

# Eigenvalue Statistics of Finite-Dimensional Random Matrices for MIMO Wireless Communications

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**Abstract**—This paper characterizes the marginal probability density function of an unordered eigenvalue of finite-dimensional random matrices of particular interest in MIMO (multiple-input multiple-output) wireless communications. Specifically, a technique is presented for deriving the eigenvalue statistics in one-side correlated Rayleigh-faded channels and in Ricean-faded channels, with or without cochannel interferers. The exact expressions found turn out to be extremely useful in calculating information-theoretic quantities. As an application, we calculate the ergodic mutual information for all the abovementioned channel fading conditions, obtaining a closed form formula for the Rayleigh case and, in turn, a series expression for the Ricean faded one.

## I. INTRODUCTION

Much of the early analytical characterizations of MIMO channels were asymptotic, either in the number of antennas or in the SNR (signal-to-noise ratio). More recently, strides have been made in finding exact nonasymptotic expressions. The ergodic mutual information, specifically, has been expressed for:

- Channels with IID (independent identically distributed) Rayleigh-faded entries [1].
- Channels with IID Ricean-faded entries [2], [3].
- Channels with various forms of antenna correlation [4], [5], [6], [7] and/or interference [8].

In [1], the ergodic capacity of channels with IID Rayleigh-faded entries was obtained by integrating over the marginal distribution of an unordered eigenvalue of a central Wishart matrix. Posterior works, however, mostly abandoned this approach due to the lack of explicit expressions for such marginal distribution in more general channels. The aim of this paper is, precisely, to present a unified procedure for calculating the marginal density distribution of an unordered eigenvalue for the most popular MIMO channel models. Via this distribution, one can then calculate the ergodic mutual information and/or the channel capacity via a simple, one-folded integral. In addition, this distribution is also relevant to other MIMO problems such as the study of adaptive transmission schemes with spatial diversity at the receiver [9] and/or at the transmitter [10] and the performance evaluation of differential MIMO transmission [11].

## II. SIGNAL AND CHANNEL MODEL

Denoting by  $n_T$  and  $n_R$  the number of transmit and receive antennas, we consider the complex frequency-flat linear model

$$\mathbf{y} = \sqrt{g}\mathbf{H}\mathbf{x} + \mathbf{n}$$

where  $\mathbf{x}$  and  $\mathbf{y}$  are the input and output vectors while  $\mathbf{n}$  is white Gaussian noise. The channel is represented by the  $(n_R \times n_T)$  zero-mean random matrix  $\mathbf{H}$  normalized such that

$$E[\text{Tr}\{\mathbf{H}\mathbf{H}^\dagger\}] = n_R n_T. \quad (1)$$

The noise is composed of both thermal noise and out-of-cell interference such that

$$\mathbf{n} = \sum_{\ell=1}^{\mathcal{L}} \sqrt{g_\ell} \mathbf{H}_\ell \mathbf{x}_\ell + \mathbf{n}_{\text{th}} \quad (2)$$

where  $\mathbf{n}_{\text{th}}$  is the thermal noise,  $\mathcal{L}$  the number of interferers,  $\mathbf{x}_\ell$  the signal transmitted by the  $\ell$ -th interferer, modeled as a Gaussian vector with unit covariance matrix, and  $\sqrt{g_\ell} \mathbf{H}_\ell$  the channel from such interferer. The number of transmit antennas at the  $\ell$ -th interferer is denoted by  $m_\ell$  while the entries of  $\mathbf{H}_\ell$  are modeled as zero-mean jointly Gaussian random variables with distribution

$$f(\mathbf{H}_\ell) = \frac{e^{-\text{Tr}\{\Theta_I^{-1} \mathbf{H}_\ell \mathbf{H}_\ell^\dagger\}}}{\pi^{m_\ell n_R} \det(\Theta_I)^{m_\ell}}$$

and such that

$$E[\text{Tr}\{\mathbf{H}_\ell \mathbf{H}_\ell^\dagger\}] = n_R m_\ell.$$

The single-sided spectral density of the noise is denoted by

$$N_0 = \frac{E[\|\mathbf{n}\|^2]}{n_R} \quad (3)$$

and its normalized conditional covariance is defined as

$$\Phi_{\mathbf{n}}(\{\mathbf{H}_\ell\}) \triangleq \frac{E[\mathbf{nn}^\dagger | \{\mathbf{H}_\ell\}]}{N_0}. \quad (4)$$

Under these assumptions, the received SNR can be computed as [12]

$$\text{SNR} = g \frac{E[\|\mathbf{x}\|^2]}{N_0} \frac{E[\text{Tr}\{\mathbf{H}\mathbf{H}^\dagger \Phi_{\mathbf{n}}^{-1}(\{\mathbf{H}_\ell\})\}]}{n_T n_R} \quad (5)$$

with expectation over  $\mathbf{x}$ ,  $\mathbf{H}$  and the set  $\{\mathbf{H}_\ell\}$ . If  $\Phi_{\mathbf{n}} = \mathbf{I}$ , (5) reduces to

$$\text{SNR} = g \frac{E[\|\mathbf{x}\|^2]}{N_0}. \quad (6)$$

Since the fading encountered by wireless systems tends to be either Rayleigh or Ricean, the entries of  $\mathbf{H}$  can be modeled as jointly Gaussian. With that, the characterization of  $\mathbf{H}$  entails simply determining the mean and correlation between its entries.

### A. MIMO channel with Rayleigh-faded Interferers

We model the out-of-cell interferers as Rayleigh-faded. By conveniently defining [12]

$$\begin{aligned} \mathcal{J}_\ell &\triangleq \mathbf{g}_\ell \frac{E[\|\mathbf{x}_\ell\|^2]}{m_\ell} \frac{E[\text{Tr}\{\mathbf{H}_\ell \mathbf{H}_\ell^\dagger\}]}{n_R} \\ &= \mathbf{g}_\ell E[\|\mathbf{x}_\ell\|^2] \end{aligned} \quad (7)$$

as the average energy per antenna received from the  $\ell$ -th interferer with expectation over both  $\mathbf{x}_\ell$  and  $\mathbf{H}_\ell$ , we can write

$$N_0 = \sum_{\ell=1}^{\mathcal{L}} \mathcal{J}_\ell + \rho_{\text{th}}, \quad (8)$$

with  $\rho_{\text{th}} = E[\|\mathbf{n}_{\text{th}}\|^2]/n_R$ . In the remainder we seek some insight by concentrating on the interference-limited scenario that is [12]

$$\frac{1}{\rho_{\text{th}}} \sum \mathcal{J}_\ell \rightarrow \infty.$$

For this scenario we can reasonably assume that  $\Phi_{\mathbf{n}}$  can be modeled as a complex Wishart matrix with  $L = \sum_{\ell=1}^{\mathcal{L}} m_\ell$  degrees of freedom and covariance matrix  $\Theta_I$ , whose distribution can be written as [13]

$$f(\Phi_{\mathbf{n}}) = \frac{\det(\Phi_{\mathbf{n}})^{L-n_R} \exp[-\text{Tr}(\Theta_I^{-1} \Phi_{\mathbf{n}})]}{\Gamma_{n_R}(L) \det(\Theta_I)^L},$$

with  $\Gamma_p(q)$ ,  $p \leq q$ , the complex multivariate Gamma function [13]

$$\Gamma_p(q) = \pi^{\frac{p(p-1)}{2}} \prod_{\ell=1}^p (q - \ell)!$$

We will also assume that  $L \geq n_R$ . Under these assumptions, we can define the positive-definite matrix

$$\mathbf{S} = \mathbf{H} \mathbf{H}^\dagger \Phi_{\mathbf{n}}^{-1}$$

whose marginal eigenvalues distribution we shall characterize in the following. Herein, the law of  $\mathbf{H}$  depends on the statistical characterization of the fading. In the remainder we focus for  $\mathbf{H}$  on two different scenarios:

- MIMO Rayleigh-faded channels with Rayleigh-faded interferers,
- MIMO Ricean-faded channels with Rayleigh-faded interferers.

1) *Rayleigh-faded Channels*: In the Rayleigh case, we adhere to the so-called *separable* model whereby

$$\mathbf{H} = \Theta_{\mathbf{R}}^{1/2} \mathbf{W} \Theta_{\mathbf{T}}^{1/2} \quad (9)$$

where  $\mathbf{W}$  is a  $(n_R \times n_T)$  matrix with IID zero-mean unit-variance complex Gaussian random entries while  $\Theta_{\mathbf{T}}$  and  $\Theta_{\mathbf{R}}$  are deterministic transmit and receive correlation matrices. In the following, we further assume that  $\Theta_{\mathbf{T}} = \mathbf{I}_{n_T}$  and  $n_R \leq n_T$ , and for ease of notation define  $\tau = n_T - n_R$ .

The entries of  $\mathbf{H}$  are jointly Gaussian with distribution [14]

$$f(\mathbf{H}) = \frac{e^{-\text{Tr}\{\Theta_{\mathbf{R}}^{-1} \mathbf{H} \mathbf{H}^\dagger\}}}{\pi^{n_T n_R} \det(\Theta_{\mathbf{R}})^{n_T}}$$

hence the distribution of  $\mathbf{S}$  is [13]

$$f(\mathbf{S}) = \prod_{\ell=1}^{n_R} \frac{\omega_\ell^{n_T} (\tau+L-\ell)!}{\omega_\ell^{n_T} (n_T-\ell)!(L-\ell)!} \frac{\det(\mathbf{S})^\tau}{\det(\mathbf{I} + \Omega^{-1} \mathbf{S})^{n_T+L}} \quad (10)$$

with  $\omega_\ell$  the  $\ell$ -th eigenvalue of  $\Omega = \Theta_{\mathbf{I}}^{-1} \Theta_{\mathbf{R}}$ .

2) *Ricean-faded Channels*: The Rice case can be modeled by incorporating an additional deterministic matrix  $\mathbf{H}_0$  containing unit-magnitude entries

$$\mathbf{H} = \sqrt{\frac{1}{K+1}} \Theta_{\mathbf{R}}^{1/2} \mathbf{W} \Theta_{\mathbf{T}}^{1/2} + \sqrt{\frac{K}{K+1}} \mathbf{H}_0 \quad (11)$$

so that

$$E[\mathbf{H}] = \sqrt{\frac{K}{K+1}} \mathbf{H}_0 \quad (12)$$

with the Ricean  $K$ -factor quantifying the ratio between the deterministic (unfaded) and the random (faded) energies.

For this channel, still assuming uncorrelated transmit antennas, namely  $\Theta_{\mathbf{T}} = \mathbf{I}_{n_T}$ , the distribution of  $\mathbf{S}$  can be written as [13]

$$\begin{aligned} f(\mathbf{S}) &= \frac{(K+1)e^{-\text{Tr}(\mathbf{M})} \det(\mathbf{S})^\tau}{\det(\mathbf{I} + (K+1)\mathbf{S})^{n_T+L}} \prod_{\ell=1}^{n_R} \frac{(\tau+L-\ell)!}{(n_T-\ell)!(L-\ell)!} \\ &{}_1F_1 \left( n_T+L, n_T, \mathbf{M} \left( \mathbf{I} + \frac{1}{K+1} \mathbf{S}^{-1} \right)^{-1} \right) \end{aligned} \quad (13)$$

with  $\mathbf{M} = K \mathbf{H}_0 \mathbf{H}_0^\dagger \Theta_{\mathbf{I}}^{-1}$  and  ${}_1F_1(a, b, \cdot)$  the confluent hypergeometric function of matrix argument [13].

### III. CHARACTERIZATION OF THE UNORDERED EIGENVALUES

This main section is devoted to the formulation of a general strategy to obtain the marginal density distribution of an unordered eigenvalue of finite-dimensional random matrices conforming to the models described in the previous section. The method is based on two main results that are stated as Lemmas in the following.

**Lemma 1** [15], [16] *A hypergeometric function of two square matrix arguments  $\mathbf{A}$  and  $\mathbf{B}$  of the same<sup>1</sup> dimension  $m$  and with all distinct eigenvalues can be expressed as a ratio of determinants*

$$\begin{aligned} {}_pF_q \left( \begin{matrix} x_1 + m, \dots, x_p + m \\ y_1 + m, \dots, y_q + m \end{matrix} ; \mathbf{A}, \mathbf{B} \right) &= \\ &= \frac{c_m \det \mathbf{F}}{\prod_{k < \ell}^m (a_k - a_\ell) \prod_{k < \ell}^m (b_k - b_\ell)} \end{aligned} \quad (14)$$

with  $a_k$  (resp.  $b_k$ ) the  $k$ -th eigenvalue of  $\mathbf{A}$  (resp.  $\mathbf{B}$ ),

$$c_m = \prod_{t=1}^{m-1} \left[ \frac{\prod_{i=1}^p (x_i + t)}{t \prod_{j=1}^q (y_j + t)} \right]^{t-m}$$

and with the  $(i, j)$ -th entry of  $\mathbf{F}$  given by

$$(\mathbf{F})_{i,j} = {}_pF_q \left( \begin{matrix} x_1 + 1, \dots, x_p + 1 \\ y_1 + 1, \dots, y_q + 1 \end{matrix} ; a_i b_j \right)$$

**Lemma 2** [17] *Let  $\mathbf{F}$  and  $\mathbf{G}$  be two  $(n \times n)$  matrices whose  $(i, j)$ -th entries are, respectively,  $(\mathbf{F})_{i,j} = f_j(w_i)$  and  $(\mathbf{G})_{i,j} = g_j(w_i)$  where  $f_j$  and  $g_j$ ,  $j = 1, \dots, n$ , are functions defined on  $\mathfrak{R}^+$ . Then, for  $b > a > 0$ ,*

$$\int_{[a,b]^n} \det \mathbf{F} \det \mathbf{G} \prod_{k=1}^n h(w_k) dw_1 \dots dw_n = n! \det \mathbf{A}$$

<sup>1</sup>For a generalization of this formula to square matrices of different dimensions, or with nondistinct eigenvalues, see [16].

where  $h$  is a function defined on  $\mathbb{R}^+$  and  $\mathbf{A}$  is another  $(n \times n)$  matrix whose  $(i, j)$ -th entry is

$$A_{i,j} = \int_a^b f_i(w)g_j(w)h(w) dw$$

Note that, in [17], the factor  $n!$  does not appear because the variables  $w_1, \dots, w_n$  are ordered. As claimed in Lemma 2, in contrast, they are unordered.

Using these lemmas, we can now state the main result of the paper.

**Theorem 1** Let  $\psi_i$  be a function defined on  $\mathbb{R}^+$  and consider a random matrix whose unordered eigenvalues admit a joint distribution of the form

$$f(\mathbf{\Lambda}) = K \prod_{k=1}^m \zeta(\lambda_k) \prod_{k < \ell}^m (\lambda_k - \lambda_\ell) \det(\mathbf{\Psi}(\mathbf{\Lambda})) \quad (15)$$

with  $\mathbf{\Lambda} = \text{diag}\{\lambda_1, \dots, \lambda_m\}$ ,  $K$  a normalization constant and  $\mathbf{\Psi}(\mathbf{\Lambda})$  a matrix whose  $(i, j)$ -th entry is  $\psi_i(\lambda_j)$ . The marginal probability density function of the unordered eigenvalues can be written as

$$f(\lambda) = \tilde{K} \sum_{i=1}^m \sum_{j=1}^m \lambda^{j-1} \zeta(\lambda) \psi_i(\lambda) \mathcal{D}(i, j) \quad (16)$$

where  $\tilde{K}$  is a proper normalizing constant and  $\mathcal{D}(i, j)$  is the  $(i, j)$ -cofactor of the  $(m \times m)$  matrix  $\mathbf{A}$  whose  $(\ell, k)$ -th entry equals

$$\int_0^\infty \lambda^{\ell-1} \zeta(\lambda) \psi_k(\lambda) d\lambda$$

**Proof:** Using the Laplace determinant expansion, (15) becomes

$$\begin{aligned} f(\mathbf{\Lambda}) &= K \sum_{i=1}^m \sum_{j=1}^m (-1)^{i+j} \lambda_1^{j-1} \zeta(\lambda_1) \psi_i(\lambda_1) \\ &\times \prod_{k=2}^m \zeta(\lambda_k) \det \tilde{\mathbf{V}}(\mathbf{\Lambda}) \det \tilde{\mathbf{\Psi}}(\mathbf{\Lambda}) \end{aligned} \quad (17)$$

with  $\tilde{\mathbf{\Psi}}(\mathbf{\Lambda})$  and  $\tilde{\mathbf{V}}(\mathbf{\Lambda})$  the  $(m-1) \times (m-1)$  matrices obtained deleting the first row and column from the matrix  $\mathbf{\Psi}(\mathbf{\Lambda})$  and  $\mathbf{V}(\mathbf{\Lambda})$ , respectively. Here,  $\mathbf{V}(\mathbf{\Lambda})$  is the matrix whose  $(i, j)$ -th entry is  $\lambda_i^{j-1}$ . Eq. (16) follows from integration of (17) over  $m-1$  eigenvalues via Lemma 2. Note that the choice of  $\lambda_1$  in (17) has no effect on the final result since we started from an unordered distribution.

Based on this general result, we can express the marginal density distributions of the unordered eigenvalues for two cases of interest in MIMO for which no analytical solutions existed:

- One-side correlated Rayleigh-faded channel with one-side correlated Rayleigh-faded interferers.
- One-side correlated Ricean-faded channel with one-side correlated Rayleigh-faded interferers.

These expressions will be stated as corollaries in the remainder<sup>2</sup>. In the absence of interference, the corresponding characterizations can be found in [7] and [3], respectively. Our

<sup>2</sup>For sake of brevity the proofs are relegated to the full version of the paper [18].

formulation in this paper unifies and generalizes those previous results and may lead to further expressions of interest.

**Corollary 1** Consider a MIMO communication where both the desired user and the interferers are affected by Rayleigh fading. Let  $\omega_1, \dots, \omega_{n_R}$  be the  $n_R$  distinct<sup>3</sup> singular values of  $\mathbf{\Omega}$ . Then, the marginal density distribution of an unordered singular value of  $\mathbf{S}$  is

$$f(\lambda) = K \sum_{i=1}^{n_R} \sum_{j=1}^{n_R} \frac{\lambda^{\tau+j-1}}{(1 + \lambda/\omega_i)^{\tau+L+1}} \mathcal{D}(i, j) \quad (18)$$

with

$$K = \frac{\prod_{\ell=1}^{n_R-1} \binom{n_T+L+\ell}{\ell}^{\ell-n_R}}{n_R \prod_{k < \ell}^{n_R} (\frac{1}{\omega_\ell} - \frac{1}{\omega_k})} \prod_{\ell=1}^{n_R} \frac{\omega_\ell^{-n_T} (\tau+L-\ell)!}{(n_T-\ell)! (n_R-\ell)! (L-\ell)!} \quad (19)$$

and with  $\mathcal{D}(i, j)$  the  $(i, j)$ -cofactor of the  $(n_R \times n_R)$  matrix whose  $(\ell, k)$ -th entry equals

$$\frac{(\tau+\ell-1)!(L-\ell)!}{(\tau+L)!} \omega_k^{\tau+\ell}.$$

**Corollary 2** Consider a MIMO communication where the desired user experiences a Rice distributed fading, while the interferers are affected by Rayleigh fading. Let us assume  $\Theta_I = \Theta_R$  and let  $\mu_1, \dots, \mu_{n_R}$  be the  $n_R$  distinct singular values of  $\mathbf{M} = \mathbf{K} \mathbf{H}_0 \mathbf{H}_0^\dagger \Theta_I^{-1}$ . The marginal density distribution of an arbitrary singular value of  $\mathbf{S}$  is

$$\begin{aligned} f(\lambda) &= K \sum_{i=1}^{n_R} \sum_{j=1}^{n_R} \frac{(K+1)^{\tau+j} \lambda^{\tau+j-1}}{(1+(K+1)\lambda)^{\tau+L+1}} \mathcal{D}(i, j) \\ &{}_1F_1 \left( \tau+L+1, \tau+1, \frac{(K+1)\mu_i \lambda}{(1+(K+1)\lambda)} \right) \end{aligned} \quad (20)$$

with

$$K = \frac{e^{-\sum_i \mu_i}}{n_R \prod_{k < \ell}^{n_R} (\mu_k - \mu_\ell)} \prod_{\ell=1}^{n_R} \frac{(n_T+L-\ell)!}{(n_T-\ell)! (n_R-\ell)! (L-\ell)!} \prod_{\ell=1}^{n_R-1} \left( \frac{\tau+L+\ell}{\ell(\tau+\ell)} \right)^{\ell-n_R} \quad (21)$$

and with  $\mathcal{D}(i, j)$  the  $(i, j)$ -cofactor of the  $(n_R \times n_R)$  matrix whose  $(\ell, k)$ -th entry equals

$$(L-\ell)! \sum_{t=0}^L \binom{L}{t} \frac{\mu_k^t (\delta+\ell-1)!}{[\delta+1]_t (\delta+L)!} {}_1F_1(\delta+\ell, \delta+L+1, \mu_k),$$

with  $\delta = \tau + t$  and  $[a]_b = \frac{(a+b-1)!}{(a-1)!}$ .

#### IV. ERGODIC MUTUAL INFORMATION

In this section, we capitalize on the unordered eigenvalue characterizations in order to calculate the ergodic mutual information with coherent reception, which can be expressed as a function of the SNR as [1]

$$\begin{aligned} \mathcal{J}(\text{SNR}) &= E \left[ \log_2 \det \left( \mathbf{I} + \frac{\text{SNR } n_R}{g E[\text{Tr}\{\mathbf{S}\}]} \mathbf{S} \right) \right] \\ &= n_R E \left[ \log_2 \left( 1 + \frac{\text{SNR } n_R}{g E[\text{Tr}\{\mathbf{S}\}]} \lambda(\mathbf{S}) \right) \right] \end{aligned} \quad (22)$$

<sup>3</sup>The assumption of distinct eigenvalues can be relaxed resorting to the results in [16]. This allows, in particular, to deal with the case  $\Theta_I = \Theta_R$ .

where  $\lambda$  denotes an arbitrary nonzero eigenvalue of  $\mathbf{S}$  and the expectation is over its distribution.

In the remaining of the Section we exploit (22) together with the newly obtained expressions to evaluate the mutual information.

**Proposition 1** *Under the hypotheses of Corollary 1, the ergodic mutual information is given by*

$$\mathcal{J}(\text{SNR}) = \frac{n_R K}{\log_e 2} \sum_{j=1}^{n_R} \sum_{i=1}^{n_R} \frac{\mathcal{D}(i,j) \omega_i^{\tau+j} (\tau+j-1)! (L-j)!}{(L+\tau)!} \quad (23)$$

$$\sum_{\nu=0}^{\tau+j-1} \frac{{}_2F_1\left(1, \tau+L-\nu; \tau+L+1; 1-\frac{1}{\gamma\omega_i}\right)}{(L+\tau-\nu)}$$

with

$$\gamma = \frac{\text{SNR } n_R (L - n_R)}{g n_T \text{Tr}\{\mathbf{\Omega}\}},$$

and with  $K$  and  $\mathcal{D}(i, j)$  as in Corollary 1.

**Example 1** *Consider a mobile terminal radiating from  $n_T=4$  transmit antennas, loosely correlated, i.e., with  $\mathbf{\Theta}_T=\mathbf{I}$ . Then, let the receiving base station have  $n_R=2$  antennas with*

$$(\mathbf{\Theta}_R)_{i,j} = e^{-0.05 d^2 (i-j)^2} \quad (24)$$

which corresponds to a  $d$ -wavelength antenna separation and a broadside (truncated) Gaussian power azimuth spectrum with  $2^\circ$  root-mean-square spread. Consider further the presence of an interferer, equipped with  $m_1 = 5$  uncorrelated transmit antennas, and assume the channel from this interferer to be  $\mathbf{H}_1 = \mathbf{\Theta}_I^{1/2} \mathbf{W}$ , with

$$(\mathbf{\Theta}_I)_{i,j} = e^{-0.03 d^2 (i-j)^2}. \quad (25)$$

For  $d=2$ , the ergodic mutual information conveyed by the channel, given by (23), is depicted in Fig 1. Also shown is the result of 2000 random realizations obtained via Montecarlo, which validates the analytical function.

The ergodic mutual information for the case of Ricean faded desired user impaired by cochannel Rayleigh interference can be encompassed by expanding the confluent hypergeometric function in (20) and following the same steps as for Proposition 1.

**Proposition 2** *Under the hypotheses of Corollary 2, the ergodic mutual information is given by*

$$\mathcal{J}(\text{SNR}) = \frac{n_R K}{\log_e 2} \sum_{j=1}^{n_R} \sum_{i=1}^{n_R} \frac{\mathcal{D}(i,j) (L-j)!}{[\tau+1]_L} \sum_{t=0}^{\infty} \frac{\mu_i^t (\delta+j-1)!}{\delta! t!}$$

$$\sum_{\nu=0}^{\delta+j-1} \frac{{}_2F_1\left(1, \delta+L-\nu; \delta+L+1; 1-\frac{\kappa+1}{\gamma}\right)}{(L+\delta-\nu)}$$

where

$$\gamma = \text{SNR} \frac{n_R (L - n_R) (K + 1)}{g n_T [n_R + K \text{Tr}\{\mathbf{H}_0 \mathbf{H}_0^\dagger \mathbf{\Theta}_I^{-1}\}]},$$

while  $\delta$ ,  $K$  and  $\mathcal{D}(i, j)$  are as in Corollary 2.

## V. CONCLUSIONS

Analytical expressions for the marginal density distribution of an unordered eigenvalue of finite-dimensional random matrices of particular interest in wireless communications are given and their validity is confirmed by matching with Montecarlo simulated statistics. The newly obtained formulas are especially useful in evaluating information theoretic performance measures of MIMO channels whose communication is impaired by cochannel interference.

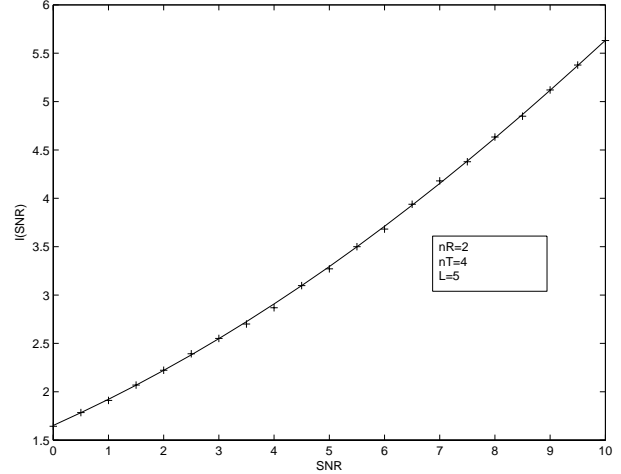


Fig. 1. Ergodic mutual information given in (23) with  $n_T=4$ ,  $n_R=2$ ,  $L=5$ , with  $\mathbf{\Theta}_T=\mathbf{I}$ ,  $\mathbf{\Theta}_R$  as in (24) and  $\mathbf{\Theta}_I$  as in (25).

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