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Joint Source and Channel Coding

Proposing a low-density parity-check code

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The objectives of this article are two-fold: First, to present the problem of joint source and channel (JSC) coding from a graphical model perspective and second, to propose a structure that uses a new graphical model for jointly encoding and decoding a redundant source. In the first part of the article, relevant contributions to JSC coding, ranging from the Slepian-Wolf problem to joint decoding of variable length codes with state-of-the-art source codes, are reviewed and summarized. In the second part, a double low-density parity-check (LDPC) code for JSC coding is proposed. The double LDPC code can be decoded as a single bipartite graph using standard belief propagation (BP) and its limiting performance is analyzed by using extrinsic information transfer (EXIT) chart approximations.

INTRODUCTION

A cornerstone of information theory is the separation principle, which states that there is nothing to be gained from joint data compression and channel transmission. This principle holds for stationary channels and sources provided that the delay is unbounded [51]. The separate design of source and channel codes allows for diverse sources to share the same digital media. However, for finite-blocklength communications, the separation principle does not apply and the channel decoder can exploit the residual redundancy left by the source code to reduce the overall error rate. Furthermore, as pointed out in [38], some communications standards (e.g., the Universal Mobile Telecommunications System) specify block sizes that are too small for the asymptotic limits to be representative. In many such cases, in practice the sources are transmitted uncoded, while state-of-the-art block channel encoders are used to protect the transmitted data against channel errors. This pragmatic design choice is made because transmission in packets of moderate fixed length is ill suited to existing data compression algorithms that are sensitive to error propagation.

The main idea behind JSC coding is that for fixed-size blocks the source (encoded or not) is redundant, and this redundancy can be exploited at the decoder side.

While much of the work in the area of JSC pertains to lossy recovery of the signal (where a nonvanishing per-letter distortion is allowed), many data transmission applications require that the data be recovered almost noiselessly despite the presence of a noisy channel. In those applications, block error rate is a more suitable reliability measure. The following are two main approaches to almost-lossless JSC coding:

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- A given source compression format (e.g., JPEG) is jointly decoded with a channel decoder (e.g., BP for a sparse-graph code) [43], [58], [42], [40], [54]. These ad hoc approaches use high-level information from the source code to drive the channel decoding process, and they are highly dependent on the source encoder.
- A Markov model for the source is assumed, and it is jointly decoded with the factor graph of the channel code [38], [18], [29], [19], [61], [26]. In some of these approaches the source statistics need not be known a priori (i.e., the transition probability matrix is estimated on the fly at the decoder side).

A CORNERSTONE OF INFORMATION THEORY IS THE SEPARATION PRINCIPLE, WHICH STATES THAT THERE IS NOTHING TO BE GAINED FROM JOINT DATA COMPRESSION AND CHANNEL TRANSMISSION.

This article focuses on the latter approach, in which the graphical model structure of the source and the code are jointly exploited. Standard fixed-to-variable length sources codes (e.g., Lempel-Ziv or arithmetic codes) are not well suited for state-of-the-art channel codes (e.g., turbo or LDPC codes) because they need long sequences to achieve the entropy of the source, and a single error would catastrophically affect the decoded source data. These peculiar features pose strict constraints: if the source encoding is restarted after each block of the channel code (to circumscribe the catastrophic behavior of residual channel errors) the compression efficiency might be low (depending on the channel block size). On the other hand, if the source encoder is not restarted for the entire source transmission (to obtain a good compression performance), a single residual error on the channel decoded stream might disrupt all the subsequent source decoded data.

We advocate a new structure for the JSC encoder, which includes a fixed-to-fixed length source encoder to compress the source, followed by an LDPC code to protect the compressed source against channel errors. As the fixed-to-fixed length source code, we also use an LDPC code. Thus, at the encoder side the structure is given by two concatenated LDPC codes, which, at the decoder side, can be represented as a single bipartite graph and decoded by means of BP. Thanks to the double LDPC structure, it is possible, by means of a design parameter, to find the desired tradeoff between the error floor at high signal-to-noise ratio, E_b/N_0, and the bit error rate (BER) at lower E_b/N_0. In particular, the error floor is due mainly to the source code, while the error rate is due to the channel code.

LITERATURE REVIEW

The results in this article draw on the ideas of many different authors. In this section, we summarize their contributions and we partially review the literature on JSC coding using graphical models. The standard solutions for JSC decoding assume that the source code leaves some residual redundancy, due to constraints on either delay or complexity, in the encoded sequence and that a joint decoder that exploits these redundancies can further reduce its error rate.

After turbo codes were proposed in the early 1990s, some authors pursued the idea of JSC by transforming the decoding process of variables length codes, such as Huffman or arithmetic codes, typically used in image and audio coding. These joint decoding schemes are based on two central ideas: the variable length encoders are not ideal and some bit streams are not admissible; and they can be soft decoded similarly to convolutional codes. In 1995, Hagenauer proposed to use a soft-output Viterbi algorithm to perform JSC decoding using a posteriori information from the source code to control the channel decoder [27]. References [26], [28], [4], and [5] propose to describe the Huffman code with a trellis structure allowing then the joint decoding with a linear channel code (in particular turbo codes are used). The authors in [25] extend this idea to arithmetic codes and in [31] to reversible variable length codes. References [33], [48], and [34] propose jointly decoding a Lempel-Ziv code and a channel code. Finally, [49] develops a maximum a posteriori probability decoder for a variable entropy decoder with a binary symmetric channel and without any channel code.

Among the authors that use a model to describe the redundancy left in the source, García-Frias et al. combine a hidden Markov source model and a turbo code for JSC decoding [18]. They extended their results to Markov sources with unknown parameters, where the decoder jointly estimates them along with the source bits [19] and [20]. Moreover, they extended their work including LDPC codes [23], and allowing nonbinary sources [56] and [55]. Other relevant contributions to solve this problem for memoryless and hidden Markov sources are proposed in [59]–[63], [30], [41], and [57]. Caire, Shamai, and Verdú [12] proposed several linear sparse-graph structures for source-channel coding: in particular, generalizing the approach taken in [10] and [9] they proposed a joint LDPC encoding structure with a joint BP decoder, which is the approach we take in this work. Reference [12] also proposed the LOTUS codes, in which a recursive convolutional encoder follows an irregular LDPC code; LOTUS codes provide a general structure for error protection that includes as particular cases: turbo codes; LDPC codes; and irregular repeat and accumulate (IRA) codes. In [12], the LOTUS codes are optimized for JSC coding and it was shown that quenched BP decoding is more robust to channel erasures than arithmetic coding.

We also highlight a recent contribution [38], which uses the discrete universal denoiser (DUDE) [52] for JSC decoding. The DUDE constructs a conditional model for each symbol using its neighboring symbols and the information of the complete source. It uses this conditional model to detect errors in the source and denoise it. In [38], the authors use the DUDE posterior estimate for each symbol to iteratively decode the received source together with an LDPC decoder.

The Slepian-Wolf problem for independently compressing two correlated sources [47] that are jointly decoded can be seen as a problem of channel coding with side information [39], and it is perhaps the best-known application of channel codes for source coding. In a nutshell, one source is compressed to its entropy and the other is channel encoded using a capacity-achieving code, in which only the redundant bits (parity checks) are used to compress the second source. At the decoder side, the first source is independently decoded. The first source can be interpreted as a noisy version of the second source. The parity check bits, which describe the second source, are appended to the decoded bits of the first source, and the second source is recovered by running the channel decoder on this channel codeword. There are several publications that address the Slepian-Wolf problem with different channel codes, such as turbo codes [3], [1], LDPC codes [23], [32], [37] or low-density generator-matrices codes [24]. Most of the approaches assume that the sources are coupled by a binary symmetric channel with known crossover probability. Others assume that this probability is unknown and it is estimated as part of the iterative decoding procedure [22], while still others use a Markov model to measure the correlation between the sources for binary [6] and nonbinary sources [21].

THE MAIN IDEA BEHIND JSC CODING IS THAT FOR FIXED-SIZE BLOCKS THE SOURCE (ENCODED OR NOT) IS REDUNDANT, AND THIS REDUNDANCY CAN BE EXPLOITED AT THE DECODER SIDE.

Let us consider an independent and identically distributed (i.i.d.) Bernoulli source with success probability $p < 1/2$ and an $\ell \times n$ parity check matrix $\mathbf{H}_{sc}$. We can compress the source using this parity check matrix

$$\mathbf{b} = \mathbf{H}_{sc}\,\mathbf{s}, \tag{1}$$

where $\mathbf{s}$ is a column vector representing the n-source bits and $\mathbf{b}$ is the ℓ-bit column-vector compressed sequence. The decoding procedure is based on approximate syndrome decoding using loopy BP, just as in the decoding for LDPC channel codes. LDPC for source decoding uses the prior probability from the source and iterates between variable and check nodes until the syndrome is $\mathbf{b}$.

This idea that we have outlined for i.i.d. Bernoulli sources can be extended to (hidden) Markov sources by appending the graphical model of the source to the source bits and running the BP algorithm over the joint graph. For other sources, [14] proposes the application of the Burrows-Wheeler transform [8], which transforms the redundant source into an approximately piecewise i.i.d. Bernoulli source [16]. These and other ideas are exploited in [11] and [13] to build a universal source encoders using LDPC codes exhibiting excellent performance relative to established universal data compression algorithms such as Lempel-Ziv.

DOUBLE LDPC CODES FOR JSC CODING

ENCODER STRUCTURE

One approach to JSC coding is to protect a source using a rate-n/m LDPC code and decode it using the source statistics. In this article, we propose a different structure for the encoder. First, we compress the source using an LDPC code (1). Second, we protect the compressed bits with another LDPC code

$$\mathbf{c} = \mathbf{G}_{cc}^{\top}\mathbf{b} = \mathbf{G}_{cc}^{\top}\mathbf{H}_{sc}\,\mathbf{s}, \tag{2}$$

where $\mathbf{G}_{cc}$ is an $\ell \times m$ LDPC generator matrix and $\mathbf{c}$ is the m-dimensional codeword to be transmitted. We refer to the source encoder as $\mathcal{C}_{sc} = (\mathbf{H}_{sc}, \mathbf{G}_{sc})$ and to the channel encoder as $\mathcal{C}_{cc} = (\mathbf{H}_{cc}, \mathbf{G}_{cc})$. In other words, we have included a source encoder between the redundant source and the channel encoder, but the overall rate remains unchanged and equal to n/m. This structure, which we have pursued in [17], was introduced in [12], but the authors did not explore it further in favor of the LOTUS codes.

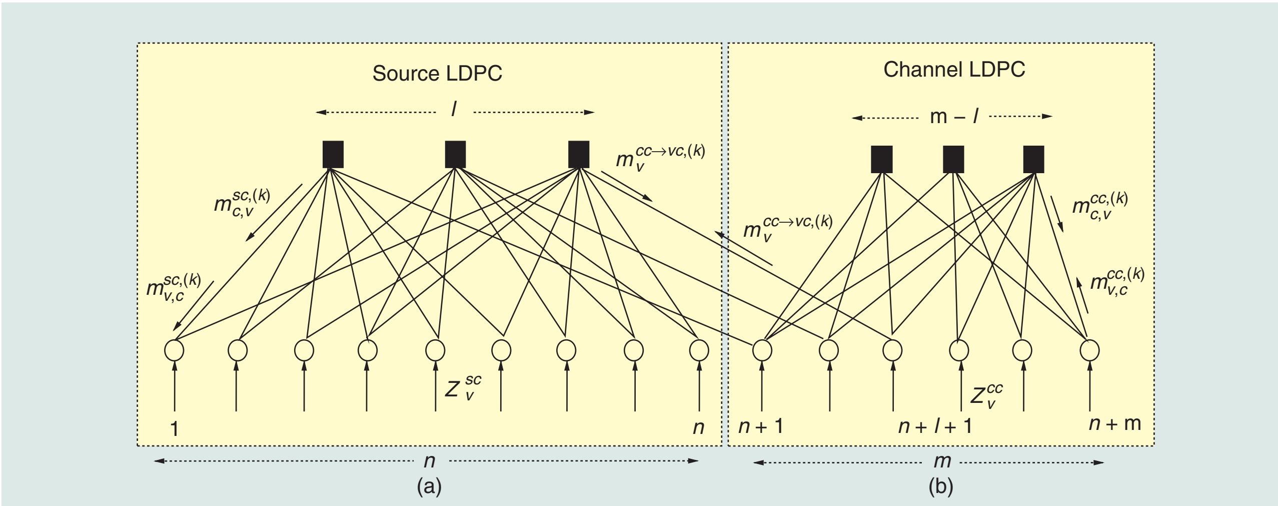
The JSC decoder considered in this article is depicted in Figure 1 as a single bipartite graph. Figure 1(a) represents the source code sparse Tanner graph and (b) represents the channel code. The solid bold lines joining the ℓ factor nodes from the source code to the first variable nodes in the channel code represent the bits compressed by the source code and the message bits at the channel code. We have not included the Markov source model over the n leftmost bits, to avoid cluttering the figure.

DECODER STRUCTURE

Two sparse bipartite graphs compose the decoder, as shown in Figure 1, where each check node of the source code (a) is connected to a single variable node of the channel code (b). The joint decoder runs in parallel. First, the variable nodes inform the check nodes about their log-likelihood ratios (LLRs) and then the check nodes respond with their LLR constraints for each variable node.

Let us consider the kth iteration of the decoder. For the sake of clarity, we describe the two decoders with separate notation:

- $m_{v,c}^{sc,(k)}$ and $m_{v,c}^{cc,(k)}$ are, respectively, the message passed from the vth variable node to the cth check node of the source code ($\mathcal{C}_{sc}$) and channel code ($\mathcal{C}_{cc}$)
- $m_{c,v}^{sc,(k)}$ and $m_{c,v}^{cc,(k)}$ are, respectively, the message passed from the cth check node to the vth variable node of $\mathcal{C}_{sc}$ and $\mathcal{C}_{cc}$
- $m_{v}^{sc\to cc,(k)}$ is the message passed from the check node in $\mathcal{C}_{sc}$ connected to the vth variable node in $\mathcal{C}_{cc}$
- $m_{v}^{cc\to sc,(k)}$ is the message passed from the vth variable node in $\mathcal{C}_{cc}$ connected to the cth check node in $\mathcal{C}_{sc}$



[FIG1] **Parts (a) and (b) show a joint Tanner graph decoding scheme.**

- Z_v^{sc} and Z_v^{cc} represent, respectively, the LLRs for the variable nodes for $v = 1, \ldots, n$ (i.e., the variable nodes of the source decoder) and for $v = n+1, \ldots, n+m$ (i.e., the variable nodes of the channel decoder).

$m_v^{sc\to cc,(k)}$ and $m_v^{cc\to sc,(k)}$ are indexed only by v, because each check node in $\mathcal{C}_{cs}$ is connected to only a single variable node in $\mathcal{C}_{cc}$.

For independent binary sources transmitted over a binary input additive white Gaussian noise (BI-AWGN) channel, $Z_v^{sc} = \log((1-p_v)/p_v)$ (where $p_v = \mathbb{P}[s_v = 1]$), and $Z_v^{cc} = 2r_v/\sigma_n^2$, where $r_v = (1-2x_v) + n_v$, and σ_n^2 is the channel noise variance. Notice that, Z_v^{sc} contains the information about the source, i.e., depends on the source statistic. For this reason if the source is modeled as a Markov process, an additional module is added to the decoder to estimate these values.

The messages between variable nodes and check nodes follow the same procedure as standard BP. First the variable nodes send their LLRs to the check nodes and the corresponding messages are given by

$$m_{v,c}^{sc,(k)} = Z_v^{sc} + \sum_{c'\neq c} m_{c',v}^{sc,(k-1)}, \tag{3}$$

$$m_{v,c}^{cc,(k)} = Z_v^{cc} + m_v^{sc\to cc,(k-1)} + \sum_{c'\neq c} m_{c',v}^{cc,(k-1)}, \tag{4}$$

$$m_v^{cc\to sc,(k)} = Z_v^{cc} + \sum_{c'} m_{c',v}^{cc,(k-1)} \tag{5}$$

and

$$m_{v,c}^{cc,(k)} = Z_v^{cc} + \sum_{c'\neq c} m_{c',v}^{cc,(k-1)}, \tag{6}$$

where (3) runs for $v = 1, \ldots, n$; (4) and (5) for $v = n+1, \ldots, \ell$; and (6) for $v = n+\ell+1, \ldots, n+m$. Notice that $m_{c',v}^{sc,(0)} = 0$, $m_{c',v}^{cc,(0)} = 0$ and $m_v^{sc\to cc,(0)} = 0$.

The messages between the check nodes and the variables nodes are given by

$$\tanh\left(\frac{m_{c,v}^{sc,(k)}}{2}\right) = \tanh\left(\frac{m_v^{cc\to sc,(k)}}{2}\right)\prod_{v'\neq v}\tanh\left(\frac{m_{v',c}^{sc,(k)}}{2}\right), \tag{7}$$

$$\tanh\left(\frac{m_v^{sc\to cc,(k)}}{2}\right) = \prod_{v'}\tanh\left(\frac{m_{v',c}^{sc,(k)}}{2}\right) \tag{8}$$

and

$$\tanh\left(\frac{m_{c,v}^{cc,(k)}}{2}\right) = \prod_{v'\neq v}\tanh\left(\frac{m_{v',c}^{cc,(k)}}{2}\right), \tag{9}$$

where (7) and (8) run for $c = 1, \ldots, \ell$, while (9) runs for $c = \ell+1, \ldots, m$.

After K iterations of the decoding process, the source bits are estimated by

$$\hat{s}_v = \begin{cases} 0 & \text{if} \quad L(s_v) \geq 0 \\ 1 & \text{if} \quad L(s_v) \leq 0, \end{cases}$$

where $L(s_v)$ is the LLR of the source bit s_v for $v = 1, \ldots, n$ computed as

$$L(s_v) = Z_v^{sc} + \sum_c m_{c,v}^{sc,(K)}.$$

ASYMPTOTIC ANALYSIS: EXIT CHARTS

The BP algorithm allows for the analysis of finite-length codes but is impractical for studying the asymptotic behavior of sparse codes. To analyze the behavior of the scheme in the limit of infinite block length, the standard analytical tool for factor graph-based codes under BP iterative decoding is the density evolution (DE) analysis [35], [45]. The DE analysis is typically computationally intense and not very well conditioned numerically. A simple and more manageable way to study the graph behavior consists of an approximation of the DE analysis called the EXIT chart analysis [7], which corresponds to the DE analysis by imposing the restriction that the message densities are of a particular form. More specifically, the EXIT chart with Gaussian approximation assumes that at every iteration, the BP messages are Gaussian having a particular symmetry condition, which imposes that $\sigma^2 = 2\mu$.

Since the decoder is composed of two separated LDPC decoders that exchange information, it is not possible to combine the evolution of the two decoders in a single input-output function. Even if they run in parallel exchanging information, we need to describe the evolution of the source and channel decoders separately.

The following notation is used in the rest of the section:

- $x_{i_{sc}}$ [$x_{i_{cc}}$] denotes the mutual information between a message sent along an edge (v, c) with "left-degree" i_{sc} [i_{cc}] and the symbol corresponding to the bitnode v for the LDPC source [channel] decoder.
- x_{sc} [x_{cc}] denotes the average of $x_{i_{sc}}$ [$x_{i_{cc}}$] over all edges (v, c).
- $y_{j_{sc}}$ [$y_{j_{cc}}$] denotes the mutual information between a message sent along an edge (c, v) with "right-degree" j_{sc} [j_{cc}] and the symbol corresponding to the bit node v for the LDPC source [channel] decoder.
- y_{sc} [y_{cc}] denotes the average of $y_{i_{sc}}$ [$y_{i_{cc}}$] over all edge (c, v).

We consider the class of EXIT functions that make use of Gaussian approximation of the BP messages, which considers the well-known fact that the family of Gaussian random variables is closed under addition (i.e., the sum of Gaussian random variables is also Gaussian, and its mean is the sum of the means of the addends). Imposing the symmetry condition and Gaussianity, the conditional distribution of each message $\mathcal{L}$ in the direction $v \to c$ is Gaussian $\sim\mathcal{N}(\mu, 2\mu)$, for some value $\mu \in \mathbb{R}_+$. Hence, letting V denote the corresponding bit node variable, we have

$$I(V;\mathcal{L}) = 1 - \mathbb{E}[\log_2(1 + e^{-\mathcal{L}})] \triangleq J(\mu),$$

where $\mathcal{L} \sim N(\mu, 2\mu)$. Notice that, by using the function $J(\cdot)$, the capacity of a BI-AWGN channel with noise variance σ_n^2 can be expressed as $C = J(2/\sigma_n^2)$.

Generally, an LDPC code is defined by $\lambda(x) = \sum_i \lambda_i x^{i-1}$ and $\rho(x) = \sum_j \rho_j x^{j-1}$, which represent the degree distribution of the variable nodes and the check nodes, respectively, from the edge perspective, i.e., λ_i (ρ_i) is the fraction of edges connected to a degree i variable (check) node. Thus $\Lambda(x) = \sum_i \Lambda_i x^i = (\int_0^x \lambda(u)du)/(\int_0^1 \lambda(u)du)$, and $P(x) = \sum_j P_j x^j = (\int_0^x \rho(u)du)/(\int_0^1 \rho(u)du)$, represent the degree distribution of the variable nodes and the check nodes, respectively, from the node perspective, i.e., Λ_i (R_i) is the fraction of variable (check) nodes with degree i. The source code is then defined by the pairs $(\lambda_{sc}(x), \rho_{sc}(x))$ (or $(\Lambda_{sc}(x), P_{sc}(x))$) while the channel code is defined by $(\lambda_{cc}(x), \rho_{cc}(x))$ (or $(\Lambda_{cc}(x), P_{cc}(x))$).

In BP, the message on (v, c) is the sum of all messages incoming to v on all other edges. The sum of Gaussian random variables is also Gaussian, and its mean is the sum of the means of the incoming messages. It follows that

$$x_i = J((i-1)\, J^{-1}(y) + J^{-1}(C)),$$

where C is the mutual information (capacity) between the bit node variable and the corresponding LLR at the (binary-input symmetric output) channel output. In the LDPC codes, the bit nodes correspond to variables that are observed through a virtual channel by the LDPC decoder. Averaging with respect to the edge degree distribution we have

$$x = \sum_i \lambda_i J((i-1)J^{-1}(y) + J^{-1}(C)).$$

As far as check nodes are concerned, we use the well-known quasi-duality approximation (proven to be exact for binary erasure channels [2], and to be accurate for additive white Gaussian noise (AWGN) channels [46]) and replace check nodes with bit nodes by changing mutual information into entropy (i.e., replacing x by $1 - x$). Then

$$y_j = 1 - J((j-1)J^{-1}(1-x)),$$

and averaging with respect to the edge degree distribution we have

$$y = 1 - \sum_j \rho_j J((j-1)J^{-1}(1-x)).$$

Following the relationship between channel coding and source coding with side information, established in [53], we can analyze our system as being composed of two channels, i.e., the transmission channel considered and a "virtual" channel associated with the source with noise distribution identical to the source statistics and capacity equal to $C = 1 - H$, where H is the source entropy. Let us consider then the "two-channel" scenario induced by the proposed JSC scheme. For the source decoder, the message x_{sc} is given by

$$x_{sc} = \sum_{i_{sc}} \lambda_{i_{sc}} J_{BSC}((i_{sc} - 1)J^{-1}(y_{sc}), p), \qquad (10)$$

where $J_{BSC}(\cdot)$ is a manipulation of the function $J(\cdot)$ to take into account that the source is binary and i.i.d. with $p = \mathbb{P}[s_v = 1]$, i.e., the equivalent channel is a binary symmetric channel (BSC) with crossover probability p (and hence capacity $1 - H(p)$). In particular, the probability density function (pdf) of the LLR output from the equivalent channel is given by $p\delta(x + L) + (1-p)\delta(x - L)$. Therefore, the function J_{BSC} can be expressed as

$$J_{BSC}(\mu, p) = (1-p)I(V;\mathcal{L}^{(1-p)}) + pI(V;L^{(p)}),$$

where $\mathcal{L}^{(1-p)} \sim N(\mu + L, 2\mu)$, and $\mathcal{L}^{(p)} \sim N(\mu - L, 2\mu)$.

The message y_{sc} is given by

$$y_{sc} = 1 - \sum_{i_{cc}, j_{sc}} \Lambda_{i_{cc}} \rho_{j_{sc}} J((j_{sc} - 1)J^{-1}(1 - x_{sc}) + J^{-1}(1 - c_{i_{cc}})), \qquad (11)$$

where $c_{i_{cc}} = J(i_{cc} J^{-1}(y_{cc})) + J^{-1}(C))$ is the message generated by a variable node of degree i_{cc} of the channel decoder. Notice that we average over all possible values of i_{cc} through $\Lambda_{i_{cc}}$.

For the channel decoder, the message x_{cc} is given by

$$x_{cc}^{(k)} = R_{cc} \sum_{j_{sc}, i_{cc}} P_{j_{sc}} \lambda_{i_{cc}} J((i_{cc}-1)J^{-1}(y_{cc}) + J^{-1}(C) + J^{-1}(c_{j_{sc}})) + (1-R_{cc}) \sum_{i_{cc}} \lambda_{i_{cc}} J((i_{cc}-1)J^{-1}(y_{cc}) + J^{-1}(C)), \quad (12)$$

where $c_{j_{sc}} = 1 - J(j_{sc}J^{-1}(1-x_{sc}))$ is the message generated by a check node of degree j_{sc} of the source decoder. We average over all possible values of j_{sc} through $P_{j_{sc}}$. Notice that (12) is composed of two parts to take into account the fact that a fraction of R_{cc} variable nodes of the channel decoder are connected to the check nodes of the source decoder (i.e., they have the extra message $\downarrow c_{j_{sc}}$), while the remaining $1-R_{cc}$ are connected only to the transmission channel. Since the data are transmitted over a BI-AWGN channel, then $C = J(2/\sigma_n^2)$, and therefore $J^{-1}(C) = 2/\sigma_n^2$.

Finally, the message y_{cc} is given by

$$y_{cc} = 1 - \sum_{j_{cc}} \rho_{j_{cc}} J((j_{cc}-1)J^{-1}(1-x_{cc})). \quad (13)$$

After K iterations we need to obtain the conditional pdf of the LLRs output by the source bits (i.e., the variable nodes of the source code) to compute the BER. Without taking into account the message generated by the equivalent channel, these LLRs are Gaussian, i.e., for a variable node with degree i_{sc}, they are $N(\mu_{i_{sc}}, 2\mu_{i_{sc}})$, where $\mu_{i_{sc}} = i_{sc}J^{-1}(y_{sc})$. Since the equivalent channel is modeled as a BSC, the pdf of the overall message is a Gaussian mixture weighted by the value of p and is given by

$$pN(\mu_{i_{sc}} - L, 2\mu_{i_{sc}}) + (1-p)N(\mu_{i_{sc}} + L, 2\mu_{i_{sc}}).$$

Averaging over all possible values we have that the BER is equal to

$$P_e = \sum_{i_{sc}} \lambda_{i_{sc}} [pQ(x_{i_{sc}}^{p}) + (1-p)Q(x_{i_{sc}}^{1-p})], \quad (14)$$

where

$$x_{i_{sc}}^{p} = \sqrt{\mu_{i_{sc}}/2} - L/\sqrt{2\mu_{i_{sc}}}$$

and

$$x_{i_{sc}}^{1-p} = \sqrt{\mu_{i_{sc}}/2} + L/\sqrt{2\mu_{i_{sc}}},$$

and where $Q(\cdot)$ is the unit Gaussian tail function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-z^2/2} dz.$$

As described in the section "Decoder Structure," in the finite length simulations both decoders run in parallel: first all the LDPC bit nodes are activated, then all the LDPC check nodes and so on. In the infinite length case we adopt a conceptually easier schedule: for each iteration of the LDPC source decoder, a large number of iterations of the channel decoder are performed to reach the fixed-point equilibrium; the generated messages are incorporated as "additive messages" to the check nodes of the source LDPC decoder; all the check nodes of the channel LDPC code are activated. This provides a complete cycle of scheduling, which is repeated an arbitrarily large number of times. The adopted schedule implies that the source and channel decoder are basically treated and studied separately. Notice that in the analysis described in this section, we adopted this scheduling instead of the practical one used in the simulation results (i.e., both source and channel decoders run in parallel at each iteration) because the EXIT chart can be seen as a multidimensional dynamical system with state variables. Since we are interested in studying the fixed points and the trajectories of this system, we need to reduce the problem to an input-output function and then reduce the number of variables. From the equations above, it is clear that this cannot be done by considering the practical scheduling. In fact, if we consider the practical scheduling, as we can see from the equations above, it is not possible to reduce the number of variables of the system since the dependencies between the variables are such that the system cannot be reduced.

Notice that the conceptually easier schedule is useful in the source code optimization procedure described in the next section: since for the channel code a fixed point is reached at each iteration, the input-output function of the channel code can be viewed as a fixed look-up table that, for a given input, returns a fixed value. As we shall see, this allows to us to reduce the optimization to a linear optimization problem.

JSC CODE OPTIMIZATION

In this section, we present an optimization procedure for maximizing the transmission rate of the proposed JSC code. We suggest a suboptimal procedure that gives optimal codes when the source and channel rates tend, respectively, to the entropy of the source and the capacity of the channel. First, we compute the optimal channel code assuming the input bits are i.i.d. and equally likely (worse case). This is the standard LDPC optimization for channel coding [44], [50]. Given the optimized channel code, we compute optimal degree distributions for the variable and check nodes in the source code.

Substituting (11) into (10), we can express the input-output function of the source code as

$$x_{sc}^{(k)} = F_{sc}(x_{sc}^{(k-1)}, p, f_{cc}(x_{sc}^{(k-1)}, C)), \quad (15)$$

where $f_{cc}(x_{sc}^{(k-1)}, C)$ is the input-output function related to the channel code and is derived by substituting (13) into (12).

In a density evolution analysis, the convergence is guaranteed if $F_{sc}(x_{sc}, p, f_{sc}(x_{sc}, \sigma^2)) > x_{sc}$ for $x_{sc} \in [0, 1]$, which ensures convergence at the fixed point $x_{sc} = 1$.

Given $(\lambda_{cc}(x), \rho_{cc}(x))$ (i.e., fixing the channel code) and $\rho_{sc}(x)$, (according to [15], we consider a concentrated right degree distribution of the form $\rho_{sc}(x) = \rho x^{k-1} + (1-\rho)x^k$ for some $k \geq 2$ and $0 \leq \rho \leq 1$) (15) is linear with respect to the coefficients of $\lambda_{sc}(x)$, and thus the optimization problem can be written as

$$\max \sum_{i_{sc} \geq 2} \frac{\lambda_{i_{sc}}}{i_{sc}} \tag{16}$$

$$\text{subject to } \sum_{i_{sc}} \lambda_{i_{sc} \geq 2} = 1, \quad 0 \geq \lambda_{i_{sc}} \leq 1 \tag{17}$$

$$F_{sc}(x_{sc}, p, f_{cc}(x_{sc}, C)) > x_{sc} \tag{18}$$

$$\lambda_2 < \frac{1}{2\sqrt{p(1-p)}} \frac{1}{\sum_{j_{sc}} \rho_{j_{sc}}(j-1)}, \tag{19}$$

where (19) represents the stability condition [15].

The optimization sketched above is based on the procedure proposed in [15]. In contrast to [15], we deal with two codes that iterate in parallel and then we add the input-output function of the channel code [i.e., $f_{cc}(x_{sc}^{(k-1)}, C)$] to the optimization.

EXPERIMENTAL RESULTS

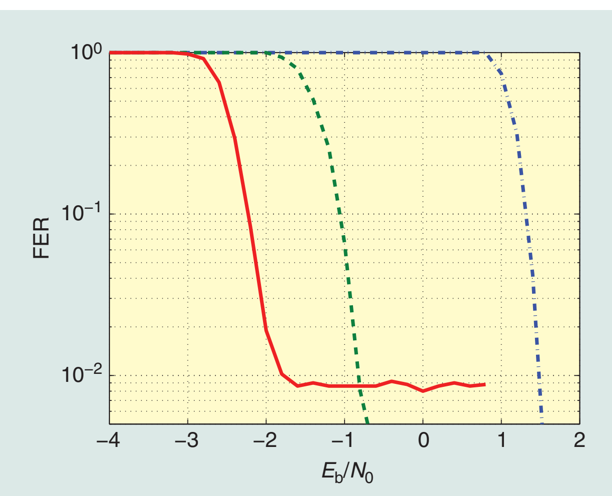
In this section, we illustrate the advantages of using two concatenated LDPC codes for joint source-channel coding instead of one structure as typically proposed in JSC coding. We have performed four sets of experiments with regular and irregular LDPC codes. In the first, we compare the double LDPC code with standard JSC decoding, in which the overall rate protects the source. In the second, we illustrate the advantage of using two concatenated LDPC codes when the overall rate compresses the source. In the third experiment, we show why joint decoding is better than cascade decoding: a channel decoder followed by an independent source decoder. In the final experiment, we explore the use of optimized irregular LDPC codes instead of regular ones. For all the experiments, we use AWGN channels and symmetric Markov sources.

In the first experiment, we consider a Markov source with two states, in which the probability of switching states is equal to 0.07. The entropy of this source is 0.366. We transmit this source through an AWGN channel using a rate 1/2 regular-LDPC code with three ones per column [36]. We first decode the sequence ignoring the source model and we plot the frame error rate (FER) with a dash-dotted line in Figure 2. We also decode the sequence incorporating the Markov source model to the factor graph of the LDPC code. We represent the FER by a dashed line in the figure. The channel encoder benefits from the information from the source to reduce its FER at a significantly lower E_b/N_0. Note that E_b refers to energy per source bit rather than nonredundant information bit for which the asymptotic limit compatible with reliable communication is -1.59 dB.

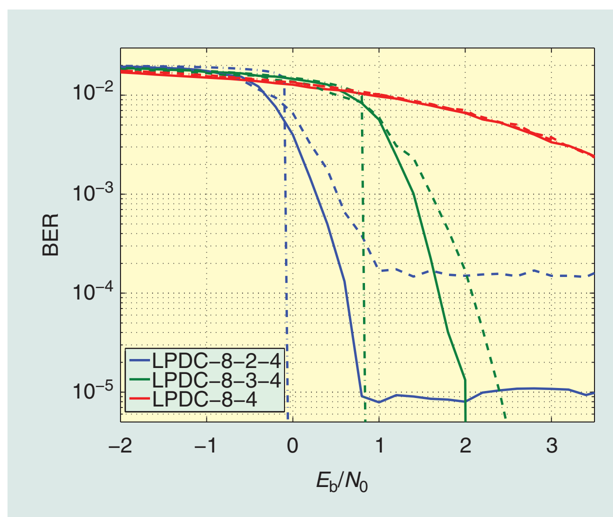
We finally apply our double LDPC encoder with a rate-1/2 regular-LDPC source code and a rate-1/4 regular-LDPC channel code. The overall rate is 1/2, as in the previous case. We depict the FER in Figure 2 with a solid line. The FER is reduced at an even lower E_b/N_0 at the expense of an error floor. The error floor is due to residual decoding errors introduced by the fixed-to-fixed source code, as some less likely words are decoded into a more likely (incorrect) word. This error floor can be reduced by either increasing the block size or by increasing the compression rate (i.e., less compression). The additional coding gain is due to the lower-rate channel encoder. The proposed double LDPC source and channel encoder has a design parameter $\ell \in (0, n)$ that trades off the coding gain and the error floor. The larger ℓ is, the smaller the coding gain and the lower the error floor are.

To illustrate the tradeoff introduced by the design parameter ℓ, for the second experiment we use three codes. The first scheme, denoted as LDPC-8-2-4, consists of two concatenated LDPC codes with rates $R_{sc} = 2/8$ and $R_{cc} = 2/4$ for source and channel coding, respectively. The second scheme, LDPC-8-3-4, consists of two concatenated LDPC codes with rates $R_{sc} = 3/8$ and $R_{cc} = 3/4$. The last scheme, LDPC-8-4, consists of a single LDPC code whose compression rate is $R_{sc} = 4/8$. This code only compresses the source and does not add any redundancy to the transmitted bits.

In Figure 3, we show the BER as a function of E_b/N_0 for a Markov source with two states and the probability of switching



[FIG2] FER for an LDPC channel decoder (dash-dotted), a standard LDPC-JSC decoder (dashed) and the double LDPC-JSC decoder (solid) $n = 3{,}200$ b.



[FIG3] BER versus E_b/N_0 for LDPC-8-2-4, LDPC-8-3-4 and LDPC-8-4.

states is equal to 0.02. For each code there is a set of three plots: the BER predicted by the EXIT chart (dash-dotted line), the BER for code words with 3,200 b (solid lines), and the BER for code words with 1,600 b (dashed lines). The three leftmost plots are for LDPC-8-2-4, the three middle plots are for LDPC-8-3-4, and the rightmost plots for LDPC-8-4.

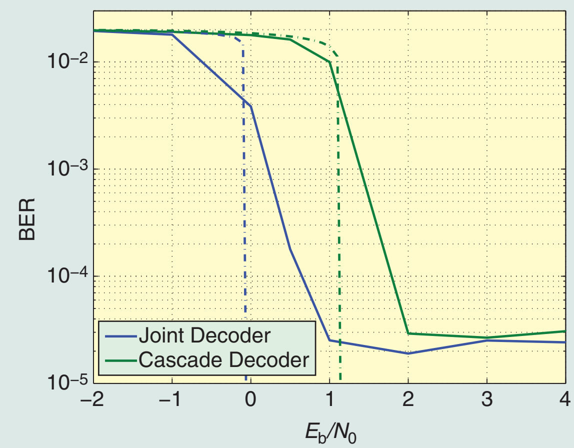
In Figure 3, we observe the standard behavior of the joint decoder for two concatenated LDPC codes, one for channel coding, and the other for source coding. As E_b/N_0 increases, there is a sharp decline in the BER due to the channel code operating below capacity and, as expected, this transition is sharper as the code length increases. There is a residual BER at high E_b/N_0, only observable for LDPC-8-2-4 in Figure 3, due to the source decoder's inability to correctly decompress every word for finite-length codes. This residual BER tends to zero as the code word length increases, because R_{sc} is above the entropy of the source. The residual BER does not show for LDPC-8-3-4, because its R_{sc} is higher.

In the plots, we observe about 1 dB gain when we compare LDPC-8-2-4 with LDPC-8-3-4 for BER values larger than the residual BER. This gain is due to the additional redundancy in LDPC-8-2-4. The price we pay for this gain is a higher residual BER. We can trade off the expected gain and the residual BER by parametrizing the source $R_{sc} = \ell/8$ and channel $R_{cc} = \ell/4$ code rates with ℓ. The value of $\ell \in (0, 4]$ is inversely proportional to the gain and to the residual BER. We can also decrease the residual BER by increasing the code length, as illustrated in Figure 3. For LDPC8-4, the BER does not improve as we increase the code length (the three lines superimpose), even when the decoder has the information about the source statistics.

In the third experiment, we decode the LDPC-8-2-4 scheme with a cascade decoder and compare it with the joint scheme proposed in this article. The cascade decoder first decodes the channel code assuming the compressed bits are equally likely and i.i.d., and then it decodes the source bits using the LLRs output by the channel decoder.

In Figure 4, we plot the BER for both decoders as a function of E_b/N_0 for a Markov source with two states, in which the probability of switching states is equal to 0.02. For each decoder there are two plots: the BER estimated by the EXIT chart (dash-dotted line) and the BER for code words with 3,200 b (solid line). The two leftmost plots are for the joint decoding and the rightmost plots for the cascade decoding.

For low E_b/N_0 neither decoding procedure is able to decode the transmitted word and both return high BER. The signal-to-noise ratio is below capacity and the redundancy is not high enough to decode the transmitted words correctly. For high values of E_b/N_0 both decoding procedures return the same residual BER. There are no errors due to the channel decoder and the residual BER is solely due to the source decoder failing to return the correct word. There is a range in E_b/N_0 between 0 and 1 dB in which the joint decoder returns the residual BER (low BER) and the cascade decoder returns chance level performance (high BER). The difference in performance in this E_b/N_0 range is



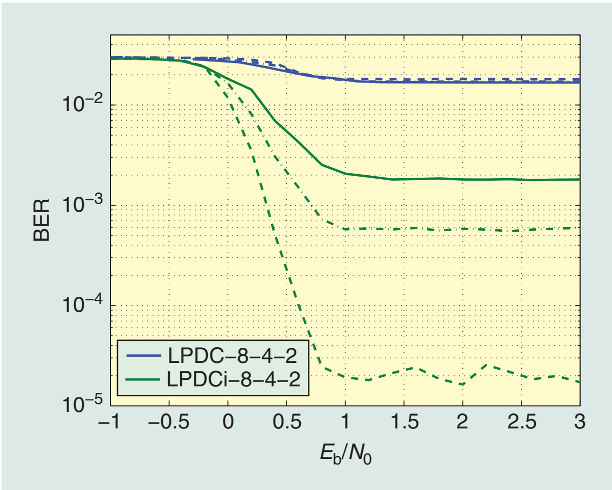
[FIG4] BER versus E_b/N_0 for the joint and cascade decoder for LDPC-8-2-4.

explained by the fact that the source and channel decoders work together to estimate the correct word. The redundancy not removed by the source encoder gives additional information to the channel decoder to return the correct word. This information is not present in the cascade decoder and therefore the channel decoder is unable to exploit the left redundancy and estimate the correct word. The joint decoder provides, in this particular example, a 1 dB gain with respect to the cascade decoder. This gain remains unchanged as the code word length tends to infinity and the performance is studied by means of EXIT chart approximation. This gain disappears only if the rates of the source and channel encoders tend to the entropy and capacity, respectively, as the code word length increases. But for finite-length codes, the rate of the source cannot approach the entropy, and we obtain a gain from a JSC.

Finally, we present the performance of irregular optimized LDPC codes obtained by using the method described in the section "JSC Code Optimization." For the channel code, we adopt the first LDPC code with $R_{cc} = 1/2$ in [50].

For the source code, we use a Markov source with two states, in which the probability of switching states is equal to 0.03. By fixing the degree distribution of the check nodes equal to $\rho(x) = 05x^{21} + 05x^{22}$ and using the optimization procedure described in the section "JSC Code Optimization," we obtain the degree distribution for the variable nodes $\lambda(x) = 0.098x + 0.274x^3 + 0.025x^7 + 0.292x^9 + 0.075x^{33} + 0.234x^{34}$.

The rate of the source code is $R_{sc} \approx 0.24$ and the overall coding rate is around 2.08. We denote this scheme as LDPCi-8-2-4 and we compare it with LDPC-8-2-4, as their rates are similar. In Figure 5, we plot the BER for the two codes as a function of E_b/N_0. For each code there is a set of three plots: the BER for code words with 3,200 b (solid lines), the BER for code words with 6,400 b (dash-dotted lines)m, and the BER for code words with 12,800 b (dashed lines). The three top plots are for LDPC-8-2-4; the three bottom plots are for LDPCi-8-2-4.



[FIG5] BER versus E_b/N_0 for the codes LDPC-8-2-4 and LDPCi-8-2-4.

In Figure 5, we observe that for the regular code as the code word length increases the residual BER remains constant, while the irregular code is able to reduce its residual BER gradually with the code length. This result is similar to the typical results for LDPC codes for channel coding. Regular codes cannot approach capacity and need a margin in their rate to be able to reduce the BER towards zero, while irregular LDPC codes can achieve capacity as the code length increases. Therefore, if the source code rate approaches the entropy of the source, optimized irregular codes are vital to reduce the BER with the code length.

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