessary and sufficient conditions for the equalizer to converge to the channel inverse are derived.

1 Introduction

Blind Equalization has presented a theoretical challenge of finding a globally convergent strategy. Recently there has been some progress towards that goal: For the case of minimum phase channels, the problem of exact equalization has been solved by Verdú, Anderson and Kennedy [1]. In the general case, their energy based cost function cannot resolve the nonminimum phase zeros of the system. A partial solution to this is presented in [2], which deals with a convex cost function approach. A different approach using fractional sampling and exploiting the resulting cyclostationarity is presented by Tong, Xu and Kailath [3]. In their work, necessary and sufficient conditions on the continuous time channel impulse response are presented which pertain to the solvability of the the blind equalization problem using second order statistics. They also present an RLS-type of approach to solve the problem provided the sufficient condition is met.

The main thrust of this paper is to reformulate the problem tackled in [3] in a very simple and intuitive way and then derive necessary and sufficient condition using elementary arguments. Our conditions are similar to those found in [3]. We also suggest an alternative to the RLS type method presented in [3]. This method may be easier to implement if we parametrize the equalizers suitably in factored form. If we implement in unfactored form it will require polynomial factorization.

The method we suggest is based on the LMS algorithm and the gradient based blind equalization algorithm of Verdú, Anderson and Kennedy [1]. The latter algorithm minimizes the energy of the equalizer output and uses an equalizer with a tap anchored at 1. The equalizer converges to a configuration that corresponds to inverting the minimum phase version of the channel. Hence if the channel is not minimum phase, the channel-equalizer combination becomes an allpass function. Indeed to quote [1], "A sensible strategy to the blind equalization of nonminimum phase channels is to decouple the equalization of the minimum phase part (which can be done exactly) and the identification of the zeros located outside the unit circle." The method presented in this paper serves to pinpoint the zeros that are outside the unit circle. For the derivation and discussion of the anchored energy algorithm, we refer the reader to [1].

We derive a set of necessary and sufficient conditions in the spirit of [3]. The conditions there are given in terms the zeros of the sampled impulse response of the channel and alternatively as a rank condition on a matrix that depends on the channel impulse response. The problem is posed as extraction of phase information from cyclostationaly statistics. In this paper, a related condition on the zeros is derived in a very simple and intuitively appealing way. We also do not assume that the channel impulse response is time limited - an assumption made in [3]. In fact, we will only require that the resulting discrete time systems be finite dimensional, as usual in system identification. This is because we are explicitly identifying the zeros of the nonminimum phase system and so we require that there be finite number of

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such zeros.

The sufficient condition is demonstrated through a novel structure for adaptation and hence is a constructive method. The only potential drawback in the method is that in principle it may require polynomial factorization - we will show that we can circumvent this by suitable parametrization of the equalizer.

We first give some intuition on why fractional sampling aids in the solution to the blind equalization problem. If we know the impulse response exactly, then matched filtering and sampling at the symbol rate gives a set of sufficient statistics. But if we do not know the impulse response exactly, we use a "nominal" matched filter and then sample at the symbol rate. However, in general this process will not result in a set of sufficient statistics. Hence fractionally sampling the received signal could potentially increase the available information and aid in the solution to the blind equalization problem. We show that this is indeed the case.

Fractional sampling is widely used for conventional adaptive equalization for other advantages it confers, such as insensitivity to sampling epochs etc. The canonical form of the front end of a digital receiver is a matched filter followed by a transversal filter. If we use fractional sampling, we can role both of these into one and in effect make the matched filter adaptive e.g. [5].

2 System Model

The baseband model we consider is the following: we transmit a stream of data over a channel with causal impulse response $h(t)$ (this is the complete resultant impulse response including the "nominal" matched filter - before any sampling process is applied). We do not assume that $h(t)$ is time limited. Usually the resultant matched filter output is sampled at the symbol rate and we will obtain a discrete-time model. As we pointed out earlier, due to our lack of knowledge of the exact channel impulse response, we cannot do exact matched filtering. In such a case sampling the "nominal" matched filter at the symbol rate results in loss of information. Hence we will assume that we take two sets of samples - both at the symbol rate of $\frac{1}{T}$ where T is the symbol duration, and these two sets of samples are shifted with respect to each other by an offset $\tau < T$. Thus our communication system can be replaced by two parallel discrete systems with a common input data stream. The problem now is to identify at least one of these two systems uniquely using only second order statistics of the outputs of the systems.

We make the assumption that the data stream is iid and that the systems we deal with have no zeros on the unit circle - or the imaginary axis in the case of Laplace transform. This is so that we can recover the data by inverting the system. We also assume that noise is negligible. These are standard assumptions made in the blind equalization literature and are satisfied in the usual applications.

3 Necessary and Sufficient Conditions

First we derive a necessary condition on the zeros of the two parallel discrete time systems, so that we can identify the systems uniquely using second order statistics of the outputs only. This condition turns out to be the following: there must exist some $0 < \tau < T$ such that the two discrete time systems do not share any zeros.

Why is this condition necessary? We know that we can identify the location of the zeros of a linear system using second order statistics alone only up to phase - i.e. we cannot tell whether the zeros are inside or outside the unit circle. Suppose we have two parallel systems with common input that share a zero and yet we could tell whether this zero is inside or outside the unit circle using second order statistics alone. We will arrive at a contradiction: Redraw the system diagram to put the common zero first and then the two parallel systems from which the zero has been factored out. Now we have a method of identifying whether this zero is inside or outside the unit circle using second order statistics - by simply cascading it with two parallel systems! This shows us that it is impossible to tell whether the common zero is inside or outside the unit circle using second order statistics alone.

Of course if we had apriori knowledge that tells us that the system is minimum phase (resp. maximum phase) then we can use second order statistics alone and solve the problem. But most practical systems obtained by sampling a continuous time system do not have this property. Even if the original continuous time system is minimum phase, the discrete time system need not be minimum phase.

Now we will present a gradient based method that shows that if the two parallel systems do not share any zeros, then we can uniquely identify the systems and hence recover the transmitted data.

Figure 1 suggests our strategy: we put an equalizer on the first system and try to approximate the output of the second system using the Least Mean Squared Error algorithm. We will parametrize this equalizer

θ_1 in the form of a cascade of second order sections. This will be useful later. If we denote the transfer function of the two systems as H_1 and H_2 respectively, this equalizer will converge to $H_1^{-1}H_2$. Now we find the nonminimum phase zeros of this equalizer, which clearly have to have come from system 2, since we assume that the systems are causal and stable. By the assumption we made about the nonoverlap of such zeros between the two systems, the nonminimum phase zeros we have found account for *all* such zeros in system 2. With system 2 we use the VAK energy algorithm [1] to update the equalizer θ_2 and this will convert system 2 in to an all pass system. We can invert this anticausally because *we know* the zeros of this all pass system, namely the nonminimum phase zeros we found for the equalizer θ_1.

There is one subtle problem with the above strategy. It is the following. We claimed that the nonminimum phase zeros of $H_1^{-1}H_2$ are exclusively due to H_2. However in a practical implementation, we will approximate this transfer function by a FIR realization. In that case some spurious nonminimum phase zeros might arise due to our attempting to approximate H_1^{-1}. Now we outline a method to overcome this problem based on the equalizer in [1]. The VAK equalizer [1] converges to a configuration such that the cascade $H_2\theta_2$ becomes an allpass function (in the general case when H_2 is nonminimum phase). In [1], the equalizer is parametrized as a cascade of second order IIR sections. Hence there is no problem finding the poles of this equalizer. All these poles are stable but some of these poles correspond to the the nonminimum phase zeros of H_2 and we have to know which ones they are. We reflect all these poles with respect to the unit circle. Now we know that the nonminimum phase zeros of H_2 also occur in the factorization of $H_1^{-1}H_2$ system. We compare this set of zeros to the reflected set of poles mentioned earlier and pick out all of those elements that are in common. Clearly these are exactly the nonminimum phase zeros of the system H_2 as we wanted.

Now we have used FIR factored form parametrization for equalizer θ_1 and IIR factored form parametrization for θ_2. The global convergence of the θ_1 is discussed in [1]. See also [4] for a detailed discussion on IIR adaptive filters.

The above procedure can also be done with $H_2^{-1}H_1$ and the VAK equalizer corresponding to H_1 in order to get the nonminimum phase zeros of H_1. This will serve to robustify the method because the gradient adaptations are completely uncoupled.

We have derived a condition on the zeros of the two parallel systems. But these two parallel systems are obtained by sampling the continuous time impulse response $h(t)$ at rate $\frac{1}{T}$ but with samples shifted with respect to each other. What does the lack of common zeros condition mean when we translate it for original channel response $h(t)$? If $H(s)$ is the Laplace transform of the continuous time impulse response, then the discrete time system obtained by sampling the continuous time system at rate $\frac{1}{T}$ and at epoch τ_1, has a zero at a point $z_1 = (\sigma_1 + j\omega_1)T$ if and only if

$$\sum_{n=-\infty}^{\infty} H(\sigma_1 + j(\omega_1 - \frac{2\pi n}{T}))e^{\frac{2\pi n \tau_1}{T}} = 0$$

But the above is equivalent to the statement

$$\tilde{h}_{\sigma_1+j\omega_1}(\tau_1)$$

$$= \sum_{n=-\infty}^{\infty} h(\tau_1 - nT)e^{-\sigma_1(\tau_1 - nT)}e^{-j\omega_1(\tau_1 - nT)} = 0 \quad (1)$$

Note that for any fixed $z \in C$ the function $\tilde{h}_z$ is a periodic function with period T.

We will now consider the case when our sampling epochs are $\tau = 0$ and $\tau = \frac{T}{2}$ which corresponds to the fact that we are taking periodic samples with rate $\frac{2}{T}$ and the even samples comprise discrete system 1 and the odd samples comprise system 2. We can see that the two parallel systems have a common zero at some point $z_1 = \sigma_1 + j\omega_1$ if and only if the periodic function $\tilde{h}_{\sigma_1+j\omega_1}(\tau)$ has zeros at $\tau = 0$ and at $\tau = \frac{T}{2}$. If we want to preclude such common zeros we should require that for all $z \in C$ either $\tilde{h}_z(0) \neq 0$ or $\tilde{h}_z(\frac{T}{2}) \neq 0$.

In general we can use a similar strategy whenever we can find a set of sampling epochs $\{\tau_1, \tau_2, \ldots, \tau_K\}$ such that the resulting K parallel discrete systems do not share a zero outside the unit circle. In the procedure outlined above we have used two sampling epochs $\{0, \frac{T}{2}\}$. Note that any proper subset of these K systems could share a zero, we only need that there be no common zeros for them taken as a whole. This is the general sufficient condition.

For most $h(t)$ (see exception below) we can find sampling epochs such that the parallel discrete systems do not share a common zero, if we take K to be large.

When $h(t)$ has a long tail then too much oversampling (which means that we have to take sampling epochs very close to each other) will make the procedure numerically unstable - by making the zeros of the parallel systems very close to each other. This is the only real limitation of the above method. In practice for most "nice" $h(t)$, $K = 2$ should be sufficient.

4 Conclusion

In this paper we have reformulated the blind equalization problem by sampling the continuous time system at a rate faster than the symbol rate. We then derive necessary and sufficient conditions similar to [3] in a very simple way. We also present a simple gradient strategy to update the equalizers.

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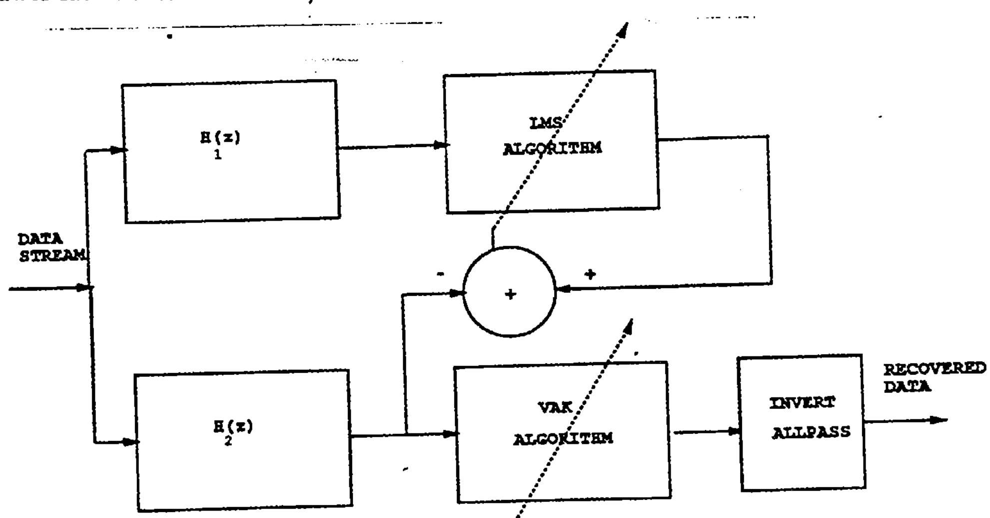

Figure 1: Equalization Strategy