

Capacity of CDMA Fading Channels

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The capacity of multiple-access channels subject to fading has attracted much recent interest exemplified by [1, and references therein].

The spectral efficiency (total capacity per chip) of the additive white Gaussian noise synchronous direct-sequence spread spectrum channel with perfect power control and randomly chosen spreading sequences was obtained in [2] as a function of the system load and signal-to-noise ratio. The loss in efficiency incurred by random spreading relative to both optimally chosen signature sequences and to multiaccess with no spreading as well as the loss in efficiency incurred by several linear multiuser detectors are studied in [2]. See also [3] and [4] for various results on suboptimal linear detectors. CDMA spectral efficiency under homogeneous (chip-independent) fading [3] was already addressed in [2].

The key aspects of the fading multiaccess model considered in the present paper are: synchronous users with common processing gain N ; random spreading sequences known at the receiver; slow independent fading, fading amplitudes remaining constant per information symbol and otherwise ergodic, with arbitrary first order distribution; the fading is independent and equally distributed for all transmitters; the realizations of the fading amplitudes are perfectly known at the receiver; additive white Gaussian noise; fixed and arbitrary system load $\beta = K/N$, while $K, N \rightarrow \infty$.

The total capacity of the multiaccess fading channel and with equal normalized power (SNR) for all users is

$$C_{opt}(SNR) = \frac{\beta}{2} \int \log(1 + SNR \cdot x) dF_\lambda(x), \quad (1)$$

where $F_\lambda(x)$ is the corresponding eigenvalue distribution accounting for the faded received power and random choice of signatures. The Stieltjes transform of F_λ ,

$$G_\lambda(z) = \int_0^\infty \frac{dF_\lambda(t)}{t - z} \quad (2)$$

plays a prominent role and it is given in this case by [5].

$$G_\lambda(z) = \int_0^\infty \left(\tau(1 - \beta - \beta z G_\lambda(z)) - z \right)^{-1} dF_f(\tau) \quad (3)$$

where $F_f(\tau)$ is the fading cdf. It can be shown that,

$$C_{opt}(SNR) = \int_0^{SNR} \frac{\beta}{2x} \left(1 - \frac{1}{x} G_\lambda\left(\frac{-1}{x}\right) \right) dx. \quad (4)$$

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We also consider the ultimate reliable rates associated with error-correction decoders that are preceded by several multiuser detection front-ends: single-user matched filter; decorrelator; linear MMSE receiver; successive canceler; decision-feedback decorrelator; and decision-feedback MMSE (DF-MMSE). For example, the capacity associated with the LMMSE is

$$C_{mmse}(SNR) = \frac{1}{2} \beta \int_0^\infty \log\left(1 + \tau \Lambda_{mmse}(SNR, \beta)\right) dF_f(\tau), \quad (5)$$

where $\Lambda_{mmse}(x, y)$ is the solution of

$$\Lambda_{mmse}^{-1}(x, y) = x^{-1} + y \int \frac{dF_f(\tau)\tau}{1 + \tau \Lambda_{mmse}(x, y)}. \quad (6)$$

The relation between the optimum detector and DF-MMSE ([2],[6], and references therein) facilitates rewriting (1) as

$$C_{opt}(SNR) = \int_0^\infty dF_f(\tau) \int_0^\beta \frac{1}{2} \log\left(1 + \tau \Lambda_{mmse}(SNR, \gamma)\right) d\gamma \quad (7)$$

In addition to spectral efficiency results, we obtain analytical results on outage capacity (cf. [7] for numerical results), thereby doing away with the ergodic assumption. We also consider in this setting the decentralized power control problem where each transmitter has access to the instantaneous value of its received attenuation [8].

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