

Channel Decoding of Systematically Encoded Unknown Redundant Sources

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Often data compression is inexistent or incomplete prior to channel encoding. If the redundancy in the data is known (or estimated) at the decoder and it has a simple structure (such as a Markov source with small number of states) certain channel decoders (Viterbi, BCJR, turbo, or belief propagation) are able to incorporate the knowledge of the statistics of the data in order to enhance performance. However, no universal approach is available which can harness both the redundancy in the channel code and the redundancy in the data without prior knowledge of the statistical structure of the data. We design practical decoders that take advantage of the source redundancy without requiring prior knowledge of the source statistics, in the case where the source is systematically encoded for transmission over a discrete memoryless channel (DMC).

A key building block in the proposed decoders is the discrete universal denoiser (DUDE), which operates as follows [1]. Letting z^n be a DMC-corrupted version of a data sequence x^n , the DUDE, operating on z^n alone, computes an estimate $\hat{P}_{X|Z^{2\kappa+1}}$ of the empirical conditional distribution of x_i given $z_{i-\kappa}^{i+\kappa}$ and then, for each i , estimates x_i using the input symbol \hat{x} minimizing $E(\Lambda(X, \hat{x})|z_{i-\kappa}^{i+\kappa})$ for some loss function Λ , where the expectation is taken with respect to $\hat{P}_{X|Z^{2\kappa+1}}$. The resulting denoising performance approaches that of the best genie aided finite window denoiser of size $2\kappa + 1$ determined with knowledge of x^n [1]. One of our decoders also relies on a generalization of the DUDE to channels with discrete inputs and continuous outputs [2]. We refer to this denoiser as DUDE*.

The following setup is common to the proposed DUDE-enhanced decoders described below, where throughout \mathcal{X} and \mathcal{Z} denote the DMC input and output alphabets. A source sequence $X^k \in \mathcal{X}^k$ is encoded using a systematic (n, k) -encoder to obtain the codeword (W^{n-k}, X^k) where W^{n-k} denotes the parity-check symbols. The codeword is passed through a channel to obtain the noisy output sequence $(Y^{n-k}, Z^k) \in \mathcal{Z}^n$, with Y^{n-k} denoting the noisy parity symbols and Z^k denoting the noisy information symbols. Three DUDE-enhanced decoders are considered.

1. *Hard-Denoising and Decoding*: The DUDE (with Λ corresponding to Hamming loss) operates on Z^k , the noisy information symbols, to generate a denoised version \hat{X}^k which is appended to Y^{n-k} , and passed through an off-the-shelf decoder, denoted by the mapping $\psi^n : \mathcal{Z}^n \rightarrow \mathcal{X}^k$, to obtain $\bar{X}^k = \psi^n(Y^{n-k}, \hat{X}^k)$, the hard DUDE-enhanced decoded in-

formation symbols.

2. *Soft-Denoising and Decoding*: Applying the DUDE to Z^k we obtain the conditional marginal input distributions $\hat{P}_{X_i} = \hat{P}_{X|Z^{2\kappa+1}}(\cdot|Z_{i-\kappa}^{i+\kappa})$. For decoding we need $\phi^n : [\mathcal{R}^{\mathcal{X}}]^n \rightarrow \mathcal{X}^k$, a soft-input decoder, capable of taking advantage of prior distributional or likelihood information concerning the code-word symbols. Let \hat{P}_{W_i} , $i = 1, \dots, n - k$, denote the parity symbol likelihood information obtained from the observations and DMC transition probability matrix. The soft DUDE-enhanced decoded symbols are then given by $\bar{X}^k = \phi^n(\hat{P}_{W_1}, \dots, \hat{P}_{W_{n-k}}, \hat{P}_{X_1}, \dots, \hat{P}_{X_k})$.

3. *Iterative Denoising and Decoding*: This architecture requires a soft-input-soft-output channel decoder (such as belief propagation (BP)) $\varphi^n : [\mathcal{R}^{\mathcal{X}}]^n \rightarrow [\mathcal{R}^{\mathcal{X}}]^n$ that outputs an estimate of the likelihood of each channel input symbol. The first iteration is identical to a run of decoder 2 above using a soft-input-soft-output decoding algorithm. In each successive iteration, the DUDE is replaced by the DUDE* operating on the likelihood information for the information symbols emitted by the soft-input-soft-output decoding algorithm in the previous iteration, as if it were the output of a discrete input/continuous output channel. After a sufficient number of iterations, MAP information symbol decisions are made using the computed likelihoods. Each run of the DUDE* algorithm requires a channel model characterizing the conditional distribution of the likelihood information emitted by the decoder for any information symbol, given the corresponding channel input symbol. It is argued that these iteration dependent conditional distributions can be modeled with a few parameters (e.g., Gaussian) that can be empirically learned off line.

Experimental results indicate that the foregoing decoders can provide substantial reductions in bit-error rates over conventional decoders that ignore redundancy in the source. One representative set of results involves a binary image encoded (in blocks) using a rate 1/4 repeat accumulate (RA) code of block length 4000 and transmitted over a binary symmetric channel with crossover probability 0.17. BP decoding alone achieves a bit error rate of 2.27×10^{-2} while DUDE-enhanced BP decoder 1 achieves 4.12×10^{-4} and DUDE-enhanced BP decoder 2 achieves 3.66×10^{-5} . Further results involving Markov sources and RA codes show that the proposed DUDE-enhanced BP decoders closely track the performance of various source distribution dependent BP decoder structures. Experiments involving Reed-Solomon codes show that DUDE-enhanced decoding can also be very effective at high rates.

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