

Error Probability of CDMA in the Poisson Channel

David Brady and Sergio Verdu
Department of Electrical Engineering
Princeton University

Recent work in the analysis of noncoherent, optical Code Division Multiple Access (CDMA) receivers has relied on the approximations of Gaussian-distributed Multiple Access Interference (MAI), cooperation among the users for chip synchronization, or direct observation of the optical intensity (also known as perfect-optical-to-electrical conversion). Until now, the accuracy of these approximations has not been addressed. In this work we derive the exact error expression for the noncoherent, optical matched-filter receiver based on the electron count in a symbol period. The analysis is valid for arbitrary quantum efficiencies and dark currents, and employs the semi-classical model of light. We do not assume perfect optical-to-electrical conversion, Gaussian-distributed MAI, or synchronism among the users. We then compare the exact error rate to those obtained from popular approximations. Using the prime codes as an example, we show that the assumption of perfect optical-to-electrical conversion, which leads to the "error-free" hypothesis test for a suitably small user group size, is a poor model for the photodetection process at moderate incident optical intensities and dark currents. We show that the combined assumptions of perfect optical-to-electrical conversion and Gaussian-distributed MAI yield an overestimate of the optimal threshold and an underestimate of the error rate for small but reasonable optical powers. The error rate expression that we derive is valid for arbitrary, i.i.d. relative delays among the users. The error rate expression is considerably simplified when the delay distribution corresponding to chip-synchronism is used. We take advantage of this fact to derive upper and lower bounds on the asynchronous error rate by using the chip-synchronous expression. The tightness of these bounds for various optical energies and signature sequence sets is discussed.