

Capacity of Channels with Uncoded-Message Side-Information

Shlomo Shamai (Shitz)¹ and Sergio Verdú²

¹Department of Electrical Engineering, Technion-Israel Inst. of Technology, Haifa 32000, Israel sshlomo@ee.technion.ac.il

²Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, U.S.A. Verdú@Princeton.edu

Abstract — Parallel independent channels where no encoding is allowed for one of the channels are studied. The Slepian-Wolf theorem on source coding of correlated sources is used to show that any information source whose entropy rate is below the sum of the capacity of the coded channel and the input/output mutual information of the uncoded channel is transmissible with arbitrary reliability. The converse is also shown. Thus, coding of the side information channel is unnecessary when its mutual information is maximized by the source distribution. An information-theoretic interpretation of Parallel-Concatenated channel codes and, in particular, Turbo codes is put forth.

I. MODEL

Consider the model depicted in the Figure 1 where two independent channels operate in parallel. If the inputs to both channels were allowed to be encoded, then Shannon's coding theorem tells us that the source is reliably transmissible provided its entropy rate is below the sum $C_1 + C_2$ of the channel capacities; conversely, if the source entropy rate exceeds $C_1 + C_2$ then reliable transmission is not possible. The new twist in the model in Figure 1 is that the information going through channel 2 is not encoded. The following practical scenarios which fit into this model are studied in this paper: an existing uncoded communication link is to be upgraded with the addition of a coded channel in order to provide reliable transmission; the receiver obtains a noisy version of the raw data in addition to the coded channel output; a single channel time-multiplexed into several independent subchannels.

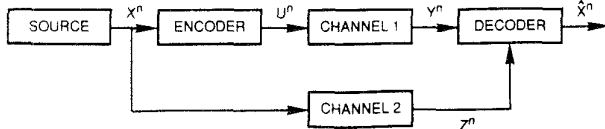


Fig. 1: Channel with Uncoded Side Information

II. CODING THEOREM

Our main result states that *the source can be transmitted reliably provided that its conditional entropy rate given the output of the uncoded channel, $H(\mathbf{X}|\mathbf{Z})$, is below the capacity C_1 of channel 1, and, conversely, it cannot be transmitted reliably if the conditional entropy rate exceeds C_1 .*

This result suggests that we view the information rate of the source as split into two nonoverlapping components, $H(\mathbf{X}) = H(\mathbf{X}|\mathbf{Z}) + I(\mathbf{X};\mathbf{Z})$. Even though the information quantified by the second term is transmitted uncoded, the source is reproducible with arbitrary reliability at the output. If, furthermore, the source is matched to the uncoded channel in the sense that it maximizes its input/output mutual information, then it is possible to transmit information at rate

$C_1 + C_2$ even though no coding is provided for the information going through one of the channels. This implies that the sum of the capacities of K independent parallel binary symmetric channels can be achieved even if only one of them is encoded. This observation is most striking when the encoded BSC has very poor crossover probability.

Our coding theorem is proved under very mild conditions on the channels and the source. The source and the output of the uncoded channel are assumed to be jointly ergodic/stationary and the coded channel is assumed to be such that its capacity is equal to the limit of maximal mutual informations.

To prove the converse part of the result we show that even if the encoder were to observe the output of the uncoded channel, it would not be possible to send information at a faster rate. The proof of the achievability part is by construction of an encoder where the source coding and channel coding operations are performed separately. The source encoder does not operate at the full entropy rate of the source. Rather it is a *Slepian-Wolf encoder* [1] operating at rate $H(\mathbf{X}|\mathbf{Z})$. In the special case of binary-input memoryless channels, optimal encoding is possible by restricting attention to linear codes.

III. PARALLEL-CONCATENATED CODES

Parallel-Concatenated codes, and in particular Turbo codes [2], exhibit favorable complexity/performance tradeoffs. They can be cast within the model of this paper by considering a single-channel time-multiplexed into several independent subchannels. For example, one subchannel transmits the uncoded raw data (the Turbo codes are systematic), and two parallel channels are driven by *partial* encoders which can be viewed as joint source-channel encoders driven by a redundant source. A practically appealing way to ensure that the information encoded by the partial encoders is nonoverlapping is by prepending a sufficiently long interleaver at the input of one of the encoders. This setup is more attractive than simply multiplexing the source because of the complexity reductions of combined source/channel coding with high compression ratios. Good component codes in Parallel-Concatenated schemes are able to trade to some extent the traditional role of reducing the uncertainty of the source given the channel outputs for the easier goal of preserving mutual information.

ACKNOWLEDGEMENTS

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REFERENCES

- [1] D. Slepian and J. K. Wolf, Noiseless Coding of correlated information sources, *IEEE Trans. Information Theory*, IT-19, pp. 471-480, July 1973
- [2] C. Berrou, A. Glavieux and P. Thitimajshima, Near Shannon Limit Error Correcting Coding and Decoding: Turbo-Codes, *Proc. International Conference on Communications*, Geneva, Switzerland, pp. 1064-1070, May 23-26, 1993