

# EAVESDROPPING SYNDICATES IN CELLULAR COMMUNICATIONS \*

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**Abstract** – We consider eavesdropping strategies for a syndicate consisting of a number of co-operative eavesdroppers employing selective diversity in a wireless communication cell. The potential for performance improvement over that of a single eavesdropper is analyzed. Performance limits are developed by considering a syndicate with an arbitrarily large supply of eavesdroppers. Optimal configurations are considered under the assumption that eavesdropper cell locations must be fixed prior to activity of the users, although the special case of an arbitrarily large syndicate can be viewed as a single mobile eavesdropper afforded the capability to optimally situate itself upon initialization of the desired user. Cells employing power control are considered along with those that do not.

## I. INTRODUCTION

Consider a circular communication cell of radius  $A$  with a centrally-located base station and  $K$  active users with independent uniformly random locations. A syndicate consisting of  $J$  eavesdroppers wishes to eavesdrop on the transmission of a single desired user. It is assumed that eavesdropper locations must be fixed without knowledge of the locations of the desired and interfering users. It is also assumed that the eavesdropping syndicate employs selective diversity, representative of the scenario where communication among the eavesdroppers is not of sufficient quality to support data sharing required by more effective diversity techniques. Signal attenuation is taken to follow a log-distance path loss model [1].

The scenario at hand is depicted in Figure 1. If user 1 is the user of interest, it is clear that eavesdropper 1 will enjoy the strongest received signal power due to its closer proximity, and would therefore be best suited to detect user 1 on behalf of the syndicate. It is clear from Figure 1 that the realization of the interfering users' locations affect eavesdropping performance when the users are assigned correlated signal waveforms; for instance, eavesdropper 1 will suffer from the near-far effect if user 2 becomes the user of interest. Although this observation suggests that the location of *all* users should be incorporated into the employed selectivity scheme, we will consider selective diversity based only on received power of the user of interest in order to maintain simplicity of both implementation and analysis, and with the hope that with reasonably high probability such a scheme will provide desirable results.

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For systems wherein users are assigned orthogonal waveforms, this is clearly the scheme of interest.

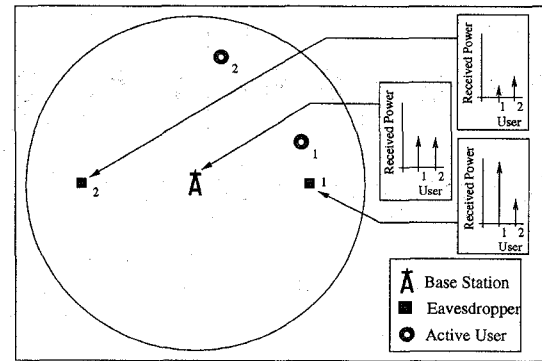


Figure 1: Eavesdropping Syndicate with Two Eavesdroppers in a Cell with Two Users

In Section II we analyze eavesdropper performance for a cell employing power control; the optimal configuration for an eavesdropping duo is given, resulting performance is compared with that achievable by larger syndicates, and performance limits are developed. Cells which do not employ power control are treated in Section III. A summary and interpretation of results is presented in Section IV.

## II. CELLS WITH POWER CONTROL

In this section, we investigate eavesdropper performance in cells wherein the base station employs power control, mandating that each active user adjust its transmit power to be received at a reference level which we take to be 0dB.

### A. Eavesdropping Duo in Cells with Power Control

Under the log-distance path loss model, the ratio of received power  $P_R$  to transmitted power  $P_T$  is given by

$$P_R/P_T = 1/d^{2\alpha} \quad (1)$$

where  $d$  is the transmitter-receiver separation and  $2\alpha$  represents the path loss exponent [1]; we take  $\alpha = 1$  throughout when numerical substitutions are warranted. Consider a single eavesdropper located a distance  $a \leq A$  from the base station. In [2] it was shown that the induced cumulative distribution function (cdf) of received power  $P$  from a uniformly randomly located user is dependent on the eavesdropper location through a single *off-centered-ness* param-

eter  $\gamma \triangleq a/A$  and is given by

$$F_P(p) \triangleq \text{Prob}(P \leq p) = \begin{cases} \frac{p^{1/\alpha} \gamma^2}{(1-p^{1/\alpha})^2}, & 0 \leq p \leq \frac{1}{(1+\gamma)^{2\alpha}} \\ \frac{1}{2} + \psi_1 + \psi_2(1-\psi_3) + \psi_4, & \frac{1}{(1+\gamma)^{2\alpha}} < p < 1 \\ \frac{1}{2} + \frac{1}{\pi} \sin^{-1}\left(\frac{\gamma}{2}\right) + \frac{\gamma}{4\pi} \sqrt{4-\gamma^2}, & p=1 \\ \frac{1}{2} + \psi_1 - \psi_2(1-\psi_3) + \psi_4, & 1 < p < \frac{1}{(1-\gamma)^{2\alpha}} \\ 1 - \frac{p^{1/\alpha} \gamma^2}{(1-p^{1/\alpha})^2}, & p \geq \frac{1}{(1-\gamma)^{2\alpha}}, \end{cases} \quad (2)$$

$$\psi_1 = \frac{1}{\pi} \sin^{-1}\left(\frac{p^{1/\alpha}(1+\gamma^2)-1}{2p^{1/\alpha}\gamma}\right)$$

$$\psi_2 = \frac{p^{1/\alpha} \gamma^2}{2(1-p^{1/\alpha})^2}$$

$$\psi_3 = \frac{2}{\pi} \sin^{-1}\left(\frac{p^{1/\alpha}(p^{1/\alpha}+1)\gamma^2 - (1-p^{1/\alpha})^2}{2p^{3/2\alpha}\gamma^2}\right)$$

$$\psi_4 = \frac{\sqrt{2p^{1/\alpha}(1+\gamma^2) - p^{2/\alpha}(1-\gamma^2)^2} - 1}{2\pi(p^{1/\alpha} - 1)}$$

where  $\text{Prob}(A)$  denotes the probability of an event  $A$  and the range of  $\sin^{-1}(\cdot)$  is taken to be  $[-\pi/2, \pi, 2]$ . In light of the dependence on cell radius  $A$  exclusively through its ratio with eavesdropper location  $a$ , we will assume throughout a normalized cell radius  $A = 1$ . Figure 2 depicts  $F_P(p)$  for various values of off-centered-ness. Expression (2) provides the necessary tool for evaluating expected performance measures when performance depends on the received powers of one or more independently located users. In this paper, we are concerned with evaluating such a performance measure for syndicates employing more than one eavesdropper, where increased proximity of at least one eavesdropper to the desire user is the anticipated advantage. As such proximity is increased, it is necessary to account for the inconsistency of the path-loss model (1) for very small transmitter-receiver separation  $d$ .

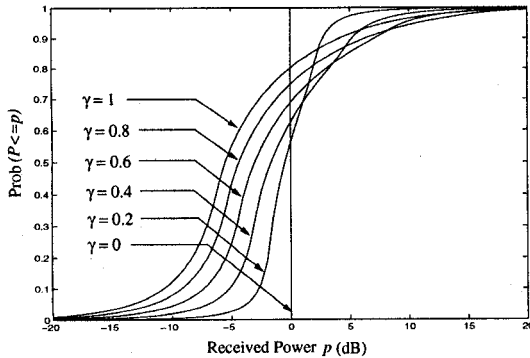


Figure 2: Cumulative Distribution Function of Received Power at a Single Eavesdropper from a Uniformly Located User in a Cell with Power Control

We will assume that the log-distance path loss model holds whenever the receiver falls in the far-field region of

the transmitting antenna as determined by a fixed far-field radius  $r$ ; received power will be taken to be location-independent within this region, taking the value  $p = 1/r^{2\alpha}$ . Hence, the augmented path loss model predicts a received-to-transmitted power ratio of

$$P_R/P_T = \max\{1/d^{2\alpha}, 1/r^{2\alpha}\}$$

given a transmitter-receiver separation  $d$ . For small values of far-field radius  $r$ , minimum and maximum received powers at an eavesdropper with off-centered-ness  $\gamma$  from an active user are given to good approximation by  $(r/\gamma)^{2\alpha}$  and  $(\gamma/r)^{2\alpha}$  respectively whenever either the base station or eavesdropper (or both) falls in the far-field region of the user's transmitting antenna (*i.e.*  $\gamma \geq 2r$ ). Hence, the adjusted cdf  $F_P^1(p)$  for received power at an eavesdropper with off-centered-ness  $\gamma$  from a single uniformly located user is given through (2) by

$$F_P^1(p) = \begin{cases} 0, & 0 \leq p \leq (r/\gamma)^{2\alpha} \\ F_P(p), & (r/\gamma)^{2\alpha} < p \leq (\gamma/r)^{2\alpha} \\ 1, & \text{otherwise.} \end{cases} \quad (3)$$

We will take  $r = .01$  under a model with cell size 10km and far-field radius 100m.

It is of interest to quantify the performance improvement that can be gained by utilizing two eavesdroppers rather than one. With selectivity based on the received power of the desired user's signal, the boundary between the regions of coverage for both eavesdroppers is determined by the perpendicular bisector. Taking for performance measure the expected received power (in dB) of the desired user, it can be shown that the optimal configuration for the eavesdroppers is realized when they share an equal off-centered-ness  $\gamma_2$  on a cell diagonal, as depicted in Figure 3. Hence, determination of the optimal configuration amounts to a single-parameter maximization of expected received power over  $\gamma_2 \in [0, 1]$ .

Figure 3 depicts several representative contours defined by equal received power from a desired user over the regions of coverage for an eavesdropping duo. The cdf  $F_P^2$  of maximum received power among the duo from a uniformly randomly located user is given by

$$F_P^2(p) = \begin{cases} 0, & 0 \leq p \leq \left(\frac{\pi}{\gamma}\right)^{2\alpha} \\ \frac{p^{1/\alpha} \gamma^2}{(1-p^{1/\alpha})^2} \left(1 - \frac{2}{\pi} \left(\sin^{-1}(p^{1/2\alpha}) + p^{1/2\alpha} \sqrt{1-p^{1/\alpha}}\right)\right), & \left(\frac{\pi}{\gamma}\right)^{2\alpha} < p \leq \frac{1}{(1+\gamma^2)^\alpha} \\ 2\psi_1 + 2\psi_2(1-\psi_3) + 2\psi_4, & \frac{1}{(1+\gamma^2)^\alpha} < p < 1 \\ \frac{2}{\pi} \sin^{-1}\left(\frac{\gamma}{2}\right) + \frac{\gamma}{2\pi} \sqrt{4-\gamma^2}, & p=1 \\ 2\psi_1 - 2\psi_2(1-\psi_3) + 2\psi_4, & 1 < p \leq \frac{1}{(1-\gamma)^{2\alpha}} \\ 1 - \frac{2p^{1/\alpha} \gamma^2}{(p^{1/\alpha}-1)^2}, & \frac{1}{(1-\gamma)^{2\alpha}} < p \leq \left(\frac{\gamma}{r}\right)^{2\alpha} \\ 1, & \text{otherwise.} \end{cases}$$

Figure 4 depicts  $F_P^2$  for various off-centered-ness values.

The performance measure under consideration is the expected maximum received power from a desired user

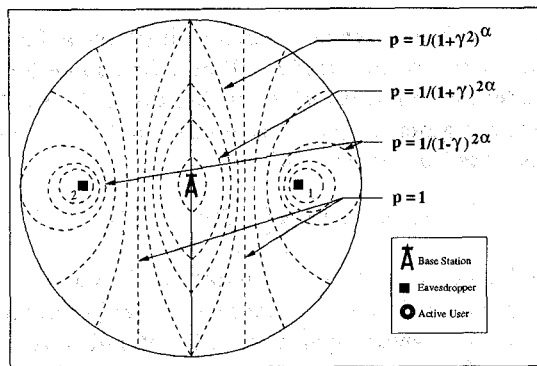


Figure 3: Equal Received Power Contours for an Eavesdropping Duo

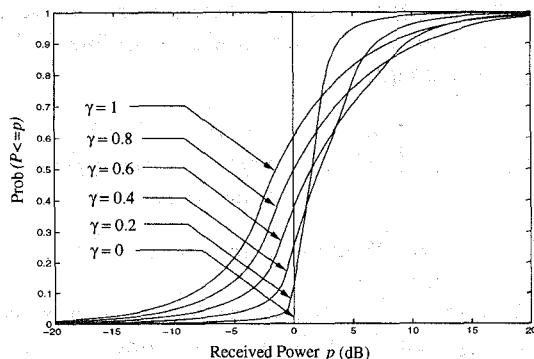


Figure 4: Cumulative Distribution Function of Maximum Received Power Among Two Eavesdroppers from a Uniformly Located User in a Cell with Power Control

among a syndicate, and is given in general by

$$E[10 \log_{10} P] = 10 \log_{10}(P_{\min}) + \frac{10}{\log 10} \int_{P_{\min}}^{P_{\max}} \frac{(1 - F_P(p))}{p} dp \quad (4)$$

where  $P_{\min}$  and  $P_{\max}$  represent minimum and maximum received power respectively,  $F_P(p)$  represents the appropriate cdf of received power, and  $\log(\cdot)$  denotes the natural logarithm. Figure 5 depicts expected received power for syndicates with one and two eavesdroppers as a function of off-centered-ness. Note that the optimal configuration for an eavesdropping duo calls for a non-zero off-centered-ness, exhibiting a gain of approximately 2.5 dB over the optimally (centrally) situated eavesdropper singleton.

### B. Larger Syndicates in Cells with Power Control

Naturally, it is of interest to quantify the potential for performance improvement through employment of larger syndicates. The results pertain not only to larger syndicates, but form an approximate analysis of an eavesdropper with restricted mobility following activation of the users. While it is difficult to determine the optimal configuration for a syndicate with more than two eavesdroppers, we can generate achievable performance curves by fixing a configura-

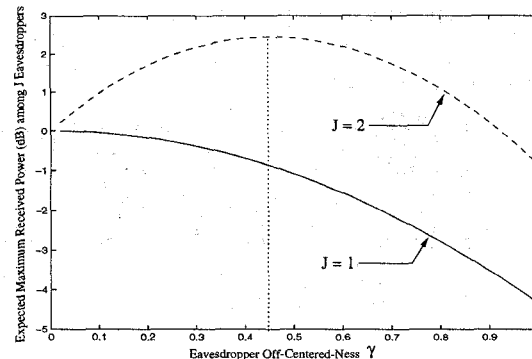


Figure 5: Expected Received Power for Syndicates with One and Two Eavesdroppers

tion; the optimal configuration for a syndicate with more than two users remains an interesting open question.

We consider the so-called *radial* configuration depicted in Figure 6, where each eavesdropper covers a similar angular arc of  $2\pi/J$  radians of the cell. Rotational invariance induced by the uniform distribution of users within the cell allows us to focus on the performance of a single eavesdropper conditioned on the event that a desired user falls within its region of coverage.

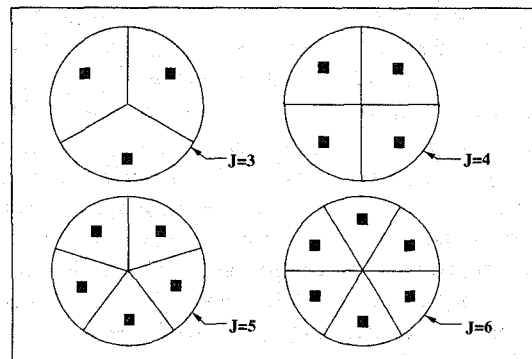


Figure 6: Radial Configuration for Eavesdropping Syndicates of Varying Size  $J$

Figure 7 depicts expected performance curves for syndicates of various sizes according to the performance measure (4) along with a performance limit developed in the following section. For  $J = 1, 2$ , performance is optimal as determined in the previous section, while the performance curves for larger syndicates are generated through simulation based on a radial configuration.

### C. Performance Limits in Cells with Power Control

We are led to ask: what is the best performance an eavesdropping syndicate can hope to achieve? Such performance is realized by letting the size of the syndicate grow unbounded, covering the cell with eavesdroppers. In this fashion, there is an eavesdropper positioned for reception with maximum possible power of a desired user with any

realized location. This scheme is equivalent to the single eavesdropper capable of situating itself optimally following activation of the desired user.

The expected performance (4) of an arbitrarily large syndicate is given by

$$E[10 \log_{10} P] = -20\alpha \log_{10}(r) - \frac{10\alpha(1-r^2)}{\log(10)}. \quad (5)$$

The performance-limiting expression (5) admits an intuitive interpretation, where the first term corresponds to maximum possible reception of a desired user situated at the edge of the cell, while the second term represents the expected loss in received power induced by a uniform distribution attributed to the desired user, where increased proximity to the base station results in a decrease in transmitted power. Figure 7 depicts expected performance for the syndicates considered thus far along with the performance limit. Note that there is a considerable gap between the limiting curve and the performance achieved even by relatively large syndicates. Most of this discrepancy is explained by the exponential gain in power enjoyed by an eavesdropper as it nears the desired user to within the far-field radius. Some of the discrepancy between the larger syndicates and the limiting case can be attributed to the sub-optimality of the radial configuration.

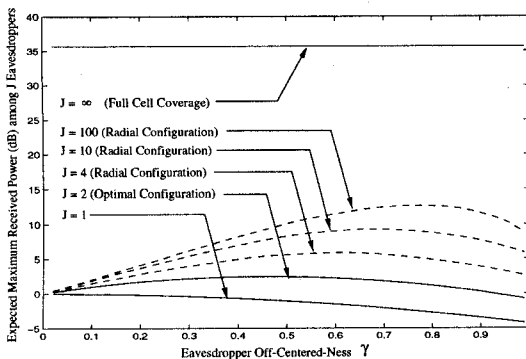


Figure 7: Expected Received Power for Syndicates of Various Sizes

If we place an off-centered-ness restriction  $\gamma_{\min}$  on an eavesdropping syndicate, amounting to a lower limit on the proximity of each eavesdropper to the base station, performance can be expected to decrease. Such a restriction represents either physical limitations, such as a base station with inaccessible surroundings, or strategic limitations, such as the required ability of an eavesdropper to move quickly between cells in order to increase coverage efficiency. Expected performance for the arbitrarily large syndicate with restricted off-centered-ness is plotted in Figure 8 along with restricted performance curves for those syndicates considered thus far. Note the tendency of the performance curves of the larger syndicates to approach the limiting curve for increasing off-centered-ness restriction, a result of the increasing efficiency of the radial configuration in this limiting case.

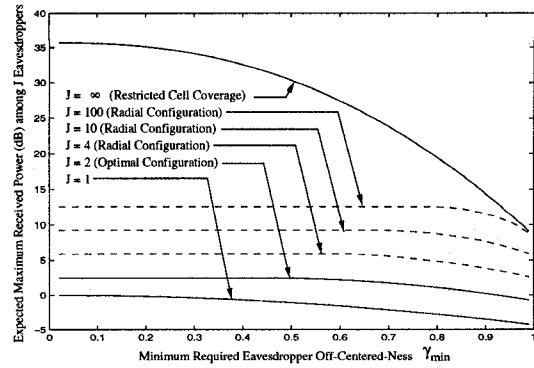


Figure 8: Expected Received Power for Syndicates of Various Sizes Satisfying an Off-Centered-ness Restriction  $\gamma_{\min}$

### III. CELLS WITHOUT POWER CONTROL

We turn attention to characterizing eavesdropping performance in communication cells wherein the base station does *not* employ power control. Such an analysis pertains to systems employing orthogonal signalling (*e.g.* FDMA and TDMA systems) along with those employing correlated waveforms (*e.g.* CDMA systems) in conjunction with multiuser detection [3].

When power control is not employed, each user transmits at a constant power level  $P_T$  independent of its location relative to the base station. An interesting observation concerning the performance measure (4) arises when we rewrite

$$E[10 \log_{10} P] = E[10 \log_{10} P_T] + E[10 \log_{10}(P/P_T)] \quad (6)$$

where  $P_T$  is the random variable representing power transmitted by a uniformly located user, and  $P/P_T$  is the random variable representing signal attenuation between user and eavesdropper, dependent only on their mutual separation under the log-distance path loss model (1). Setting the constant transmitted power  $P_T$  in a cell without power control to the value which equates the first term in (6) with that of a cell with power control (given by  $P_T = \exp\{-\alpha(1-r^2)\}$ ), we note that the second term depends only on the relative locations of the eavesdropper and desired user; since the employment of power control only affects the transmitted powers of the active users, we conclude that *expected performance is equivalent in cells with and without power control* when average transmitted power is held constant and the measure of performance is received power. Of course, this does *not* imply that overall performance is equivalent in both systems; notably, the second order statistics are considerably different: for instance, the received power at a single centrally-located eavesdropper is constant in a cell with power control, which is clearly not the case when power control is absent (Figure 9). In anticipation of a more detailed analysis of eavesdropper performance, including sensitivity to the realization of user locations, we present the cdfs of received power for an eavesdropping singleton and duo in the next section.

### A. Eavesdropping in Cells Without Power Control

The induced cdf  $G_p^1$  of received power at a single eavesdropper with off-centered-ness  $\gamma$  from a uniformly situated user is given by

$$G_p^1(p/P_T) = \begin{cases} 0, & 0 \leq p \leq \frac{1}{(1+\gamma)^{2\alpha}} \\ \frac{1}{2} + \psi_1 - \varphi_1(1-\varphi_2) + \varphi_3, & \frac{1}{(1+\gamma)^{2\alpha}} < p < \frac{1}{(1-\gamma)^{2\alpha}} \\ \frac{p^{1/\alpha}-1}{p^{1/\alpha}}, & \frac{1}{(1-\gamma)^{2\alpha}} \leq p \leq \frac{1}{\gamma^{2\alpha}} \\ 1, & \text{otherwise,} \end{cases}$$

$$\varphi_1 = \frac{1}{2p^{1/\alpha}}$$

$$\varphi_2 = \frac{2}{\pi} \sin^{-1} \left( \frac{p^{1/\alpha}(\gamma^2 - 1) + 1}{2p^{1/2\alpha}\gamma} \right)$$

$$\varphi_3 = \frac{\sqrt{2p^{1/\alpha}(1+\gamma^2) - p^{2/\alpha}(1-\gamma^2)^2} - 1}{2\pi p^{1/\alpha}}$$

For an eavesdropping duo with optimal configuration, requiring that both eavesdroppers be situated on a cell diagonal with equal off-centered-ness  $\gamma$ , the cdf of maximum received power among the duo is given by

$$G_p^2(p/P_T) = \begin{cases} 0, & 0 \leq p \leq \frac{1}{(1+\gamma^2)^{\alpha}} \\ 2\psi_1 + 2\varphi_1(\varphi_2 - \varphi_4) + 2\varphi_3 - \varphi_5, & \frac{1}{(1+\gamma^2)^{\alpha}} < p \leq \min\left\{\frac{1}{\gamma^{2\alpha}}, \frac{1}{(1-\gamma)^{2\alpha}}\right\} \\ 2\psi_1 - 2\varphi_1(1-\varphi_2) + 2\varphi_3, & \min\left\{\frac{1}{\gamma^{2\alpha}}, \frac{1}{(1-\gamma)^{2\alpha}}\right\} < p \leq \frac{1}{(1-\gamma)^{2\alpha}} \\ \frac{p^{1/\alpha}-2}{p^{1/\alpha}} + 2\varphi_1(1-\varphi_4) - \varphi_5, & \min\left\{\frac{1}{\gamma^{2\alpha}}, \frac{1}{(1-\gamma)^{2\alpha}}\right\} < p \leq \frac{1}{\gamma^{2\alpha}} \\ \frac{p^{1/\alpha}-2}{p^{1/\alpha}}, & \max\left\{\frac{1}{\gamma^{2\alpha}}, \frac{1}{(1-\gamma)^{2\alpha}}\right\} < p \leq \frac{1}{\gamma^{2\alpha}} \\ 1, & \text{otherwise,} \end{cases}$$

$$\varphi_4 = \frac{2}{\pi} \sin^{-1} \left( p^{1/2\alpha}\gamma \right), \quad \varphi_5 = \frac{2\gamma}{\pi} \sqrt{\frac{1-p^{1/\alpha}\gamma^2}{p^{1/\alpha}}}$$

Figures 9 and 10 depict  $G_p^1$  and  $G_p^2$  respectively for various off-centered-ness values.

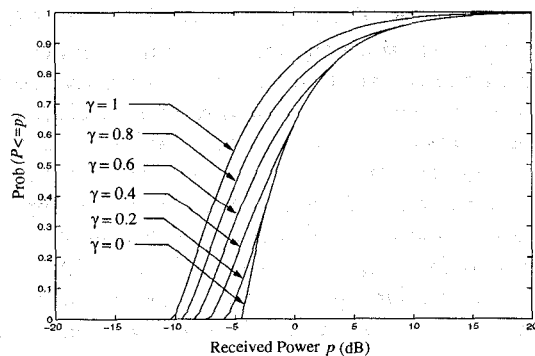


Figure 9: Cumulative Distribution Function of Received Power at a Single Eavesdropper from a Uniformly Located User in a Cell without Power Control

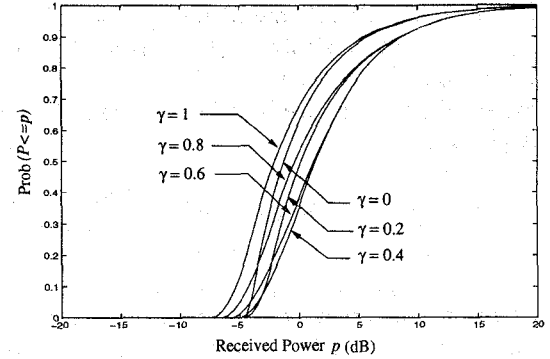


Figure 10: Cumulative Distribution Function of Maximum Received Power Among Two Eavesdroppers from a Uniformly Located User in a Cell without Power Control

### IV. CONCLUSIONS

We have quantified expected performance for eavesdropping syndicates when the measure of performance is received power from a uniformly randomly situated active user of interest. Such a measure is the indicator of interest in a communication cell employing orthogonal waveforms, and provides a useful evaluation for cells employing correlated waveforms, especially those systems maintaining low inter-user cross-correlations. Significant improvement is enjoyed by syndicates consisting of multiple eavesdroppers over the performance of a single eavesdropper. An optimally configured duo exhibits a gain of approximately 2.5dB over an optimally (centrally) situated singleton, and hence over the base station. However, even the performance of a duo falls well below that of a syndicate with full cell coverage, or equivalently that of the single eavesdropper afforded the capability to optimally situate itself subsequent to activation of the desired user. Syndicates with larger numbers of users were shown to approach performance limits, but improved results hinge on the development of improved configurations.

While overall performance will differ in cells with and without power control, expected received power was shown to be equivalent in both systems. Cumulative distribution functions were developed to facilitate further analysis of performance sensitivity and outage susceptibility.

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