Efficient Signaling for Low-Power Rician Fading Channels *

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Abstract

Transmission of information over a discrete-time memoryless Rician fading channel is considered where neither the receiver nor the transmitter knows the fading coefficients and the input amplitude is subject to both second and fourth moment constraints. The main focus is on the power-limited (i.e., wideband, low-power) regime. First, the structure of capacity achieving input signals is investigated. It is shown that uniform phase is optimal if there is a line of sight component. A sufficient and necessary condition for an input amplitude distribution to be optimal is provided. Using this condition, it is shown that for any fixed kurtosis constraint, the capacity-achieving input amplitude distribution is discrete in the low-power regime. The spectral-efficiency/bit-energy tradeoff in the power-limited regime is examined and it is shown that the minimum bit energy is not always achieved at zero spectral efficiency. In this case, it is argued that one should avoid operating in the region where the spectral efficiency is lower than the spectral efficiency of the minimum bit energy point. Finally, we propose a signaling scheme, OOQPSK, which is shown to be optimally efficient in the low-power limit.

1 Introduction

Recently, the information theoretic analysis of fading channels has received much attention [9]. This interest is motivated by the rapid advances in wireless technology and the need to use the scarce resources such as bandwidth and power as efficiently as possible under severe fading conditions. Providing the ultimate performance, information theoretic measures such as capacity, spectral efficiency and error exponents can be used as benchmarks to which we can compare the performance of practical communication systems.

A significant amount of effort has been expended to study fading channel models where side information about the fading is available at either the receiver or the transmitter or both (see [9] and references therein). However, under fast fading conditions noncoherent communications, where neither party knows the fading, becomes often the

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only available alternative. Richters [3] considered the problem of communicating over average power limited discrete-time memoryless Rayleigh fading channels without any channel side information. He conjectured that the capacity-achieving amplitude distribution is discrete with a finite number of mass points. Recently, Abou-Faycal *et al.* [5] gave a rigorous proof of Richters' conjecture. This result shows that when the fading is known by neither the transmitter nor the receiver, the optimal amplitude distribution has a notably different character than that of unfaded Gaussian channels.

Another characterization for the capacity achieving input signals can be given in the low-power regime. Verdú [1] has recently shown that when the fading is not known, increasingly peaky input signals whose kurtosis grows without bound are required as $\text{SNR} \to 0$. However, these peaky signals are not feasible in communication systems with strict peak-to-average ratio requirements. Furthermore, in some systems CDMA-type white signals which spread their energy over the available bandwidth are used because of their anti-jamming and low probability of intercept capabilities. Hence, it is of interest to investigate the effect upon the capacity of imposing a finite kurtosis constraint, especially in the low-power regime.

Médard and Gallager [6] considered unknown broadband fading channels and limited the peakedness of the input signals by imposing a fourth moment input constraint. Then they show that such a constraint forces the mutual information to zero inversely with increasing bandwidth. Similar results are obtained by Telatar and Tse [8] and Subramanian and Hajek [7]. These results show the deep impact on the spectral efficiency in the power-limited regime of using signals with limited peakiness.

In this paper, we study the capacity and the spectral efficiency of the complex-scalar discrete-time memoryless Rician fading channel subject to second and fourth moment input constraints and investigate efficient signaling schemes in the low-power regime.

2 Channel Model

We consider the following complex-scalar discrete-time memoryless Rician fading channel model

$$y_i = mx_i + a_i x_i + n_i \tag{1}$$

where a_i and n_i are i.i.d. circular zero mean complex Gaussian random variables independent of each other, and of the input, with variances $E\{|a_i|^2\} = \gamma^2$ and $E\{|n_i|^2\} = N_0$, m is a deterministic complex constant, x_i is the complex channel input and y_i is the complex channel output. $\{a_i\}$ and $\{n_i\}$ represent sequences of fading coefficients and background noise samples respectively.

The Rician fading channel model is particularly appropriate when there is a direct propagating line of sight (LOS) component in addition to the faded component arising from multipath propagation. Moreover, the Rician model includes both the unfaded Gaussian channel and the Rayleigh fading channel as two special cases. Hence, results obtained for this model provide a unifying perspective.

In the channel model (1), fading is assumed to be flat and hence has a multiplicative effect on the channel input. This is a valid assumption if the delay spread of the channel is much smaller than the symbol duration. Moreover, frequency selective fading channels can be decomposed into parallel non-interacting flat fading subchannels using orthogonal multicarrier techniques.

Note also that the fading coefficients assume independent realizations at every symbol period. Under such fast fading conditions, reliable estimation of the fading coefficients may be quite difficult because of the short duration between independent fades. Therefore, we consider the noncoherent scenario where neither the receiver nor the transmitter knows the fading coefficients $\{a_i\}$.

We impose second and fourth moment constraints on the input amplitude:

$$E\{|x_i|^2\} \leq P_{av} \qquad \forall i \tag{2}$$

$$E\{|x_i|^4\} \leq \kappa P_{av}^2 \qquad \forall i \tag{3}$$

where P_{av} is the average power constraint and $1 < \kappa < \infty$. Since the average power constraint (2) is always active for capacity achieving sources, the fourth moment constraint is identical to $\frac{E\{|x_i|^4\}}{(E\{|x_i|^2\})^2} \leq \kappa$. Hence imposing a fourth moment constraint (3) is equivalent to putting a limitation on the kurtosis, which is a measure of the peakedness of the input signal.

3 Channel Capacity and Optimal Input Distribution

In this section, we will elaborate on the structure of the capacity achieving input distribution. For the Rician channel model (1), the capacity is the supremum of the mutual information between the channel input and the output over the set of all input distributions satisfying the constraints (2) and (3) [1]

$$C = \sup_{\substack{F_x(\cdot) \\ E\{|x|^2\} \leq P_{av} \\ E\{|x|^4\} \leq \kappa P_{av}^2}} \int_{\mathbb{C}} \int_{\mathbb{C}} f_{y|x}(y|x) \ln \frac{f_{y|x}(y|x)}{f_y(y)} \, \mathrm{d}y \, \mathrm{d}F_x(x) \tag{4}$$

where the conditional density of the output given the input,

$$f_{y|x}(y|x) = \frac{1}{\pi(\gamma^2|x|^2 + N_0)} \exp\left(-\frac{|y - mx|^2}{\gamma^2|x|^2 + N_0}\right) , \tag{5}$$

is circular complex Gaussian. Moreover $f_y(y) = \int_{\mathbb{C}} f_{y|x}(y|x) \, \mathrm{d}F(x)$ is the marginal output density, F_x is the distribution function of the input and $\mathbb{C}$ denotes the complex domain.

First we can prove the following result on the capacity achieving phase distribution.

Proposition 3.1 *In the Rician fading channel model (1) with input constraints (2) and (3), if there is a line of sight component, i.e.,* $|m| > 0$, *then uniformly distributed phase that is independent of the amplitude is optimal* [2].

Note that if $|m| = 0$, (1) reduces to the Rayleigh fading channel model where phase cannot be used to convey information when the channel is unknown. However, the above proposition shows that if there is a line of sight component, phase can indeed carry information and the uniform distribution maximizes the transmission rate. After

[1] Since the channel is memoryless, without loss of generality we can drop the index i.

[2] The result holds in wider generality: the feasible set defined by (2) and (3) can be replaced by any set of constraints that are imposed only on the input magnitude.

this characterization, capacity for the channel model in (1) can be formulated as the supremum of a functional that depends only on the distribution of the input amplitude

$$C(\alpha, \kappa, \mathsf{K}) = \sup_{\substack{F_r(\cdot) \\ E\{r^2\} \leq \alpha \\ E\{r^4\} \leq \kappa\alpha^2}} I(F_r) \tag{6}$$

where

$$r = \frac{\gamma}{\sqrt{N_0}}|x| \ , \ R = \frac{|y|^2}{N_0} \ , \ \alpha = \gamma^2 \frac{P_{av}}{N_0} = \gamma^2 \text{SNR} \ , \ \mathsf{K} = \frac{|m|^2}{\gamma^2} \ , \tag{7}$$

$$I(F_r) = -\int_0^\infty f_R(R; F_r) \ln f_R(R; F_r) \, \mathrm{d}R - \int_0^\infty \ln(1 + r^2) \, \mathrm{d}F_r(r) - 1 \ , \tag{8}$$

$$f_R(R; F_r) = \int_0^\infty g(R, r) \, \mathrm{d}F_r(r) \ , \tag{9}$$

$$g(R, r) = \frac{1}{1 + r^2} \exp\left(-\frac{R + \mathsf{K}r^2}{1 + r^2}\right) I_0\left(\frac{2\sqrt{\mathsf{K}}r\sqrt{R}}{1 + r^2}\right) . \tag{10}$$

In the above formulation, r is the normalized input amplitude, F_r is the distribution function of r, R is the normalized output power and f_R is the probability density function of R. The Rician factor, K, gives the relative strength of the specular component to the faded component and α is the normalized SNR. Also note that $g(R, r)$ defined in (10) is a non-central chi-square probability density function in R with parameters $(\frac{1+r^2}{2}, \mathsf{K}r^2)$ and it can regarded as the conditional density of R given r. Next we provide a sufficient and necessary condition for an amplitude distribution to be optimal. The techniques employed to obtain this condition closely follow the approach used in [4] and [5].

Proposition 3.2 *(Kuhn-Tucker Condition) For the Rician channel model (1) and input constraints (2), (3), F_0 is a capacity achieving distribution if and only if there exist $\lambda_1 \in \mathbb{R}$ and $\lambda_2 \geq 0$ such that the following is satisfied*

$$\ln(1 + r^2) + \lambda_1(r^2 - \alpha) + \lambda_2(r^4 - \kappa\alpha^2) + 1 + C(\alpha, \kappa, \mathsf{K}) + \int_0^\infty g(R, r) \ln f_R(R, F_0) \, \mathrm{d}R \geq 0 \ , \ \forall r \geq 0 \ , \tag{11}$$

with equality if $r \in E_0$ where E_0 is the set of points of increase of F_0.

Using Proposition 3.2, we have the following result on the optimal amplitude distribution.

Proposition 3.3 *For the Rician fading channel (1) with input amplitude constraints (2), (3), if the fourth moment constraint (3) is active then the optimal input amplitude distribution is discrete with a finite number of mass points*[3].

Proposition 3.3 easily specializes to the Rayleigh and the unfaded Gaussian channels. For the unfaded Gaussian channel, if the fourth moment constraint is inactive, it is well known that a Rayleigh distributed amplitude is optimal and it has kurtosis $\kappa = 2$.

[3]The result holds in a more general setting where the fourth moment constraint (3) is replaced by a constraint in the following form: $E|x|^{2+\delta} \leq M$ for some $\delta > 0$ and $M < \infty$.

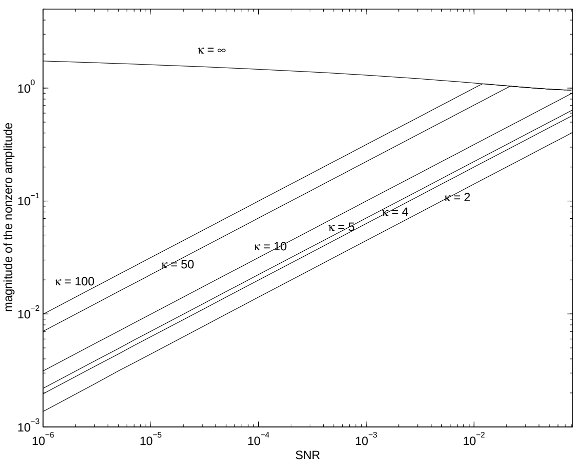

Figure 1: Magnitude of the nonzero amplitude vs. SNR in the Rician channel $\mathsf{K} = 1$.

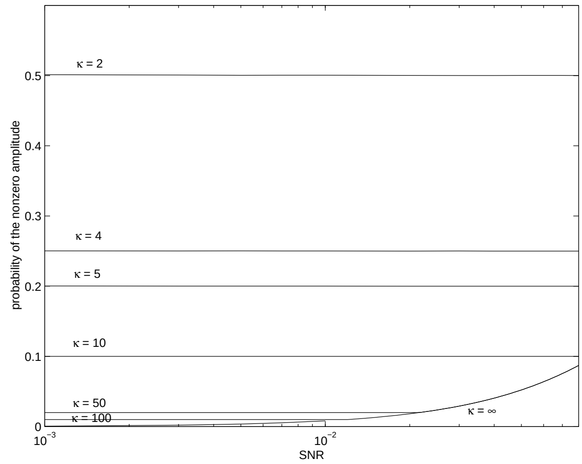

Figure 2: Probability of the nonzero amplitude vs. SNR in the Rician channel $\mathsf{K} = 1$.

Therefore the fourth moment constraint being active (i.e., $1 < \kappa < 2$) is also a necessary condition for the discrete nature in that case. In [5], the optimum amplitude for the average power limited unknown Rayleigh fading channel is shown to be discrete with a finite number of levels. The above proposition proves that this discrete character does not change when we have an additional kurtosis constraint. Moreover, for the unknown Rayleigh and Rician fading channels, any finite kurtosis constraint will eventually be active for sufficiently small SNR because if there is no such constraint, the required kurtosis grows without bound as $\text{SNR} \to 0$. Therefore, Proposition 3.3 establishes the discrete nature in the low-power regime, which is the focus of this paper.

In general, the number of mass points of the optimum discrete distribution and their locations and probabilities depend on the SNR, and obtaining analytical expressions for the capacity and the optimal distribution as a function of SNR seems unlikely. Therefore, we resort to numerical methods to examine this behavior. The numerical algorithm used here is similar to the ones employed in [4] and [5]. In particular, we start with a sufficiently small SNR and maximize the mutual information over the set of two-mass-point discrete distributions satisfying the input constraints. Then we test the maximizing two-mass-point discrete distribution with the Kuhn-Tucker condition. If this distribution satisfies the necessary and sufficient Kuhn-Tucker condition then it is optimal and the mutual information achieved by it is the capacity. As we increase the SNR, the required number of mass points monotonically increases and therefore to obtain the optimum distribution, we repeat the same procedure for discrete distributions with more and more mass-points.

Numerical results indicate that for sufficiently small SNR values, a particular two-mass-point discrete distribution with the following location and probabilities

$$\begin{array}{ll} |x_1| = 0 & p_1 = 1 - \frac{1}{\kappa} \\ |x_2| = \sqrt{\kappa N_0 \text{SNR}} & p_2 = \frac{1}{\kappa} \end{array} \tag{12}$$

is optimal. Note that this distribution does not depend on the Rician factor K. Figures 1 and 2 plot the magnitude and the probability of the nonzero amplitude respectively as a function of SNR ($N_0 = 1$) for various values of κ. We immediately notice the significant impact of imposing a finite kurtosis constraint. When there is no kurtosis constraint, the nonzero amplitude migrates away from the origin as $\text{SNR} \to 0$ while its probability decreases sufficiently fast to satisfy the average power constraint. Indeed for a general class of average power limited fading channels, Verdú [1] has recently shown that if there are no constraints other than average power, flash signaling, a class of unbounded peak-to-average ratio inputs, is necessary to achieve the capacity as $\text{SNR} \to 0$ when the channel is unknown. However, as we see from the figures, if there is any finite kurtosis constraint, then the behavior is quite different. The nonzero amplitude approaches the origin as $\text{SNR} \to 0$ while its probability is kept constant.

4 Spectral Efficiency vs. Bit Energy in the Power-Limited Regime

In the power-limited regime, insightful results are obtained by examining the spectral-efficiency/bit-energy tradeoff which reflects the fundamental tradeoff between bandwidth and power. We will denote the spectral efficiency (as a function of bit energy) by $\mathsf{C}\left(\frac{E_b}{N_0}\right)$. If we assume without loss of generality that one complex symbol occupies $1\text{s} \times 1\text{Hz}$ time-frequency slot, then the maximum achievable spectral efficiency can be obtained from the Shannon capacity (bits/symbol)

$$\mathsf{C}\left(\frac{E_b}{N_0}\right) = C(\text{SNR}) \quad \text{bits/s/Hz} \tag{13}$$

where

$$\frac{E_b}{N_0} = \frac{\text{SNR}}{C(\text{SNR})} \tag{14}$$

is the bit energy normalized to the noise power.

For average power limited channels, the bit energy required for reliable communications decreases monotonically with decreasing spectral efficiency, and the minimum bit energy is achieved at zero spectral efficiency. Hence for fixed rate transmission, reduction in the required power comes only at the expense of increased bandwidth. Recently, Verdú [1] has analyzed the spectral-efficiency/bit-energy function in the power limited regime for a general class of average power limited fading channels and has shown that the minimum bit energy is $\log_e 2 = -1.59$ dB as long as the additive background noise is Gaussian. This minimum bit energy is achieved only in the asymptotic regime of infinite bandwidth. If one is willing to spend more power, then reliable communication over a finite bandwidth is possible. Achieving minimum bit energy is not a sufficient criterion for finite bandwidth analysis. Verdú [1] has defined the wideband slope as the slope of spectral efficiency curve $\mathsf{C}\left(\frac{E_b}{N_0}\right)$ in bits/s/Hz/3dB at zero spectral efficiency as

$$S_0 \stackrel{\text{def}}{=} \lim_{\frac{E_b}{N_0} \downarrow \frac{E_b}{N_0}\big|_{\mathsf{C}=0}} \frac{\mathsf{C}\left(\frac{E_b}{N_0}\right)}{10\log_{10}\frac{E_b}{N_0} - 10\log_{10}\frac{E_b}{N_0}\Big|_{\mathsf{C}=0}} 10\log_{10} 2 = \frac{2\left(\dot{C}(0)\right)^2}{-\ddot{C}(0)}, \tag{15}$$

where $\dot{C}(0)$ and $\ddot{C}(0)$ denote the first and second derivatives of capacity in nats. The wideband slope closely approximates the growth of the spectral efficiency curve in the power-limited regime and hence is proposed as a new tool providing insightful results when bandwidth is a resource to be conserved.

Under quite general conditions on the input and the channel, Prelov and Verdú [2, Theorem 2] obtained the exact asymptotic second-order behavior of the input-output mutual information for vanishing SNR. Using this asymptotic expression, we can prove the following proposition.

Proposition 4.1 *For the Rician channel model (1) with input constraints (2) and (3), the first and second derivatives of capacity at* $\text{SNR} = 0$ *are*

$$\dot{C}(0) = |m|^2 \quad \textit{and} \quad \ddot{C}(0) = \kappa\gamma^4 - (|m|^2 + \gamma^2)^2 \,. \tag{16}$$

From the above proposition, one can easily get closed form expressions for the bit energy at zero spectral efficiency and the wideband slope. Hence we have the following result.

Corollary 4.1 *For the Rician fading channel (1) subject to input constraints (2) and (3), the normalized received bit energy* $\frac{E_b^r}{N_0}$ *required at zero spectral efficiency and wideband slope are*

$$\left.\frac{E_b^r}{N_0}\right|_{\mathsf{C}=0} = \left(1 + \frac{\gamma^2}{|m|^2}\right) \log_e 2 \tag{17}$$

and

$$S_0 = \frac{2\frac{|m|^4}{\gamma^4}}{\left(1 + \frac{|m|^2}{\gamma^2}\right)^2 - \kappa} \,. \tag{18}$$

From (17), we immediately see that for the Rayleigh fading channel where $|m| = 0$, the bit energy required at zero spectral efficiency is infinite. Therefore reliable communications is not possible at this point. This is in sharp contrast with the behavior observed in average power limited channels where the bit energy required at zero spectral efficiency is indeed the minimum one. For Rician fading channels where $|m| > 0$, (17) is finite and interestingly does not depend on the kurtosis constraint κ. Moreover (17) monotonically decreases to -1.59 dB with increasing Rician factor $\mathsf{K} = \frac{|m|^2}{\gamma^2}$. This is intuitively appealing because the channel is becoming more Gaussian where the bit energy at zero spectral efficiency is -1.59 dB.

For average power limited channels, the wideband slope is always nonnegative. In the unknown Rician fading channel subject to second and fourth moment input limitations, we again observe a markedly different behavior. From (18), we see that if $\kappa > \left(1 + \frac{|m|^2}{\gamma^2}\right)^2$, then the wideband slope is negative, leading to the conclusion that the minimum bit energy is achieved at a nonzero spectral efficiency. If $\kappa \leq \left(1 + \frac{|m|^2}{\gamma^2}\right)^2$, the wideband slope is positive and, based on numerical evidence, we conjecture that the minimum bit energy is achieved at zero spectral efficiency and is equal to (17).

Figures 3 and 4 plot the $\frac{E_b^r}{N_0}$ (dB) vs. $\mathsf{C}(\frac{E_b^r}{N_0})$ bits/s/Hz curves for the Rayleigh and Rician ($K = 1$) channels, respectively, for various values of κ. In the Rayleigh fading channel, for any finite kurtosis constraint κ, the bit energy curve is bowl-shaped, achieving its minimum at a nonzero spectral efficiency C^*. One should avoid operating in the region where the spectral efficiency is lower than C^* because decreasing the spectral efficiency further (i.e., increasing the bandwidth for fixed rate transmission) only increases the required power. In the Rician fading channel ($\mathsf{K} = 1$), we observe the same behavior when $\kappa > \left(1 + \frac{|m|^2}{\gamma^2}\right)^2 = 4$. The minimum bit energy is achieved at a nonzero spectral

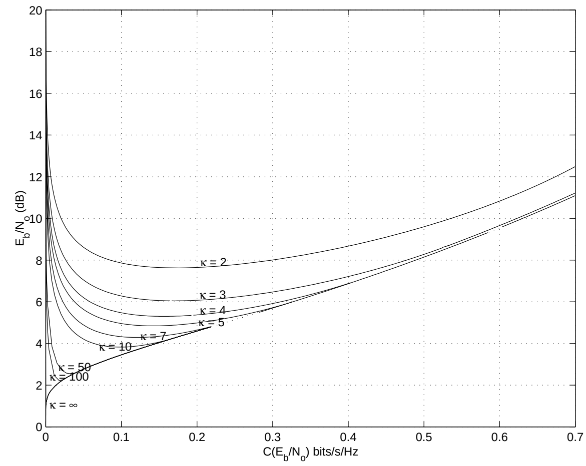

Figure 3: $\frac{E_b^r}{N_0}$ (dB) vs. $\mathsf{C}(\frac{E_b^r}{N_0})$ bits/s/Hz for the Rayleigh Channel

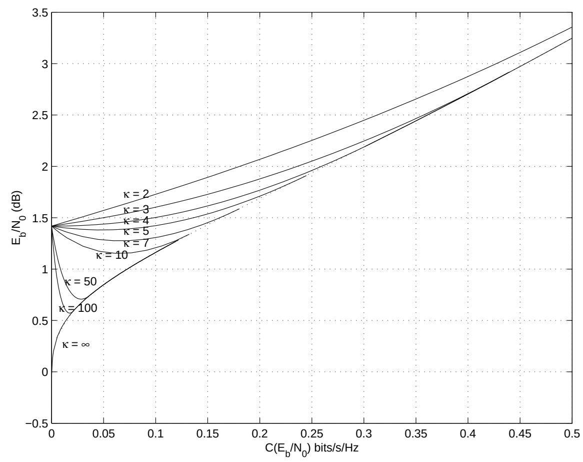

Figure 4: $\frac{E_b^r}{N_0}$ (dB) vs. $\mathsf{C}(\frac{E_b^r}{N_0})$ bits/s/Hz for the Rician Channel with $\mathsf{K} = |m|^2/\gamma^2 = 1$

efficiency. However note that now the bit energy required at zero spectral efficiency is finite and is the same for all finite κ. If $\kappa \leq 4$, the bit energy decreases monotonically with decreasing spectral efficiency and the minimum bit energy is achieved at zero spectral efficiency. Therefore in this case, the bandwidth-power tradeoff is the usual one that we encounter in average power limited channels; i.e., for fixed rate transmission, increasing the bandwidth decreases the power required for reliable communications.

Finally, we note that if the fourth moment constraint (3) is replaced by a peak power constraint, $|x|^2 \leq \kappa P_{av}$ a.s., in Proposition 4.1, we obtain the same first and second derivative expressions (16) and hence the same bit energy at zero spectral efficiency (17) and wideband slope (18). Therefore all the above remarks and the efficient signaling techniques mentioned in the next section apply also to the average and peak power limited Rician fading channels.

5 Efficient Signaling in the Power-Limited Regime

Having analyzed the optimum input structure and the spectral-efficiency/bit-energy tradeoff in the power-limited regime, we will now investigate efficient signaling schemes. Verdú [1] defines an input distribution to be *first-order optimal* if it satisfies the input constraints and achieves the first derivative of the capacity at zero SNR, and *second-order optimal* if in addition it achieves the second derivative of capacity at zero SNR. So, a first order optimal input achieves the bit energy at zero spectral efficiency and a second order optimal input achieves both the bit energy at zero spectral efficiency and the wideband slope. We define the following signaling schemes which are PSK overlaid on OOK (on-off keying).

Definition 5.1 *An OOBPSK signal, parametrized by* $0 < p \leq 1$, *has the following constellation points with the corresponding probabilities*

$$\begin{array}{lll} x_1 = 0 & \textit{with prob.} & 1-p \\ x_2 = +\sqrt{P_{av}/p} & '' & p/2 \\ x_3 = -\sqrt{P_{av}/p} & '' & p/2 \end{array} \tag{19}$$

where P_{av} *is the average power of the signal.*

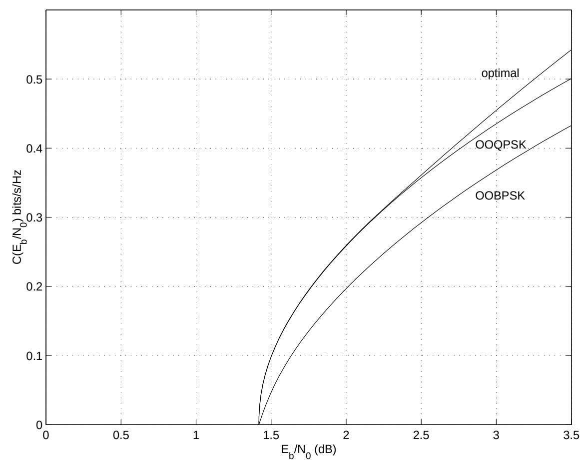

Figure 5: $\mathsf{C}(\frac{E_b^r}{N_0})$ bits/s/Hz vs. $\frac{E_b^r}{N_0}$ (dB) curves for optimal, OOQPSK and OOBPSK signaling in the Rician Channel with $\mathsf{K} = 1$ and $\kappa = 4$.

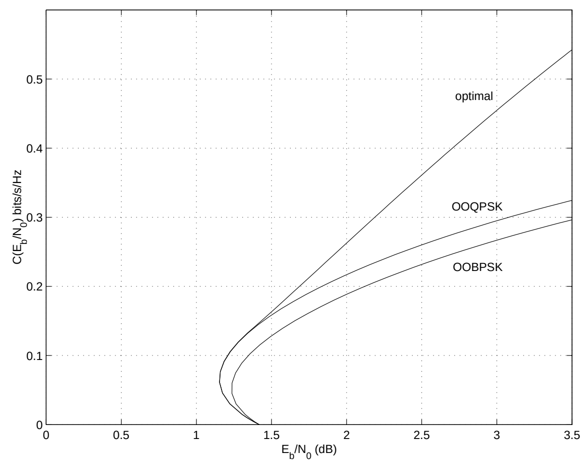

Figure 6: $\mathsf{C}(\frac{E_b^r}{N_0})$ bits/s/Hz vs. $\frac{E_b^r}{N_0}$ (dB) curves for optimal, OOQPSK and OOBPSK signaling in the Rician Channel with $\mathsf{K} = 1$ and $\kappa = 10$.

Definition 5.2 *An OOQPSK signal, parametrized by* $0 < p \leq 1$*, has the following constellation points with the corresponding probabilities*

$$\begin{array}{llll} x_1 = 0 & \textit{with prob.} & 1-p & \\ x_i = \sqrt{\frac{P_{av}}{2p}}(\pm 1 \pm j) & \quad '' & p/4 & i = 2,3,4,5 \end{array} \tag{20}$$

where P_{av} *is the average power of the signal.*

Note that in the above definitions, $1/p$ is the kurtosis of the signals. We have the following result on the optimality of these signaling schemes in the power-limited regime.

Proposition 5.1 *For the Rician fading channel model (1) where the input amplitude has average power constraint* P_{av} *and kurtosis constraint* κ*, an OOBPSK input with average power* P_{av} *and* $\frac{1}{\kappa} \leq p \leq 1$ *is first-order optimal and an OOQPSK input with average power* P_{av} *and* $p = \frac{1}{\kappa}$ *is second-order optimal.*

Note that ordinary BPSK (OOBPSK with $p = 1$) signaling is enough for first order optimality. Hence in the asymptotic regime of zero spectral efficiency, only the phase can be used to carry information which explains the fact that in this regime reliable communications over the unknown Rayleigh fading channel is not possible.

Figure 5 plots the spectral efficiency-bit energy curve for optimal, OOQPSK and OOBPSK signaling in the Rician fading channel (Rician factor $\mathsf{K} = 1$) where the kurtosis constraint is $\kappa = 4$. Note that for this value of κ, the wideband slope is positive. Both OOBPSK and OOQPSK achieve the minimum bit energy (first order optimality). Being second-order optimal, OOQPSK also achieves the wideband slope and is very close to the optimal curve in the power-limited regime. Therefore, we conclude that OOQPSK signaling is a very efficient transmission scheme in the low-power regime. Note that OOBPSK achieves a smaller slope and hence for fixed rate and power it requires more bandwidth. In Fig. 6 the kurtosis constraint is increased to $\kappa = 10$. In this case, the wideband slope is negative and the minimum bit energy is achieved at a nonzero spectral efficiency, which implies that the very low-power regime ought to be avoided. However, we observe that On/Off-QPSK is still an efficient scheme achieving very close to the minimum bit energy.

6 Conclusions

We have considered transmission over a discrete-time memoryless Rician fading channel and have analyzed the effects upon the capacity and the spectral efficiency in the power limited regime of using input signals having limited peakedness, which is achieved by imposing a fourth moment input constraint.

First, we focused on the structure of the optimal input distribution. We showed the optimality of uniform phase if there is a line of sight component. Then, using a sufficient and necessary condition, we proved that the optimal input amplitude is discrete with a finite number of levels in the low-power regime.

We then analyzed the spectral-efficiency/bit-energy tradeoff in the low-power regime. In contrast to the behavior observed in average power limited channels, the minimum bit energy is not always achieved at zero spectral efficiency. In this case, we identified a forbidden region where one should not operate. We also established that replacing the fourth moment constraint with a peak constraint does not affect the second order asymptotics of the capacity and hence the achievable low-power performance. We note that the analytical determination of $\frac{E_b}{N_0}_{\min}$ under a kurtosis or peak constraint is an open problem.

Finally we defined OOBPSK and OOQPSK signaling schemes and showed that OOQPSK is a very efficient signaling scheme for low-power Rician channels.

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