Design of MMSE Multiuser Detectors using Random Matrix Techniques

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Abstract— **Reduced-rank MMSE receivers using asymptotic weights reduce receiver complexity while maintaining good performance in long-sequence DS-CDMA systems. In this paper, we analyze such receivers in multipath fading channels and extend their design to multicarrier CDMA (both uplink and downlink). An explicit expression is obtained for the asymptotic eigenvalue moments of the interference autocorrelation matrix and for the asymptotic weights derived therefrom and used in the reduced-rank receiver. The full-rank MMSE receiver is also considered for multicarrier CDMA and a fixed point equation of the asymptotic maximum output SINR is derived which particularizes to the Tse-Hanly fixed point equation for the special case of DS-CDMA. An explicit expression of the MMSE spectral efficiency is proposed for multicarrier CDMA.**

I. INTRODUCTION

Asymptotic analysis of DS-CDMA systems with random spreading has received much attention because it helps to understand the behavior of multiuser receivers in large (in the sense that the processing gain and the number of users are large) DS-CDMA systems [1], [2], [11]. Particular attention has been paid to the analysis of the linear minimum mean-squared error (MMSE) receiver. In order to reduce its complexity, the reduced-rank MMSE receiver, which uses a polynomial expansion to approximate the matrix inversion, was proposed in [3]. The weights used in the reduced-rank receiver depend on the spreading sequences. Therefore, in a long-sequence CDMA system, they need to be computed and updated from symbol to symbol, which hampers real-time implementation.

To solve this problem, [5] proposed an asymptotic reduced-rank MMSE receiver which replaces the actual weights by their asymptotic values. The asymptotic weights depend only on the system load (ratio of the number of users to the processing gain) and the power profile of the interfering users but not on the realization of the spreading sequences.

In this paper, we first extend our techniques in [5] to multipath (frequency selective) channels, and then to multicarrier CDMA (MC-CDMA), both uplink and downlink. Finally, the full-rank MMSE receiver is also analyzed for uplink and downlink MC-CDMA.

Our analysis is carried out under the random signature sequence model. The chip values of the signature sequences are i.i.d. complex Gaussian random variables with zero mean and variance $1/N$, where N is the processing gain. The sequences are independently chosen for different users. The large system limit is taken when the processing gain and the number of users (denoted by K) go to infinity with their ratio fixed. Perfect channel state information at the receiver is assumed, which implies that in the full-rank or reduced-rank MMSE receiver, the interference autocorrelation matrix is conditioned on the fading coefficients.

II. DS-CDMA IN MULTIPATH FADING CHANNELS

In this section, we present our results on the asymptotic reduced-rank MMSE receiver for the DS-CDMA signal transmitted over multipath fading channels. Suppose the channel is of order L, i.e., the transmitted signal of each user propagates through L resolvable discrete paths to the receiver. The output of the chip-matched filter at the receiver is

$$\mathbf{r} \;=\; \sum_{l=1}^{L}\sum_{k=1}^{K} A_{k,l} b_k \mathbf{s}_{k,l} + \mathbf{n} = \mathbf{S}\mathbf{A}\mathbf{b} + \mathbf{n}, \qquad (1)$$

where $A_{k,l}$ is the complex fading coefficient for path l of user k, b_k is the transmitted symbol of user k assumed i.i.d. across users and with $\mathrm{E}[|b_k|^2]{=}1$, $\mathbf{s}_{k,l}$ is the signature sequence received from path l of user k, $\mathbf{n}$ is an additive noise vector distributed as $\mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$, σ^2 is the noise power, $\mathbf{S}{=}(\tilde{\mathbf{S}}_1, \ldots, \tilde{\mathbf{S}}_K)$, $\tilde{\mathbf{S}}_k{=}(\mathbf{s}_{k,1}, \ldots, \mathbf{s}_{k,L})$, $\mathbf{a}_k{=}(A_{k,1}, \ldots, A_{k,L})^T$,

$$\mathbf{A}_{(KL)\times K} = \begin{bmatrix} \mathbf{a}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{a}_K \end{bmatrix},$$

and $\mathbf{b}{=}(b_1, \ldots, b_K)^T$.

Assuming that the spreading sequences and the fading channel coefficients are known at the receiver, the MMSE receiver for user 1 is

$$\mathbf{c} \;=\; (\mathbf{S}_1\mathbf{A}_1\mathbf{A}_1^H\mathbf{S}_1^H + \sigma^2\mathbf{I})^{-1}\tilde{\mathbf{S}}_1\mathbf{a}_1, \qquad (2)$$

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where $\mathbf{S}_1=(\tilde{\mathbf{S}}_2,\ldots,\tilde{\mathbf{S}}_K)$, and

$$\mathbf{A}_1=\begin{bmatrix}\mathbf{a}_2 & \cdots & \mathbf{0}\\ \vdots & \ddots & \vdots\\ \mathbf{0} & \cdots & \mathbf{a}_K\end{bmatrix}.$$

The rank-D reduced-rank MMSE receiver is

$$\mathbf{c}_D=\sum_{m=0}^{D-1} w_m(\mathbf{S}_1\mathbf{A}_1\mathbf{A}_1^H\mathbf{S}_1^H+\sigma^2\mathbf{I})^m\tilde{\mathbf{S}}_1\mathbf{a}_1, \qquad (3)$$

where the weight vector $\mathbf{w}=(w_0,\ldots,w_{D-1})^T$ is chosen to minimize the mean-squared error and is given by [8], [10]

$$\mathbf{w}=\mathbf{Q}\begin{bmatrix}\mathcal{H}_0\\ \vdots\\ \mathcal{H}_{D-1}\end{bmatrix}, \qquad (4)$$

where $\mathbf{Q}$ is a $D\times D$ matrix whose (i,j)-entry is $\mathcal{H}_{i+j-1}+\mathcal{H}_{i-1}\mathcal{H}_{j-1}$ with

$$\mathcal{H}_m=\mathbf{a}_1^H\tilde{\mathbf{S}}_1^H\left(\mathbf{S}_1\mathbf{A}_1\mathbf{A}_1^H\mathbf{S}_1^H+\sigma^2\mathbf{I}\right)^m\tilde{\mathbf{S}}_1\mathbf{a}_1. \qquad (5)$$

As proposed in [5], the asymptotic reduced-rank MMSE receiver is implemented using the large system limit of the weights instead of the actual weights themselves. The advantage of using asymptotic weights is that they do not depend on the realization of the spreading sequences and of the fading coefficients. In a system using long sequences, this reduces complexity and is desirable in real-time implementation of the receiver. From (4), the large system limit of the weights boils down to the calculation of the asymptotic values of $\mathcal{H}_m$.

To simplify our analysis, we make two assumptions. First, we assume that the signature sequences received for different paths of a given user are independent. Although it is more realistic to model these sequences as shifted (cyclic or with partial overlap, i.e. with zeros preceding the shifted sequence) versions of each other, as long as the delay spread (measured in chips) remains much smaller than the processing gain, it can actually be shown analytically that all these cases are asymptotically equivalent. This assumption is also supported by our numerical results in [6]. Therefore, $\tilde{\mathbf{S}}_1$ and $\mathbf{S}_1$ consist of i.i.d. entries under our assumption. Second, we assume that the asymptotic eigenvalue distribution of $\mathbf{S}_1\mathbf{A}_1\mathbf{A}_1^H\mathbf{S}_1^H$ has bounded support. It can be shown that if $\mathbf{A}_1\mathbf{A}_1^H$ has bounded support, then $\mathbf{S}_1\mathbf{A}_1\mathbf{A}_1^H\mathbf{S}_1^H$ also has bounded support [6]. The bounded support assumption on $\mathbf{A}_1\mathbf{A}_1^H$ is motivated by physical considerations. Under this assumption, Lemma A.1 and Corollary A.2 in [1] can be used as in the proof of Theorem A.4 in [1] to show that $\tilde{\mathbf{S}}_1^H\left(\mathbf{S}_1\mathbf{A}_1\mathbf{A}_1^H\mathbf{S}_1^H+\sigma^2\mathbf{I}\right)^m\tilde{\mathbf{S}}_1$ converges almost surely elementwise to an $L\times L$ diagonal matrix $\gamma_m\mathbf{I}$, where

$$\gamma_m=\int(\lambda+\sigma^2)^m dG(\lambda), \qquad (6)$$

with $G(\lambda)$ the non-random limit, as $K,N\to\infty$ but their ratio is kept constant, of the empirical eigenvalue distribution of $\mathbf{S}_1\mathbf{A}_1\mathbf{A}_1^H\mathbf{S}_1^H$ [9]. From (5) and (6), the large system limit of $\mathcal{H}_m$ is

$$\mathcal{H}_m^\infty \triangleq \lim_{\substack{N,K\to\infty\\ \frac{K}{N}=\beta}}\mathcal{H}_m=\|\mathbf{a}_1\|^2\gamma_m. \qquad (7)$$

We have the following result of the m-th moment of $G(\lambda)$ (the proof can be found in [6]): the m-th moment of the limiting eigenvalue distribution of $\mathbf{S}_1\mathbf{A}_1\mathbf{A}_1^H\mathbf{S}_1^H$ (denoted by $G(\lambda)$) is

$$\int\lambda^m dG(\lambda)=\sum_{k=1}^{m}\beta^k\sum_{\substack{m_1+\cdots+m_k=m\\ m_1\le\cdots\le m_k}}c(\mathbf{m}_k^m)\cdot P_{(m_1)}\cdots P_{(m_k)}, \qquad (8)$$

with $\beta=L\beta'$, $\beta'=\lim_{K\to\infty}\frac{K}{N}$, and

$$c(\mathbf{m}_k^m)=\frac{m!}{(m-k+1)!\cdot f(\mathbf{m}_k^m)}, \quad P_{(j)}=\frac{1}{L}\int P^j dH(P)$$

where $H(P)$ is the limiting empirical distribution of $\|\mathbf{a}_2\|^2,\ldots,\|\mathbf{a}_K\|^2$, $\mathbf{m}_k^m=[m_1,\ldots,m_k]$ is a k-dimensional vector whose sum element (for simplicity, in the following we denote by $\mathbf{x}_p^q$ a p-dimensional vector whose elements add up to q) is equal to m and $f(\mathbf{m}_k^m)$ is defined as follows: suppose the vector of k integers $\mathbf{m}_k^m=[m_1,\ldots,m_k]$ is partitioned into n equivalence classes under the equivalence relation $a=b$, and the cardinalities of the equivalence classes are $f_1,\ldots,f_n$, then $f(\mathbf{m}_k^m)\triangleq f_1!\cdots f_n!$. For example, $f(1,1,4,2,1,2)=3!\cdot 2!\cdot 1!$.

As seen in (6), γ_m can be obtained by binomial expansion from the expression of the moments of $G(\lambda)$. Therefore, $\mathcal{H}_m^\infty=\|\mathbf{a}_1\|^2\gamma_m$ can be calculated. So by plugging $\mathcal{H}_m$ into (4), we obtain the asymptotic weights.

III. MULTICARRIER CDMA

In a MC-CDMA system, all users share the N evenly spaced subcarriers, where N is the processing gain. The signature sequence of each user is first multiplied by its information bit, then N chip values are modulated and transmitted in parallel through the N subcarriers. The overall transmitted signal is the sum of all users' signals. In the following two sections, we present our results for both the uplink and downlink MC-CDMA systems.

A. Uplink MC-CDMA

Assuming synchronous users, at the output of the chip-matched filter, the received vector is

$$\mathbf{r}=\sum_{k=1}^{K}A_k b_k\tilde{\mathbf{s}}_k+\mathbf{n}=\tilde{\mathbf{S}}\mathbf{A}\mathbf{b}+\mathbf{n}, \qquad (9)$$

where $\tilde{\mathbf{s}}_k{=}\mathbf{H}_k\mathbf{s}_k$, $\mathbf{s}_k$ is the spreading sequence of user k, $\mathbf{H}_k{=}\mathrm{diag}\{H_k^1,\ldots,H_k^N\}$, H_k^i $(1{\le}i{\le}\ N)$ is the fading channel gain of the ith subcarrier of user k, $A_k{>}0$ is the transmitted amplitude of user k, b_k is the transmitted symbol of user k assumed i.i.d. across users and with $\mathrm{E}[|b_k|^2]{=}1$, $\mathbf{n}$ is an additive noise distributed as $\mathcal{N}(\mathbf{0},\sigma^2\mathbf{I})$, σ^2 is the noise power, $\tilde{\mathbf{S}}{=}(\tilde{\mathbf{s}}_1,\ldots,\tilde{\mathbf{s}}_K)$, $\mathbf{A}{=}\mathrm{diag}\{A_1,\ldots,A_K\}$, and $\mathbf{b}{=}(b_1,\ldots,b_K)^T$. Here we assume that the fading coefficients H_k^i's $(1{\le}i{\le}\ N{,}1{\le}k{\le}\ K)$ are independent across i and k, and we assume

$$\mathrm{E}[|H_k^i|^2]=C_i. \tag{10}$$

This means that the users experience independent—but statistically identical—fading channels. The fading at different subcarriers might also be statistically identical, but in interference environments the noise may not be and such differences can be accounted for by scaling the fading. Thus, we keep the model general and allow for C_i to depend on i. Our results on the asymptotic moments of the autocorrelation matrix of the received vector and the asymptotic weights of the reduced-rank receiver are presented as follows.

A.1 Asymptotic Moments: **Proposition 1.** *Suppose that the empirical distribution of* $C_1,\ldots,C_N$ *converges to a limiting distribution* $E_1(C)$ *as* $N{\to}\infty$, *and the empirical distribution of* $P_1{=}A_1^2,\ldots,P_K{=}A_K^2$ *converges to a limiting distribution* $F(P)$ *as* $K{\to}\infty$ *then as* $K,N{\to}\infty$ *with* $K/N{=}\beta$, *the m-th moment,* $\mu_m{=}\lim_{K\to\infty}\frac{1}{N}\mathrm{Tr}\left\{(\tilde{\mathbf{S}}\mathbf{A}\mathbf{A}^H\tilde{\mathbf{S}}^H)^m\right\}$, *of* $\tilde{\mathbf{S}}\mathbf{A}\mathbf{A}^H\tilde{\mathbf{S}}^H$ *converges almost surely to*

$$\sum_{k=1}^{m}\beta^k\sum_{\substack{m_1+\cdots+m_k=m\\ m_1\le\cdots\le m_k}}\ \sum_{\substack{n_1+\cdots+n_{m+1-k}=m\\ n_1\le\cdots\le n_{m+1-k}}}B(\mathbf{m}_k^m,\mathbf{n}_{m+1-k}^m)\cdot P_{(m_1)}\cdots P_{(m_k)}C_{(n_1)}\cdots C_{(n_{m+1-k})}, \tag{11}$$

where

$$C_{(i)}\triangleq\int C^i dE_1(C),\qquad P_{(i)}\triangleq\int P^i dF(P), \tag{12}$$

$\mathbf{m}_k^m{=}[m_1,\ldots,m_k]$ *is a k-dimensional vector of integers whose sum is equal to m,* $\mathbf{n}_{m+1-k}^m{=}[n_1,\ldots,n_{m+1-k}]$*is a (m+1−k)-dimensional vector of integers that add up to m, and the coefficient*

$$B(\mathbf{m}_k^m,\mathbf{n}_{m+1-k}^m)=\frac{m(m-k)!(k-1)!}{f(\mathbf{m}_k^m)\cdot f(\mathbf{n}_{m+1-k}^m)} \tag{13}$$

is the number of non-crossing partitions π *on* $\{1,\ldots,m\}$ *satisfying the following conditions:*

(i) the cardinalities of the subsets in π *are (in increasing order)* $m_1,\ldots,m_k$,

(ii) the cardinalities of the subsets in $K(\pi)$, *which is the complementation map (see [4]) of* π, *are (in increasing order)* $n_1,\ldots,n_{m+1-k}$.

The $f(\cdot)$ *function in (13) is as defined in Section II.*

Proof: For proof, please refer to Appendix B in [6].

A.2 Asymptotic Weights: The rank-D MMSE receiver for the uplink MC-CDMA system is

$$\mathbf{c}_D=\sum_{m=0}^{D-1}w_m(\tilde{\mathbf{S}}_1\mathbf{D}_1\tilde{\mathbf{S}}_1^H+\sigma^2\mathbf{I})^m\tilde{\mathbf{s}}_1, \tag{14}$$

where $\tilde{\mathbf{S}}_1{=}(\tilde{\mathbf{s}}_2,\ldots,\tilde{\mathbf{s}}_K)$, $\mathbf{D}_1{=}\mathrm{diag}\{A_2^2,\ldots,A_K^2\}$, and the weight vector $\mathbf{w}{=}(w_0,\ldots,w_{D-1})^T$ is chosen to minimize the mean-squared error and is given by (4) with

$$\mathcal{H}_m=A_1^2\tilde{\mathbf{s}}_1^H\left(\tilde{\mathbf{S}}_1\mathbf{D}_1\tilde{\mathbf{S}}_1^H+\sigma^2\mathbf{I}\right)^m\tilde{\mathbf{s}}_1. \tag{15}$$

Suppose that the empirical distribution of $C_1,\ldots,C_N$ converges to a limiting distribution $E_1(C)$ as $N{\to}\infty$ and the empirical distribution of $P_2{=}A_2^2,\ldots,P_K{=}A_K^2$ converges to a limiting distribution $F(P)$ as $K{\to}\infty$, then we need to calculate the large system limit of the $\mathcal{H}_m$'s and plug them into (4) to obtain the asymptotic weights. In what follows, we calculate the expected large system limit of $\mathcal{H}_m$, namely

$$\lim_{K\to\infty}\mathrm{E}[\mathcal{H}_m], \tag{16}$$

where the expectation is taken with respect to the random spreading sequences and the subcarrier fading channel coefficients. As discussed in [6], it is possible to show that $\mathcal{H}_m$'s themselves converge almost surely. However, to simplify our derivation, here we concentrate on the calculation of the large system limit of the expectation of $\mathcal{H}_m$ (16) which coincides with the almost sure limit of $\mathcal{H}_m$. Since

$$\lim_{K\to\infty}\mathrm{E}[\tilde{\mathbf{s}}_1^H\tilde{\mathbf{s}}_1]=\lim_{K\to\infty}\mathrm{E}[\mathbf{s}_1^H H_1^H H_1\mathbf{s}_1]=\frac{1}{N}\sum_{i=1}^{N}C_i\to C_{(1)}, \tag{17}$$

where $C_{(1)}{=}\int CdE_1(C)$. From (15), the calculation of $\lim_{K\to\infty}\mathrm{E}[\mathcal{H}_m]$ reduces to the binomial expansion and to the calculation of

$$\delta_m\triangleq\lim_{K\to\infty}\mathrm{E}\left[\tilde{\mathbf{s}}_1^H\left(\tilde{\mathbf{S}}_1\mathbf{D}_1\tilde{\mathbf{S}}_1^H\right)^m\tilde{\mathbf{s}}_1\right]. \tag{18}$$

An algorithm for calculating δ_m for any given m is proposed in Appendix C of [6].

B. Downlink MC-CDMA

In a downlink MC-CDMA system, the fading channel gains of the N subcarriers are common to all users. If those gains are $H_1,\ldots,H_N$, the received vector at the output of the chip-matched filter is

$$\mathbf{r}\ =\ \sum_{k=1}^{K}A_kb_k\mathbf{H}\mathbf{s}_k+\mathbf{n}=\mathbf{H}\mathbf{S}\mathbf{A}\mathbf{b}+\mathbf{n} \tag{19}$$

where $A_k{>}0$ is the transmitted amplitude of user k, b_k is the transmitted symbol of user k which is assumed to be i.i.d. across users and to have unit energy as in the previous sections, $\mathbf{H}{=}\mathrm{diag}\{H_1,\ldots,H_N\}$,

$\mathbf{s}_k$ is the signature sequence of user k, $\mathbf{n}$ is an additive white Gaussian noise vector distributed as $\mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$, $\mathbf{S}{=}(\mathbf{s}_1, \ldots, \mathbf{s}_K)$, $\mathbf{A}{=}\mathrm{diag}\{A_1, \ldots, A_K\}$, and $\mathbf{b}{=}(b_1, \ldots, b_K)^T$. We let $C_i{=}|H_i|^2$ $(1{\le}i{\le}N)$. In what follows, we present our results on the asymptotic moments of the autocorrelation matrix of the received vector and the asymptotic weights of the reduced-rank receiver.

B.1 Asymptotic Moments: **Proposition 2.** *Suppose that the empirical distribution of $C_1, \ldots, C_N$ converges to a limiting distribution $E_2(C)$ as $N{\to}\infty$, and the empirical distribution of $P_1{=}A_1^2, \ldots, P_K{=}A_K^2$ converges to a limiting distribution $F(P)$ as $K{\to}\infty$, then the m-th moment, $\mu_m{=}\lim\limits_{K\to\infty}\dfrac{1}{N}\mathrm{Tr}\left\{(\mathbf{HSAA}^H\mathbf{S}^H\mathbf{H}^H)^m\right\}$, of $\mathbf{HSAA}^H\mathbf{S}^H\mathbf{H}^H$ almost surely converges, as $K, N{\to}\infty$ with $K/N{=}\beta$, to (11) with $E_1(C)$ replaced by $E_2(C)$.*

Proof: See Appendix D in [6].

B.2 Asymptotic Weights: The rank-D MMSE receiver for the downlink MC-CDMA system is

$$\mathbf{c}_D = \sum_{m=0}^{D-1} w_m(\mathbf{HS}_1\mathbf{D}_1\mathbf{S}_1^H\mathbf{H}^H + \sigma^2\mathbf{I})^m\mathbf{Hs}_1, \tag{20}$$

where $\mathbf{S}_1{=}(\mathbf{s}_2, \ldots, \mathbf{s}_K)$, $\mathbf{D}_1{=}\mathrm{diag}\{A_2^2, \ldots, A_K^2\}$, and the weight vector $\mathbf{w}{=}(w_0, \ldots, w_{D-1})^T$ is chosen to minimize the mean-squared error and is given by (4) with

$$\mathcal{H}_m = A_1^2\mathbf{s}_1^H\mathbf{H}^H\left(\mathbf{HS}_1\mathbf{D}_1\mathbf{S}_1^H\mathbf{H}^H + \sigma^2\mathbf{I}\right)^m\mathbf{Hs}_1. \tag{21}$$

As in an uplink system, the asymptotic weights are obtained from the expectation of $\mathcal{H}_m$'s with respect to the spreading sequences. However, it is possible to show that $\mathcal{H}_m$'s themselves converge almost surely to the same limit, validating the use of the asymptotic weights [6]. Our result on the asymptotic weights is:

Proposition 3. *Suppose that the empirical distribution of $C_1, \ldots, C_N$ converges to a limiting distribution $E_2(C)$ as $N{\to}\infty$, and the empirical distribution of $P_2{=}A_2^2, \ldots, P_K{=}A_K^2$ converges to a limiting distribution $F(P)$ as $K{\to}\infty$, then the expected large system limit of $\mathcal{H}_m$, and therefore the asymptotic weights, are the same as in an uplink system with $E_1(C)$ replaced by $E_2(C)$.*

Proof: For proof, please refer to Appendix E of [6].

C. Asymptotic Performance Analysis of Full-rank MMSE Receiver

Finally, we consider the full-rank MMSE receiver and analyze its asymptotic (in N, K) output signal-to-interference-plus-noise ratio (SINR) and spectral efficiency for MC-CDMA.

Let us recall that for downlink and uplink MC-CDMA, A_k is the transmitted amplitude of user k. Let $P_k{\triangleq}A_k^2$. We know that SINR_k is proportional to P_k, therefore,

$$\mathsf{SINR}_k = P_k\xi_k, \tag{22}$$

and ξ_k is a normalized output SINR for user k. As discussed in [6], ξ_k converges almost surely to a limit that does not depend on the user index k, and we denote that limit by ξ^∞:

$$\lim_{\substack{K\to\infty \\ \frac{K}{N}=\beta}} \xi_k = \xi^\infty. \tag{23}$$

In [6], we prove the following fixed point equation satisfied by ξ^∞ for MC-CDMA:

$$\xi^\infty = \mathrm{E}_{E_i}\left[\frac{C}{\sigma^2 + \beta C\,\mathrm{E}_F\left[\frac{P}{1+\xi^\infty P}\right]}\right], \tag{24}$$

and the following equation relating ξ^∞ to the asymptotic eigenvalue distribution $G(\lambda)$ of the interference autocorrelation matrix:

$$1 - \sigma^2\int\frac{1}{\lambda+\sigma^2}dG(\lambda) = \beta\xi^\infty\,\mathrm{E}_F\left[\frac{P}{1+\xi^\infty P}\right], \tag{25}$$

where E_{E_i} is the expectation taken with respect to $E_i(C)$, E_F is the expectation taken with respect to $F(P)$, and $E_i(C)$ is equal to $E_1(C)$ or $E_2(C)$ for uplink and downlink MC-CDMA, respectively. From (24), it is easy to see that the asymptotic maximum output SINR of uplink MC-CDMA is equal to that of downlink MC-CDMA given that their asymptotic subcarrier fading profiles ($E_i(C)$ with i=1,2) are equal. The above equivalence can also be seen from (25) considering Proposition 2, which states that the moments of $G(\lambda)$ of the uplink and downlink systems are identical.

In addition to the output SINR, another performance measure of interest is the spectral efficiency. It is defined as the total number of bits per chip that can be transmitted arbitrarily reliably. In [7], the following explicit expression of the asymptotic (in N, K) MMSE spectral efficiency for MC-CDMA is proven in terms of the normalized output SINR ξ^∞ that satisfies (24).

$$\mathsf{C}^{\mathsf{mmse}} = \beta\,\mathrm{E}_F\left[\log\left(1+\xi^\infty P\right)\right]. \tag{26}$$

Analysis on the spectral efficiency of MMSE receiver and other receivers (jointly optimum receiver, decorrelator, and single-user matched filter) for MC-CDMA can be found in [7].

IV. NUMERICAL RESULTS

Let us compare the output SINR of two receivers: (i) the reduced-rank MMSE receiver that uses the actual weights; (ii) the asymptotic reduced-rank MMSE receiver that uses the asymptotic weights. The output SINR of the matched filter is also plotted for comparison. In the simulation, the channel is assumed perfectly known at the receiver.

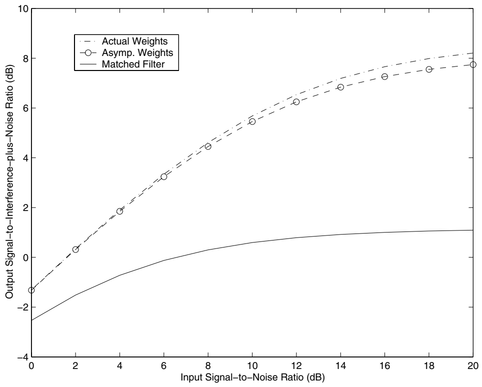

Fig. 1. Downlink MC-CDMA: N=64, K=33.

In the implementation of the asymptotic reduced-rank receiver, we use the moments of the empirical distribution of $P_2, \ldots, P_K$ when K is finite to approximate the moments of the limiting empirical distribution $F(P)$, i.e.

$$P_{(m)} = \int P^m dF(P) \approx \frac{1}{K-1} \sum_{k=2}^{K} P_k^m. \qquad (27)$$

Similarly, we use $\frac{1}{N}\sum_{n=1}^{N} C_n^m$ to approximate $C_{(m)}$.

Figure 1 shows the performance of the receivers for downlink MC-CDMA with $N = 64, K = 33$. The transmitted amplitudes of the interfering users are uniformly distributed and the interfering users have the same transmitted power as the desired user on average. The limiting empirical distribution of $C_1, \ldots, C_N$ is a Rayleigh distribution, and a 3-stage reduced-rank receiver is used in the simulation. We point out that significant gain in the output SINR of the reduced-rank receivers can be achieved if a few more stages are used. In Figure 1 we see that the performance loss due to the use of asymptotic weights is very small for the input SNR considered.

V. CONCLUSION

Reduced-rank MMSE receivers, which use polynomial expansion to approximate matrix inversion, reduce the computational complexity of the full-rank MMSE multiuser receiver for DS-CDMA.

The weights in a reduced-rank MMSE receiver, however, still depend on the spreading sequences. This implies that, in long-sequence CDMA systems, they must be re-computed for each symbol, which seriously hampers real-time implementation.

The central idea in this paper is that those weights can be replaced by the values they take asymptotically (i.e. in the limit of infinite number of users and processing gain) with little loss in performance. In contrast with the actual weights, the asymptotic weights depend on neither the realization of the spreading sequences nor the realization of the channels and hence they do not need to be updated from symbol to symbol. Therefore, real-time implementation becomes feasible.

Using combinatorics on non-crossing partitions and random matrix techniques, we design single- and multi-antenna DS-CDMA receivers in frequency-flat and multipath (frequency-selective) fading channels. Furthermore, we extend the receiver design to multicarrier CDMA. In addition, the performance of full-rank MMSE receiver is analyzed for MC-CDMA. A fixed point equation is obtained for the maximum output SINR. We also prove close equivalences between the asymptotic moments and weights of uplink and downlink MC-CDMA. Numerical results show that, for practical processing gains, the performance loss due to the use of asymptotic—rather than actual —weights is very small for input SNR levels of interest.

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