Optimal Decentralized Control in the Random Access Multipacket Channel

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Abstract—**A decentralized control algorithm is sought that maximizes the stability region of the infinite-user slotted multipacket channel and is easily implementable. To this end, the perfect state information case where the stations can use the instantaneous value of the backlog to compute the retransmission probability is studied first. The best throughput possible for a decentralized control protocol is obtained, as well as an algorithm that achieves it. Those results are then applied to derive a control scheme when the backlog is unknown, which is the case of practical relevance. This scheme, based on a binary feedback, is shown to be optimal given some restrictions on the channel multipacket reception capability.**

I. Introduction

MOST studies on random access communications rely on the assumption that when two or more packets overlap, all the information that was sent is irremediably lost, hence the need to repeat all transmissions at some later time. This is actually a pessimistic point of view, since there are many examples of random access systems where one or more packets may be successful in the presence of other simultaneous transmissions. In order to represent such random access systems, a model for a channel with multipacket reception capability has been developed in [6]–[8]. We consider a slotted channel with an infinite population of users, and we assume that the probability of having k successes in a slot where there are n transmissions depends only on the collision size n

$$\epsilon_{nk} = P[k \text{ packets are correctly received} \mid n \text{ are transmitted}] \quad (n \geq 1,\ 0 \leq k \leq n).$$

We define the reception matrix as

$$E = \begin{bmatrix} \epsilon_{10} & \epsilon_{11} & & & \\ \epsilon_{20} & \epsilon_{21} & \epsilon_{22} & 0 & \\ \cdot & \cdot & & & \\ \epsilon_{n0} & \epsilon_{n1} & & \epsilon_{nn} & \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \end{bmatrix}.$$

This model can be applied to channels with capture [1]–[3], [10], [16], [18], [20], [23], [26], [28], [34] and to systems using CDMA [22], [24], [29]. It is also relevant for many other applications, such as systems with multiuser detectors [33] or, for instance, the channel studied in [17], [31]. For more details about this model, the reader is referred to [6] and [8]. Denoting by $C_n = \Sigma_{k=1}^{n} k\epsilon_{nk}$ the average number of packets correctly received in collisions of size n, we assume that the limit $C = \lim_{n\to\infty} C_n$ exists, as is usually the case with models of practical interest. It has been proved in [8] that the Aloha random access algorithm has a maximum stable throughput $\eta_0 = C$ in the multipacket channel.

Decentralized control strategies have been shown [11], [12], [19], [25], [30] to stabilize the slotted Aloha algorithm in the case of the usual collision channel, hence, it is reasonable to expect that when those strategies are used in the multipacket channel, the resulting throughput will be higher than η_0. We consider schemes of the form

$$p_n = F(S_n)$$
$$S_{n+1} = G(S_n, Z_n) \tag{1}$$

where p_n is the retransmission probability in slot n, S_n is an estimate of the backlog X_n at the beginning of slot n, and Z_n is the feedback at the end of slot n. The number of new packets arriving during slot n, A_n is assumd to form a sequence of i.i.d. random variables with probability distribution $P[A_n = k] = \lambda_k (k \geq 0)$, such that the mean arrival rate $\lambda = \Sigma_{n=1}^{\infty} n\lambda_n$ is finite. Each of the A_{n-1} new packets that arrived during slot $n-1$ is transmitted in slot n with probability p_n.

As in the case of conventional channels, it is useful to study first the case of control with perfect state information where the value of the backlog is given to the users prior to the selection of the retransmission probability. To keep track of the exact value of the backlog, a central controller is usually necessary, which is an unreasonable requirement for most practical random access channels. However, the study of the perfect state information case allows us to determine an upper bound to the best throughput η_c achievable by any decentralized control of the form (1), and suggests a simple implementation. Those results are in turn helpful to derive control protocols in the case where the backlog is unknown. This is done in Section III where we consider a backlog estimate which is recursively updated using the binary feedback empty/nonempty. In addition, it is assumed throughout the paper that each station is informed when its packet is successfully received. It is proved that provided a certain condition on the reception matrix holds, the throughput achievable with this type of feedback is the same as the perfect state information throughput. This condition is verified for most multipoint-to-point channels of practical interest.

In a paper whose translation appeared only very recently [19] (after our work [7]), Mikhailov has derived sufficient conditions for stability and instability of two-dimensional Markov chains. Although this was meant to be used for decentralized control schemes in the usual collision channel, this approach is powerful enough to be applied to the multipacket channel. In Section IV we show by using Mikhailov's result that the scheme presented in Section III is stable under weaker assumptions. However, only a weaker form of stability can be proved in this way.

Manuscript received April 18, 1988; revised January 20, 1989 and May 7, 1989. Paper recommended by Associate Editor, X. R. Cao. This work was supported in part by the Office of Naval Research under Contracts N00014-87-K-0054, by the Army Research Office under Contract DAAL-03-87-K-0062, and by the New Jersey Commission on Science and Technology under Grant 85-990660-6.

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IEEE Log Number 8930788.

0018-9286/89/1100-1153$01.00

II. Control of the Multipacket Channel with Perfect State Information

In this section we assume that all the users know the value of X_n at the beginning of slot n, and we let the retransmission probability be a function of the exact value of the backlog, i.e., $p_n = F(X_n)$. In this ideal case, the system is much simpler to analyze than in the general case (1) since $(X_n)_{n\geq 0}$ is a homogeneous Markov chain. Our goal is to determine the optimal control function F^* that yields the largest ergodicity region, and the corresponding throughput, denoted by η_c. For instance, it is well known [4] that for the usual collision channel with the access rule in effect here, $F^*(X_n) = 1/X_n$ is the retransmission probability that minimizes the drift at each step, resulting in an ideal throughput of $\eta_c = e^{-1}$.

First note that all the results herein are valid provided that the backlog Markov chain $(X_n, S_n)_{n\geq 0}$ corresponding to a control (1) is irreducible and aperiodic. It can be easily checked that for both access rules considered in this paper (see below), as well as all the algorithms, a simple set of sufficient conditions for irreducibility and aperiodicity is

a) $\lambda_0 \neq 0$

b) $\lambda_0 + \sum_{n=1}^{\infty} \lambda_n \epsilon_{nn} < 1$

c) $\epsilon_{10} \neq 0$

which are analogous to the conditions for the open-loop system studied in [6]. The theorem below gives the best throughput possible for a control protocol (1).

Theorem 1: There exists a retransmission probability p_n^* that minimizes the expected backlog increase when the backlog is equal to n.

With such a retransmission probability, the system is stable for $\lambda < \eta_c$ and unstable for $\lambda > \eta_c$, with

$$\eta_c = \sup_{x\geq 0} e^{-x} \sum_{n=1}^{\infty} C_n \frac{x^n}{n!}.$$

Proof of Theorem 1: The proof is based on standard drift analysis techniques. $(X_n)_{t\geq 0}$ is a homogeneous Markov chain which evolves according to

$$X_{t+1} = X_t + A_t - \Sigma_t \tag{2}$$

where Σ_t is the number of packets successfully transmitted in slot t. The system is defined to be stable if $(X_t)_{t\geq 0}$ is ergodic and unstable otherwise. Let d_n be the drift of X_t at state n: $d_n = E[X_{t+1} - X_t | X_t = n]$. We have $0 \leq \Sigma_t \leq X_t$, and if we denote by p the retransmission probability used in slot t, then for $n \geq 1$, the probability of having k successes is given by

$$P[\Sigma_t = k | X_t = n] = \sum_{j=k}^{n} \binom{n}{j} p^j (1-p)^{n-j} \epsilon_{jk} \qquad (1 \leq k \leq n). \tag{3}$$

It then follows from (2) that the backlog drift at state $n \geq 1$ is given by

$$d_n = \lambda - \sum_{k=1}^{n} k \sum_{j=k}^{n} \binom{n}{j} p^j (1-p)^{n-j} \epsilon_{jk}$$

$$= \lambda - \sum_{j=1}^{n} \binom{n}{j} p^j (1-p)^{n-j} C_j \tag{4}$$

which becomes $d_n(p) = \lambda - t_n(p)$ if we define $t_n(p)$ to be the average number of successes given the backlog n and the retransmission probability p

$$t_n(p) = \sum_{j=1}^{n} \binom{n}{j} p^j (1-p)^{n-j} C_j. \tag{5}$$

Since $t_n(p)$ is a polynomial on the compact [0,1], it achieves its maximum and we can define

$$p_n^* = \arg\max_{p\in[0,1]} t_n(p) = \arg\min_{p\in[0,1]} d_n(p).$$

We now proceed to compute the limit of the drift when the retransmission probability p_n^* is used. We show that

$$\lim_{n\to\infty} t_n(p_n^*) = \sup_{x\geq 0} e^{-x} \sum_{n=1}^{\infty} C_n \frac{x^n}{n!} = \sup_{x\geq 0} t(x). \tag{6}$$

Let us first assume that $C < +\infty$.

Property 1:

$$\lim_{x\to\infty} t(x) = C.$$

We have for $n > M$

$$|t(x) - C| \leq e^{-x}C + e^{-x} \sum_{n=1}^{M} \frac{x^n}{n!} |C_n - C| + \sum_{n=M+1}^{\infty} \frac{x^n}{n!} |C_n - C|. \tag{7}$$

Pick $\epsilon > 0$ and fix M such that $|C_n - C| < \epsilon$ for $n > M$. Then if B_c is an upper bound on the sequence $(C_n)_{n\geq 1}$, (7) yields

$$|t(x) - C| \leq e^{-x}C + 2B_c e^{-x} \sum_{n=1}^{M} \frac{x^n}{n!} + \epsilon$$

and the right-hand side of this last equation goes to zero as x goes to infinity.

Property 2: For all $\epsilon > 0$, there exists $A > 0$ such that for all $np > A$, $|t_n(p) - C| < \epsilon$. We have

$$|t_n(p) - C| \leq \sum_{j=1}^{n} \binom{n}{j} p^j (1-p)^{n-j} |C_j - C| + (1-p)^n C.$$

Choosing M as for Property 1 we get

$$|t_n(p) - C| \leq 2B_c \sum_{j=0}^{n} \binom{n}{j} p^j (1-p)^{n-j} + \epsilon.$$

Let us denote by R_n the random variable corresponding to the number of retransmissions in a slot given that the backlog is equal to n. We have

$$\sum_{j=0}^{M} \binom{n}{j} p^j (1-p)^{n-j} = P[R_n \leq M] \leq P\left[\left|\frac{R_n}{n} - p\right| > \frac{p}{2}\right]$$

for $np > 2M$. Then from the Chebyshev inequality

$$P[R_n \leq M] \leq \frac{4}{np} \tag{8}$$

and Property 2 follows.

Property 3: $t_n(x/n)$ converges uniformly to $t(x)$ on any compact $[0, A]$.

Fix $\epsilon > 0$ and choose M such that $\sum_{j=M}^{\infty} A^j C_j / j! < \epsilon$. Then for $n > M + 1$ and $x \in [0, A]$

$$\left| t_n\left(\frac{x}{n}\right) - t(x) \right| \le \sum_{j=1}^{M} A^j \left| \frac{C_j}{j!} \right| e^{-x} - \frac{n(n-1)\cdots(n-j+1)}{n^j}\left(1-\frac{x}{n}\right)^{n-j} \Bigg| + 2\epsilon.$$

Since $\lim_{n\to\infty} n(n-1)\cdots(n-j+1)/n^j = 1$ for $1 \le j \le M$, it is enough to show that $(1 - x/n)^{n-j}$ converges uniformly to e^{-x} for $1 \le j \le M$. We have

$$\left(1-\frac{x}{n}\right)^{n-j} - e^{-x} \le e^{-x}[e^{xj/n} - 1] \le e^{AM/n} - 1. \tag{9}$$

On the other hand, for $n > A$,

$$\left(1-\frac{x}{n}\right)^{n-j} - e^{-x} \ge \left(1-\frac{x}{n}\right)^{n} - e^{-x} \ge e^{-x}[e^{A+n\log(1-A/n)} - 1]$$
$$\ge e^{A}\left(1-\frac{A}{n}\right)^{n} - 1 \tag{10}$$

and uniform convergence follows from (9) and (10).

Property 4: $t_n(x/n)$ converges uniformly to $t(x)$ for $x \ge 0$.

Fix $\epsilon > 0$. From Properties 1 and 2 we can fix A such that:
i) for all $np > A$, $|t_n(p) - C| < \epsilon$,
ii) for all $x > A$, $|t(x) - C| < \epsilon$.

Then we distinguish two cases. If $x \in [0, A]$, then from Property 3 there exists N such that for all $n \ge N$, $|t_n(x/n) - t(x)| < \epsilon$. If on the other hand $x \in (A, +\infty)$, we have

$$\left| t_n\left(\frac{x}{n}\right) - t(x) \right| \le \left| t_n\left(\frac{x}{n}\right) - C \right| + |t(x) - C| \le 2\epsilon \tag{11}$$

from i) and ii).

Thus, we have shown that when C is finite, $t_n(x/n)$ converges uniformly to $t(x)$ for $x \ge 0$. It follows that $\lim_{n\to\infty} \sup_{x\ge 0} t_n(x/n) = \sup_{x\ge 0} t(x)$ and so (6) is proved.

Finally, we show that (6) holds when $C = +\infty$. Choose Δ arbitrarily large and M such that $C_n > \Delta$ for $n > M$. Then for $n > M$

$$t_n\left(\frac{x}{n}\right) \ge \Delta \sum_{j=M+1}^{n} \binom{n}{j}\left(\frac{x}{n}\right)^j\left(1-\frac{x}{n}\right)^{n-j} \ge \Delta(1 - P[R_n \le M]).$$

From (8) $P[R_n \le M]$ is arbitrarily small for $nx/n = x$ large enough. Therefore, $\sup_{x\ge 0} t_n(x/n) = +\infty$ and $\lim_{n\to\infty} t_n(p_n^*) = +\infty$. Since it is clear that if $C = +\infty$, then $\sup_{x\ge 0} t(x) = +\infty$, (6) holds.

From the equality $\lim_{n\to\infty} d_n(p_n^*) = \lambda - \sup_{x\ge 0} t(x)$ and Pakes Lemma in [21], it follows that if $\lim_{n\to\infty} C_n = +\infty$, then $\lim_{n\to\infty} d_n(p_n^*) = -\infty$, and the system is always stable, whereas if $\lim_{n\to\infty} C_n < +\infty$, then $(X_n)_{n\ge 0}$ is ergodic for $\lambda < \eta_c = \sup_{x\ge 0} t(x)$. Also, it is shown in the Appendix that Kaplan's condition holds for this system when the sequence $(C_n)_{n\ge 1}$ is bounded, thus from Kaplan's result [13], the backlog Markov chain is nonergodic when $\lambda > \eta_c$. □

It is intuitively obvious that no decentralized control algorithm of the form (1) can have a maximum stable throughput larger than η_c. The theorem below gives a rigorous proof of this fact and also shows that this throughput can be achieved with a control which is much simpler than p_n^*.

Theorem 2: The best throughput achievable by a decentralized control algorithm (1) is $\eta_c = \sup_{x\ge 0} e^{-x} \sum_{n=1}^{\infty} x^n/n!\, C_n$. If $\eta_c > C = \lim_{n\to\infty} C_n$, then there exists a constant $A > 0$ such that the control $p_t = A/X_t$ for $X_t > A$ yields the optimal throughput η_c.

Proof of Theorem 2: To prove the first part of the theorem we use a result of [27] which is a generalization of Kaplan's Theorem. If $p_t = F(S_t)$ and $S_{t+1} = G(S_t, Z_t)$, consider the Markov chain (X_t, S_t) and the Lyapunov function $V(n, s) = n$. Assume that $\lambda > \eta_c$. Then

$$E[V(X_{t+1}, S_{t+1}) - V(X_t, S_t) | X_t = n, S_t = s]$$
$$= \lambda - \sum_{j=1}^{n} \binom{n}{j} F(s)^j (1 - F(s))^{n-j} C_j$$
$$\ge d_n(p_n^*) \ge \frac{\lambda - \eta_c}{2} \tag{12}$$

for all n large enough and all s. Therefore, the drift of V is strictly positive outside a finite subset of the state space. Since it is shown in the Appendix that the generalized Kaplan's condition is verified, it is enough to conclude that (X_t, S_t) is nonergodic. Hence, η_c is indeed the best throughput achievable by any decentralized control algorithm of the form (1).

To prove the second part of the theorem, we need the following property.

Property 5: If for all $x \ge 0$, $t(x) < \sup_{x\ge 0} t(x)$, then $\sup_{x\ge 0} t(x) = C$.

If $\sup_{x\ge 0} t(x) = +\infty$, it is easily seen that $C = +\infty$. If $\sup_{x\ge 0} t(x) < +\infty$, then $C < +\infty$. Consider a sequence $(x_n)_{n\ge 1}$ of nonnegative reals such that $\lim_{n\to\infty} t(x_n) = \sup_{x\ge 0} t(x)$. If $(x_n)_{n\ge 1}$ was bounded above by $K < +\infty$, we would have for all $n \ge 1$, $t(x_n) \le \sup_{x\in[0,K]} t(x)$, and in the limit $\sup_{x\ge 0} t(x) = \sup_{x\in[0,K]} t(x)$. Then there would exist $x_0 \in [0, K]$ such that $t(x_0) = \sup_{x\ge 0} t(x)$, which is a contradiction. Therefore, $(x_n)_{n\ge 1}$ is unbounded, and one can build a subsequence $(x_{nk})_{k\ge 1}$ such that $\lim_{k\to\infty} x_{n_k} = +\infty$. We still have, of course, $\lim_{k\to\infty} t(x_{n_k}) = \sup_{x\ge 0} t(x)$, but on the other hand, we have $\lim_{k\to\infty} t(x_{n_k}) = \lim_{x\to\infty} t(x)$. From Property 1 in the proof of Theorem 1, $\lim_{x\to\infty} t(x) = C$ and Property 5 follows.

Thus, if $\eta_c > C$, then $t(x)$ achieves its supremum at some finite positive real A. Let us consider the control $p_t = A/X_t$ for $X_t \ge A$. (Note that the value of the retransmission probability is left unspecified for $X_t < A$ because it does not affect the throughput.) Then from (4) $d_n = \lambda - t_n(A/n)$, and from Property 3 in the proof of Theorem 1 $\lim_{n\to\infty} d_n = \lambda - t(A)$. Then it follows from [21] that $(X_t)_{t>0}$ is ergodic if $\lambda < t(A)$ and from [13] and the Appendix that $(X_t)_{t\ge 0}$ is nonergodic if $\lambda > t(A)$. Thus, the maximum stable throughput of the system is $t(A) = \sup_{x\ge 0} t(x) = \eta_c$. □

Note that the closed-loop throughput obtained in Theorems 1 and 2 can be interpreted as $\eta_c = \sup_{N\sim P(x), x>0} E[C_N]$, that is as the supremum over x of the expected value of C_N if N is a Poisson distributed random variable with mean x. Note that if we were to follow the popular approximation [1], [2], [10], [16], [18], [24], [26] that assumes that the number of transmissions in each slot, N, is Poisson distributed, and if we could choose any positive number as the mean of N by regulating the retransmission probability, the throughput would be equal to the average number of successes per slot, $E[C_N]$, maximized over the mean of N. As in the usual collision channel, a wrong analysis leads to a correct conclusion. Several examples are gathered in Table I (see [8] for details).

Probably the most important conclusion of this section is that in general it is not necessary to compute the exact value of p_n^*, which would require a large amount of on-line computations, and seriously hinder any application of Theorem 1 to the case where the backlog is unknown. Two cases may occur. If $t(x)$ does not attain its supremum, from Property 5 in the proof of Theorem 2, we have $\eta_c = \eta_0 = C$ (e.g., this happens in the model developed in [6] for mobile users with pairwise transmissions). In this case no throughput improvement can be achieved by varying the retransmission probability, and therefore it is enough to restrict attention to the open-loop strategy studied in [8]. On the other

TABLE I
OPEN-LOOP AND CLOSED-LOOP THROUGHPUTS FOR SEVERAL MULTIPACKET CHANNELS

	C_n	$\eta_0 = \lim_{n \to \infty} C_n$	$\eta_c = \sup_{A>0} e^{-A} \sum_{n=1}^{\infty} C_n \frac{A^n}{n!}$
conventional collision channel	$1 \quad n = 1$; $0 \quad n > 1$	0	e^{-1}
q-frequency frequency hopping [6]	$n(1 - \frac{1}{q})^{n-1}$	0	$q\ e^{-1}$
mobile users with pairwise transmission [6]	1	1	1
capture - power discrimination [6]	$1 \quad n = 1$; $\frac{1}{\beta^2} \quad n > 1$	$\frac{1}{\beta^2}$	$\frac{1}{\beta^2} + (1-\frac{1}{\beta^2}) \exp(-\frac{\beta^2}{\beta^2-1})$
capture - timing discrimination [3]	$1 \quad n = 1$; $(1-Q)^n \quad n > 1$	0	$\max_{A>0} \{ (AQ-1) e^{-A} + e^{-AQ} \}$

hand, if there exists A, $0 < A < +\infty$, such that $t(A) = \sup_{x \geq 0} t(x)$, then we have shown in the proof of Theorem 2 that the control $p_t = A/X_t$ for $X_t \geq A$ yields a maximum stable throughput $t(A) = \eta_c$, meaning that the system is optimal. Hence, only A has to be computed, and this can be done before starting the operation of the system.

Although in most practical applications $(C_n)_n \geq 1$ does have a limit, it is worth noticing that Theorem 1 can be generalized to the case where C does not exist. It can be shown [9] that if the drift is minimized at each step, then the system is stable for $\lambda < \sup_{x \geq 0} t(x)$ and unstable for $\lambda > \sup_{x \geq 0} t(x) + \lim_{n \to \infty} \sup C_n - \lim_{n \to \infty} \inf C_n$. As in the open-loop system when $(C_n)_{n \geq 1}$ does not have a limit, nothing more can be said about the throughput without further information on the sequence $(C_n)_{n \geq 1}$. But the main drawback in such a case is that there may not exist any control $p_n = A/X_n$ that yields the optimal throughput.

The access rule for new packets that we have been considering so far is usually referred to as delayed first transmission (DFT). With this access rule, newly arrived packets are treated exactly in the same way as backlogged packets. Let us now examine what happens when on the contrary an immediate first transmission (IFT) rule is used, that is when new packets are transmitted with probability one in the slot immediately following their arrival. It has been proved in [8] that the open-loop throughput is the same for both first transmission rules. The closed-loop throughput on the other hand depends on the access rule. For instance, it is well known [4] that for the usual collision channel in the IFT case, the optimal retransmission probability is $p_n^* = \lambda_0 - \lambda_1/\lambda_0 n - \lambda_1$, yielding an optimal throughput $\lambda_0 e^{\lambda_1/\lambda_0} e^{-1}$, in contrast to the throughput $\eta_c = e^{-1}$ for the DFT case. In the multipacket channel with the IFT rule, the optimal throughput depends not only on the mean but on the whole distribution of new packet arrivals. Interestingly enough, it can be proved that both throughputs coincide when the new packet arrivals are Poisson distributed. Still with the same method as in the proof of Theorem 1, it can be easily shown that there exists a retransmission probability that minimizes the drift d_n at state n. With such a retransmission probability, the system with IFT rule is stable for $\lambda < \sup_{x \geq 0} T(x)$ and unstable for $\lambda > \sup_{x \geq 0} T(x)$, with $T(x) = e^{-x} \sum_{n=0}^{\infty} x^n/n! \sum_{j=0}^{\infty} \lambda_j C_{n+j}$, where we have defined $C_0 = 0$ for notational convenience. It can also be proved that a control of the form $p_n = A/X_n$ yields a maximum stable throughput $T(A)$. Since $\sup_{x \geq 0} T(x)$ depends on the whole new packet arrival distribution $(\lambda_n)_{n \geq 0}$, this result is not as conclusive as in the DFT case. This is because the stability region $\lambda < \sup_{x \geq 0} T(x)$ is actually given in the form of an implicit equation in λ, which cannot be solved in general without further specifications on the distribution $(\lambda_n)_{n \geq 0}$. For instance, this stability region could be empty. Consider, for example, the usual collision channel with possibly some added noise $0 < C_1 \leq 1$ and $C_n = 0$ for $n \geq 2$. Then $T(x) = C_1 e^{-x}(\lambda_1 + \lambda_0 x)$ and $T'(x) = C_1 e^{-x}(\lambda_0 - \lambda_1 - \lambda_0 x)$. Therefore, for any distribution such that $\lambda_0 < \lambda_1$, $T(x)$ is maximum at $T(0) + C_1\lambda_1$, and the stability region is empty since $C_1\lambda_1 \leq \lambda_1 \leq \lambda$. Note that in this sense, the immediate first transmission does not perform as well as the delayed first transmission with which the system can always be stabilized.

If there are solutions to $\lambda < \sup_{x \geq 0} T(x)$, then the best throughput achievable by the class of algorithms in (1) is $\nu_c = \sup \{\lambda : \lambda < \sup_{x \geq 0} T(x)\}$. This is what happens, for instance, when the new packet arrivals are Poisson distributed.

Theorem 3: If the new packet arrivals are Poisson distributed, the best throughput achievable with an IFT rule is the same as in the DFT case, $\nu_c = \sup_{x \geq 0} t(x)$.

Proof of Theorem 3: If $\lim_{n \to \infty} C_n = +\infty$, then $\eta_c = \nu_c = +\infty$. Assume now that $C < +\infty$. We get

$$T(x) = e^{-(x+\lambda)} \sum_{n=0}^{\infty} \frac{x^n}{n!} \sum_{k=0}^{\infty} \sum_{k!}^{\lambda^k} C_{n+k}$$

$$= e^{-(x+\lambda)} \sum_{n=1}^{\infty} \frac{C_n}{n!} (x+\lambda)^n. \tag{13}$$

Thus, in this case, $T(x)$ depends only on λ, and to clarify the proof below, we denote it by $T_\lambda(x)$

$$T_\lambda(x) = t(x+\lambda). \tag{14}$$

Assume that $t(x)$ does not achieve its supremum. Then from Property 5 in the proof of Theorem 2, we have $\eta_c = C = \lim_{x \to \infty} t(x)$. It follows from (14) that for any $\lambda > 0$, $\lim_{x \to \infty} T_\lambda(x) = C$. Therefore, for all $\lambda > 0$, $\sup_{x \geq 0} T_\lambda(x) \geq C$. Hence, for all $\lambda > 0$, $\sup_{x \geq 0} T_\lambda(x) = \sup_{x \geq 0} t(x)$, and by definition of ν_c, we finally get $\nu_c = \sup_{x \geq 0} t(x)$. Note that T_λ does not achieve its supremum, in the sense that if there existed $\lambda \in (0, \nu_c)$ and $x_\lambda \geq 0$ such that $\nu_c = T_\lambda(x_\lambda)$, we would have $\sup_{x \geq 0} t(x) = t(\lambda + x_\lambda)$.

Assume now that $t(x)$ does achieve its supremum: there exists $x_0 \geq 0$ such that $\sup_{x \geq 0} t(x) = t(x_0)$. Then for all λ in $[0, x_0]$: $T_\lambda(x_0 - \lambda) = \sup_{x \geq 0} t(x) \geq \sup_{x \geq 0} T_\lambda(x)$. Thus, for all $\lambda \in [0, x_0]$

$$\sup_{x \geq 0} T_\lambda(x) = \sup_{x \geq 0} t(x) = T_\lambda(x_0 - \lambda). \tag{15}$$

We have for all $x \geq 0$ $t(x) \leq x$, therefore $\sup_{x \geq 0} t(x) \leq x_0$. Together with (15), it follows that for all $\lambda \in (0, \sup_{x \geq 0} t(x))$, $\lambda < \sup_{x \geq 0} T_\lambda(x)$, and therefore $\nu_c \geq \sup_{x \geq 0} t(x) = \eta_c$. Since from (14) $\sup_{x \geq 0} T_\lambda(x) \leq \sup_{x \geq 0} t(x) = \eta_c$ for all λ, we get $\nu_c \leq \eta_c$ and finally $\nu_c = \eta_c = \sup_{x \geq 0} t(x)$. Note that from (14), T_λ reaches its supremum too, since for all $\lambda < \nu_c$, there exists $x_\lambda \geq 0$ such that $T_\lambda(x_\lambda) = \nu_c$.

Note that we have also shown in this proof that $T(x)$ reaches its supremum iff $t(x)$ does, which means that η_c can be achieved with a control of the form $p_n = A/X_n$ iff ν_c can. □

III. OPTIMAL CONTROL FOR THE MULTIPACKET CHANNEL

It is assumed from now on that the users do not have access to the value of the backlog, so the problem becomes one of control of the Markov chain with partial state information provided by the channel feedback. We build a backlog estimate S_t with feedback which is such that $Z_t = 0$ if slot t was empty, and $Z_t = \bar{0}$ otherwise. The results of the previous section strongly suggest that we should use as a retransmission probability $p_t = A/S_t$, where A is a point at which $t(x)$ achieves its supremum (according to Property 5, A is assumed to be finite). We show that the resulting control algorithm achieves the optimal maximum stable throughput η_c. This holds provided that the following assumption on the reception matrix is verified.

C0: There exists $\theta > 0$ and B such that for all $n \geq 1$, $\sum_{k=1}^{n} e^{\theta k} \epsilon_{nk} \leq B$.

The purpose of condition C0 is to bound the probability of having large numbers of simultaneous successes. Unbounded numbers of successes per slot are difficult to deal with because they may result in very large instantaneous errors in the backlog estimate. Note that condition C0 is likely to hold in most multipoint-to-point channels because of practical limitations on the receiver capabilities, and that it is verified for all the examples in Table I.

Theorem 4: Assume that there exists $A \in (0, +\infty)$ such that $t(A) = \sup_{x \geq 0} t(x)$, that the new packet arrivals $(A_t)_{t \geq 0}$ are exponential type[1], and that condition C0 holds. If $\alpha < 0$ and $\beta < 0$ verify the following two conditions[2]:

C1: $\beta > \lambda$

C2: $\beta(1 - e^{-A}) + \eta_c - \lambda + \alpha e^{-A} = 0$

then the control algorithm (cf. the control laws proposed in [15], [19], and [25])

$$p_t = \frac{A}{S_t}$$

$$S_{t+1} = \max\{A, S_t + \alpha I(Z_t = 0) + \beta I(Z_t = \bar{0})\}$$

has maximum stable throughput equal to η_c.

Proof of Theorem 4: The proof is based on the method developed in [30]. The idea is to use the properties of the homogeneous two-dimensional vector Markov chain of the backlog and its estimate $M_t = (X_t, S_t)$ to build a Lyapunov function whose drift is negative in the first quadrant of the (n, s) plane when $\lambda < \eta_c$. It turns out that this fails to hold in two cones of the state space, but it can be proved that the J-step drift of the Lyapunov function is negative for some integer J, and that this is enough to ensure that M_t is geometrically ergodic. It follows from Theorem 2 that M_t is nonergodic if $\lambda > \eta_c$. For substantial portions of the proof, the reader is referred to [9] because of space limitations.

Denote by $\tilde{X}_t = S_t - X_t$ the error in the backlog estimate. The first part of the proof mainly consists of computing and approximating the drifts of X_t and $\tilde{X}_t$ which are the basic building blocks for the Lyapunov function.

Denote by $c(n, s) = E[X_{t+1} - X_t | M_t = (n, s)]$ the backlog drift at state (n, s), and by $d(n, s) = E[\tilde{X}_{t+1} - \tilde{X}_t | M_t = (n, s)]$ the drift of the backlog error. For technical reasons, what we most often use in the proof are the truncated drifts, which correspond to the value of the drifts restricted to those paths where the variation in the backlog is bounded by some integer J, that is $c(n, s, J) = E[(X_{t+1} - X_t)I(|X_{t+1} - X_t| \leq J)|M_t = (n, s)]$ and $d(n, s, J) = E[(\tilde{X}_{t+1} - \tilde{X}_t)I(|X_{t+1} - X_t| \leq J)|M_t = (n, s)]$. Clearly, these truncated drifts will be good approximations of $c(n, s)$ and $d(n, s)$, respectively, when J is large. It will turn out that the drifts depend primarily on the ratio $x = n/s$ for large values of n or s. Thus, it is convenient to define the following two regions in the (n, s) plane:

$$C(\lambda_0, \lambda_1) = \{(n, s) : n \geq 0, s \geq 0, 1 + \lambda_0 \leq \frac{n}{s} \leq 1 + \lambda_1\}$$

$$U_M = \{(n, s) : n \geq M \text{ or } s \geq M\}$$

where λ_0 and λ_1 are such that $-\infty \leq \lambda_0 \leq \lambda_1 \leq +\infty$. The aim of the first part of the proof is to show Proposition 1 below which summarizes all the properties of the drifts that are needed for our purposes (see Fig. 1).

[1] A_t is exponential type if there exists $d > 0$ such that $E[e^{dA_t}]$ is finite. For instance, this is true if A_t is Poisson distributed.

[2] Conditions C1 and C2 define half a straight line in the plane, and therefore an infinite number of possible estimation schemes, all of them yielding the same throughput.

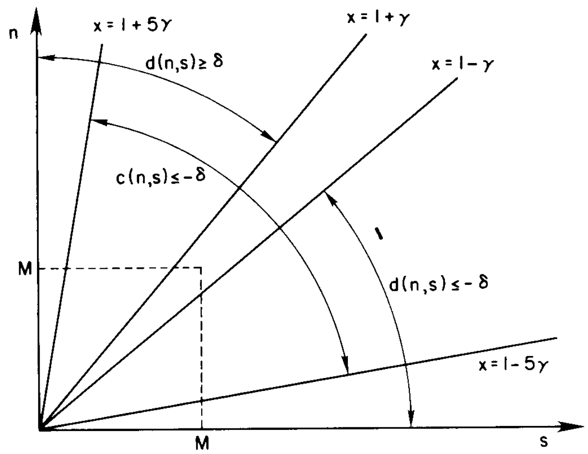


Fig. 1. Drift properties (Proposition 1).

Proposition 1: There exist $\gamma \in (0, 1/5)$, $\delta > 0$, and an integer $J_0 > 0$ such that for all $J \geq J_0$:

i) for all $(n, s) \in C(-5\gamma, 5\gamma) \cap U_M$, $c(n, s) \leq -\delta$ and $c(n, s, J) \leq -\delta + \nu(J)$;

ii) for all $(n, s) \in C(-\infty, -\gamma) \cap U_M$, $d(n, s) \leq -\delta$ and $d(n, s, J) \leq -\delta + \nu(J)$;

iii) for all $(n, s) \in C(\gamma, +\infty) \cap U_M$, $d(n, s) \geq \delta$ and $d(n, s, J) \geq \delta - \nu(J)$

where $\nu(J)$ is a nonnegative function which goes to zero a J goes to infinity.

The detailed proof of Proposition 1 can be found in [9]. After computing the value of the drifts

$$c(0, s) = \lambda \tag{16a}$$

$$c(n, s) = \lambda - \sum_{j=1}^{n} \binom{n}{j} \left(\frac{A}{s}\right)^j \left(1 - \frac{A}{s}\right)^{n-j} C_j \quad (n \geq 1) \tag{16b}$$

$$d(0, s) = \max\{A - s, \alpha\} - \lambda \tag{17a}$$

$$d(n, s) = \beta - \lambda + (\max\{A - s, \alpha\} - \beta)\left(1 - \frac{A}{s}\right)^n + \sum_{j=1}^{n} \binom{n}{j} \left(\frac{A}{s}\right)^j \left(1 - \frac{A}{s}\right)^{n-j} C_j \quad (n \geq 1) \tag{17b}$$

we work out upper and lower bounds by truncating the sums (16) and (17) to a fixed number of terms, and then we approximate those bounds as a function of the sole variable n/s. The main idea is that the dynamic behavior of the Markov vector $M_t = (X_t, S_t)$ depends essentially on the ratio X_t/S_t. For instance, if x is nearly equal to 1, the backlog estimate is close to its ideal value, and we should have $c(n, s) < 0$ since the backlog drift is negative in the perfect state information case. Also, a well-behaved estimate should be such that if $x < 1$, then the error $s - n$ is positive, and therefore should have a negative drift $d(n, s) < 0$ (see [15]). In the same way, we expect to have $d(n, s) > 0$ for $x > 1$.

Let us define the following Lyapunov function:

$$V(n, s) = \max\left\{n, \frac{1 + 3\gamma}{3\gamma}(n - s), \frac{1 - 3\gamma}{3\gamma}(s - n)\right\}$$

where the constants have been chosen so that V is continuous. $V(n, s)$ is equal to the first, second, and third term inside the bracket when (n, s) is in $C(-3\gamma, 3\gamma)$, $C(3\gamma, +\infty)$, and $C(-\infty, -3\gamma)$, respectively. Notice that V is defined so as to take the best advantage of the drift properties listed in Proposition 1. For instance, when $V(n, s)$ is equal to n, then the Markov chain M_t belongs to $C(-3\gamma, 3\gamma)$ which is included in $C(-5\gamma, 5\gamma)$ where the backlog drift is negative provided that either n or s is sufficiently large. Similar comments can be made about the other two regions. Unfortunately, this does not enable us to conclude that the drift of the Lyapunov function is negative in U_M because M_{t+1} may well be in a different region than M_t. However, this change of region becomes unlikely if we exclude a small zone around the lines $x = 1 \pm 3\gamma$ where V changes definition and indeed the second part of this proof consists of showing that the Lyapunov function has a negative drift in the remainder of the state space.

Proposition 2: There exist $M_0 \geq 0$ and $\delta_0 > 0$ such that for all $N \geq M_0$ and for all $(n, s) \in U_N \cap [C(-\infty, -4\gamma) \cup C(-2\gamma, 2\gamma) \cup C(4\gamma, \infty)]$,

$$E[V(M_{t+1}) - V(M_t) | M_t = (n, s)] < -\delta_0.$$

Proof of Proposition 2: We consider separately likely and unlikely events

$$\begin{aligned} E[V(M_{t+1}) - V(M_t) | M_t = (n, s)] &= E[(V(M_{t+1}) - V(M_t)) I(|A_t - \Sigma_t| \leq J) | M_t = (n, s)] \\ &+ E[(V(M_{t+1}) - V(M_t)) I(|A_t - \Sigma_t| > J) | M_t = (n, s)]. \end{aligned} \quad (18)$$

We start by showing that the first term, which corresponds to likely events, is negative when J is large by using the properties of the truncated drifts from Proposition 1 and a simple geometric result. The lemma below, whose proof is in [9], gives a measure of how much a cone $C(\lambda_0, \lambda_1)$ expands if each of its points is allowed to move of some distance that cannot exceed B in absolute value along each axis.

Lemma: Consider $\gamma > 0$, $B > 0$, and $\gamma - 1 < \lambda_0 < \lambda_1 < +\infty$; and assume that $|n - n'| \leq B$, $|s - s'| \leq B$, and $Q \geq B/\gamma (1 + |\lambda_1|)(\lambda_1 + 2 + \gamma)$. Then:

1) $(n, s) \in C(\lambda_0, \infty) \cap U_Q$
$\Rightarrow (n', s') \in C(\lambda_0 - \gamma, \infty) \cap U_{Q-B}$

2) $(n, s) \in C(-\infty, \lambda_1) \cap U_Q$
$\Rightarrow (n', s') \in C(-\infty, \lambda_1 + \gamma) \cap U_{Q-B}$

3) $(n, s) \in C(\lambda_0, \lambda_1) \cap U_Q$
$\Rightarrow (n', s') \in C(\lambda_0 - \gamma, \lambda_1 + \gamma) \cap U_{Q-B}$.

Set $B(J) = \max\{J, |\alpha| + \beta\}$, and define $Q(J)$ to be any real such that $Q(J) \geq \max\{B(J) + M, B(J)/\gamma(1 + 4\gamma)(2 + 3\gamma)\}$. We have $|S_{t+1} - S_t| \leq |\alpha| + \beta \leq B(J)$, and if $|A_t - \Sigma_t| \leq J$, then $|X_{t+1} - X_t| \leq J \leq B(J)$. From the lemma, $Q(J)$ is such that

$$M_t \in C(-2\gamma, 2\gamma) \cap U_{Q(J)} \Rightarrow M_{t+1} \in C(-3\gamma, 3\gamma) \cap U_M \quad (19)$$

$$M_t \in C(4\gamma, \infty) \cap U_{Q(J)} \Rightarrow M_{t+1} \in C(3\gamma, \infty) \cap U_M \quad (20)$$

$$M_t \in C(-\infty, -4\gamma) \cap U_{Q(J)} \Rightarrow M_{t+1} \in C(-\infty, -3\gamma) \cap U_M \quad (21)$$

where M has been defined in Proposition 1. Assume, for instance that M_t belongs to $C(-2\gamma, 2\gamma) \cap U_{Q(J)}$. From (19), $M_{t+1} \in C(-3\gamma, 3\gamma) \cap U_M \cap C(-5\gamma, 5\gamma) \cap U_M$. Hence, if $J \geq J_0$, we can apply Proposition 1 i):

$$E[(V(M_{t+1}) - V(M_t)) I(|A_t - \Sigma_t| \leq J) | M_t = (n, s)] = c(n, s, J) \leq -\delta + \nu(J).$$

If M_t belongs to the other two regions, $C(4\gamma, \infty) \cap U_{Q(J)}$ or $C(-\infty, -4\gamma) \cap U_{Q(J)}$, a similar argument holds, using Proposition 1 iii) and ii), respectively, along with (20) and (21). It follows that for all $J \geq J_0$ and for all $(n, s) \in U_{Q(J)} \cap [C(-\infty, -4\gamma) \cup C(-2\gamma, 2\gamma) \cup C(4\gamma, \infty)]$

$$E[(V(M_{t+1}) - V(M_t)) I(|A_t - \Sigma_t| \leq J) | M_t = (n, s)] \leq -\delta_1 + \nu_1(J) \quad (22)$$

with $\delta_1 = \min\{1, 1 - 3\gamma/3\gamma\}\delta$ and $\nu_1(J) = \nu(J)\, 1 + 3\gamma/3\gamma$.

To deal with the second term on the right-hand side of (18), we consider the further decomposition

$$\begin{aligned} E[(V(M_{t+1}) - V(M_t)) I(|A_t - \Sigma_t| > J) | M_t = (n, s)] &= E[(V(M_{t+1}) - V(M_t)) I(|A_t > \Sigma_t + J) | M_t = (n, s)] \\ &+ E[(V(M_{t+1}) - V(M_t)) I(\Sigma_t > A_t + J) | M_t = (n, s)]. \end{aligned} \quad (23)$$

Let us denote by $T_1(n, s, J)$ and $T_2(n, s, J)$ the two terms on the right-hand side of (23). The first term $T_1(n, s, J)$ corresponds to a case where the variation in the backlog is bounded below, and can be shown to vanish as J increases by using the sole fact that the mean arrival rate λ is finite. Consider now $T_2(n, s, J)$. If $M_t = (n, s)$ belongs to a region such that $x = n/s > x_0$, then x_0 can be chosen large enough so that if M_{t+1} belongs to $C(-\infty, -3\gamma)$, then the error in the backlog estimate which results from the large number of successes just compensates the initial error $n - s \gg 0$. On the other hand, when M_t belongs to any region such that x is bounded above, then $E[\Sigma_t I(\Sigma_t > J) | M_t = (n, s)]$ goes to zero uniformly in (n, s) and $T_2(n, s, J)$ can be dealt with by using the following rather crude bound for the variation of V:

$$|V(M_{t+1}) - V(M_t)| \leq \max\left\{1, \frac{1+3\gamma}{3\gamma}, \frac{1-3\gamma}{3\gamma}\right\} \cdot (|\alpha| + \beta + |A_t - \Sigma_t|) \leq R(1 + |A_t - \Sigma_t|) \quad (24)$$

where R is some positive constant. It is shown in [9] that

$$E[(V(M_{t+1}) - V(M_t)) I(|A_t - \Sigma_t| > J) | M_t = (n, s)] \leq \nu_2(J) + \epsilon_J(n, s) \quad (25)$$

where $\lim_{J \to \infty} \nu_2(J) = 0$, and $\epsilon_J(n, s)$ is a nonnegative function that depends on J, and goes to zero as either n or s goes to infinity.

By using (22), (25), and the decomposition (18), we get the desired result that the drift of V is negative in this part of the state space: fix an integer $J_{\min}$ such that $J_{\min} \geq J_0$ and that for all $J \geq J_{\min}$, $\nu_1(J) + \nu_2(J) \leq \delta_1/3$. Then from (22) and (25), we have for all $(n, s) \in U_{Q(J_{\min})} \cap [C(-\infty, -4\gamma) \cup C(-2\gamma, 2\gamma) \cup C(4\gamma, \infty)]$,

$$E[V(M_{t+1}) - V(M_t) | M_t = (n, s)] \leq -\frac{2}{3}\delta_1 + \epsilon_{J_{\min}}(n, s).$$

Then we can choose an $M_0 > Q(J_{\min})$ which is large enough so that $\epsilon_{J_{\min}}(n, s) < \delta_1/3$ for all (n, s) in U_{M_0}. □

This concludes the second part of the proof. Unfortunately, it is not always true that the drift of V is negative outside a finite subset of the state space. For instance, we have proved that in the case of the usual collision channel with Poisson new packet arrivals, there exist constants $B_{ex} > 0$ and M_{ex} such that for all $(n, s) \in U_{M_{ex}}$ for which $x = 1 \pm 3\gamma$, and for all α and β verifying C1 and C2, $E[V(M_{t+1}) - V(M_t) | M_t = (n, s)] > B_{ex}$. However, discontinuities around the lines $x = 1 \pm 3\gamma$ cancel out when one waits long enough, and in the last part of this proof we show that the J-step drift of V, $E[V(M_{t+J}) - V(M_t) | M_t = (n, s)]$ is negative for some integer J.

Proposition 3: There exist $J_f > 0$, $\rho > 0$, and $M_f > 0$ such that for all $(n, s) \in U_{M_f}$

$$E[V(M_{t+J_f}) - V(M_t) | M_t = (n, s)] \le -\rho.$$

Proof of Proposition: One of the main problems in dealing with the J-step drift of V is to control the changes of regions between M_t and M_{t+J}. To this end, we define the stopping time

$$\tau_J = \min\left\{ s \ge 0, \left| \sum_{k=0}^{s} (A_{t+k} - \Sigma_{t+k}) \right| > J^3 \right\}.$$

If $\tau_J \ge J$, then for $1 \le k \le J$, $|X_{t+k} - X_t| \le J^3$ and $|S_{t+k} - S_t| \le J(|\alpha| + \beta)$. Thus, if we define $B'(J) = \max\{J(|\alpha| + \beta), J^3\}$, and $Q'(J)$ to be any integer such that $Q'(J) \ge B'(J) + \max\{M_0, M\}$ and $Q'(J) \ge 2B'(J)/\gamma(1 + 9/2\gamma)(5\gamma + 2)$, then, still assuming that $\tau_J \ge J$, we get from the lemma for $0 \le k \le J$

$$M_t \in C\left(-\infty, -4\gamma - \frac{\gamma}{2}\right) \cap U_{Q'(J)} \Rightarrow M_{t+k} \in C(-\infty, -4\gamma) \cap U_{M_0} \quad (26)$$

$$M_t \in C\left(-2\gamma + \frac{\gamma}{2}, 2\gamma - \frac{\gamma}{2}\right) \cap U_{Q'(J)} \Rightarrow M_{t+k} \in C(-2\gamma, 2\gamma) \cap U_{M_0} \quad (27)$$

$$M_t \in C\left(4\gamma + \frac{\gamma}{2}, \infty\right) \cap U_{Q'(J)} \Rightarrow M_{t+k} \in C(4\gamma, \infty) \cap U_{M_0} \quad (28)$$

$$M_t \in C\left(-4\gamma - \frac{\gamma}{2}, -2\gamma + \frac{\gamma}{2}\right) \cap U_{Q'(J)} \Rightarrow M_{t+k} \in C(-5\gamma, -\gamma) \cap U_M \quad (29)$$

$$M_t \in C\left(2\gamma - \frac{\gamma}{2}, 4\gamma + \frac{\gamma}{2}\right) \cap U_{Q'(J)} \Rightarrow M_{t+k} \in C(\gamma, 5\gamma) \cap U_M. \quad (30)$$

In other words, we have partitioned the plane into two zones

$$Z_N = C\left(-\infty, -4\gamma - \frac{\gamma}{2}\right) \cup C\left(-2\gamma + \frac{\gamma}{2}, 2\gamma - \frac{\gamma}{2}\right) \cup C\left(4\gamma + \frac{\gamma}{2}, \infty\right),$$

and

$$Z_P = C\left(-4\gamma - \frac{\gamma}{2}, -2\gamma + \frac{\gamma}{2}\right) \cup C\left(2\gamma - \frac{\gamma}{2}, 4\gamma + \frac{\gamma}{2}\right).$$

Then we have chosen $Q'(J)$ such that if M_t belongs to Z_N which is slightly smaller than the region in which the drift of the Lyapunov function is negative, and if $\tau_J \ge J$, then the Markov chain remains in the region in which Proposition 2 applies up to time $t + J$ (see (26)–(28) and Fig. 2). $Q'(J)$ is also such that if M_t is in Z_P and if $\tau_J \ge J$, then up to time $t + J$ the chain stays in a region such that two out of the three properties of Proposition 1 hold at each step (see (29), (30), and Fig. 3).

We start by showing that the J-step drift of V is negative at (n, s) when (n, s) belongs to Z_N. We decompose the J-step drift of V as follows:

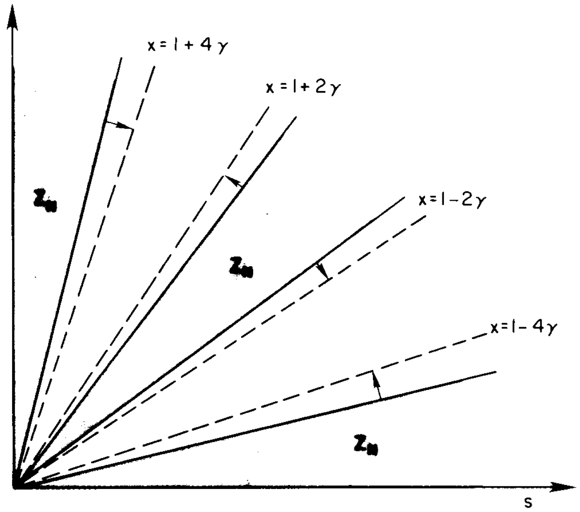


Fig. 2. If $M_t \in Z_N \cap U_{Q'(J)}$ and if $\tau_J \ge J$, then M_{t+1} belongs to the region where the drift of V is negative.

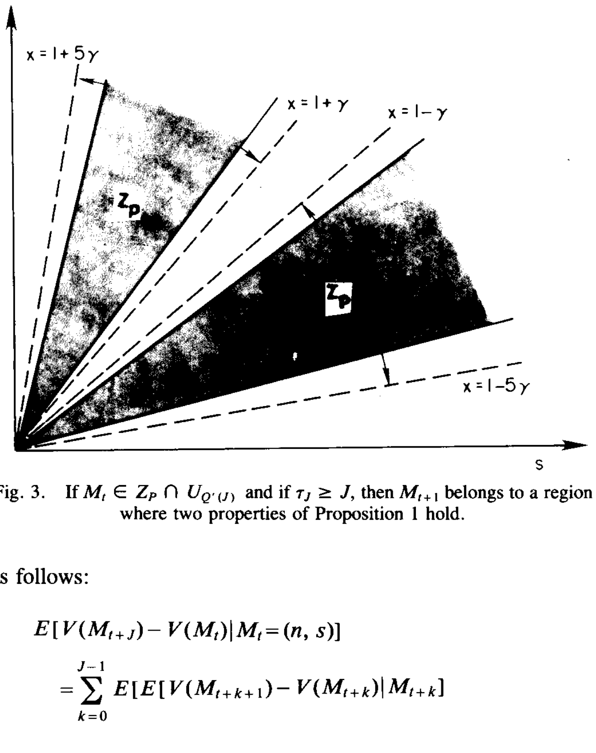


Fig. 3. If $M_t \in Z_P \cap U_{Q'(J)}$ and if $\tau_J \ge J$, then M_{t+1} belongs to a region where two properties of Proposition 1 hold.

$$\begin{aligned} E[V(M_{t+J}) - V(M_t) | M_t = (n, s)] &= \sum_{k=0}^{J-1} E[E[V(M_{t+k+1}) - V(M_{t+k}) | M_{t+k}] \\ &\cdot I(\tau_J \ge J) | M_t = (n, s)] + \sum_{k=0}^{J-1} E[E[V(M_{t+k+1}) \\ &- V(M_{t+k}) | M_{t+k}] I(\tau_J < J) | M_t = (n, s)]. \end{aligned} \quad (31)$$

Denote by $U_1(J, n, s)$ and $U_2(J, n, s)$ the two sums on the right-hand side of (31). If $\tau_J \ge J$, then (26)–(28) hold, and therefore we can apply Proposition 2

$$U_1(J, n, s) \le -J\delta_0 P[\tau_J \ge J | M_t = (n, s)]. \quad (32)$$

Let us now show that $\tau_J < J$ is indeed an unlikely event, the probability of which goes to zero as $1/J$ uniformly in (n, s)

$$P[\tau_J<J\,|\,M_t=(n,s)] \le \sum_{k=0}^{J-1} P\left[\left|\sum_{l=0}^{k}(A_{t+l}-\Sigma_{t+l})\right|>J^3\,|\,M_t=(n,s)\right] \le \sum_{k=0}^{J-1} P\left[\sum_{l=0}^{k}A_{t+l}>J^3\right] + \sum_{k=0}^{J-1} P\left[\sum_{l=0}^{k}\Sigma_{t+l}>J^3\,|\,M_t=(n,s)\right].$$

From Markov's inequality we have

$$P[\tau_J<J\,|\,M_t=(n,s)]\le \frac{1}{J^3}\sum_{k=0}^{J-1}(k+1)\lambda + \frac{1}{J^3}\sum_{k=0}^{J-1}\sum_{l=0}^{k}E[\Sigma_{t+l}\,|\,M_t=(n,s)].$$

Denoting by B_η an upper bound on the sequence $t_n(p_n^*)$, it follows from Section II that $E[\Sigma_{t+l}|M_t = (n, s)] = E[E[\Sigma_{t+l}|M_{t+1}]|M_t = (n, s)] \le B_\eta$, so we get

$$P[\tau_J<J\,|\,M_t=(n,s)]\le \frac{\lambda+B_\eta}{2}\,\frac{J+1}{J^2}\le\frac{B_\tau}{J} \tag{33}$$

where B_τ is some positive constant. From (24), it is easy to check that the drift of V is bounded by some positive constant B_V, so that

$$U_2(J,n,s)\le JB_V P[\tau_J<J\,|\,M_t=(n,s)]. \tag{34}$$

Considering (31), (32), (33), and (34), we get

$$E[V(M_{t+J})-V(M_t)\,|\,M_t=(n,s)]\le -\delta_0 J+(B_V+\delta_0)B_\tau.$$

Therefore, there exist constants $\mu_1 > 0$ and $J_1 > 0$ such that for all $J \ge J_1$ and for all $(n, s) \in U_{Q'(J)} \cap Z_N$,

$$E[V(M_{t+J})-V(M_t)\,|\,M_t=(n,s)]\le -J\mu_1. \tag{35}$$

We now proceed to show that the J-step drift of the Lyapunov function is negative in the remaining part of the state space Z_P consisting of the two cones around $x = 1 \pm 3\gamma$. This is done in two steps. We first show that the J-step drift of V restricted to likely events $\{\tau_J \ge J\}$ goes to $-\infty$ as J increases, and then we prove that the J-step drift of V restricted to unlikely events $\{\tau_J < J\}$ is bounded above independent of J.

Assume, for instance, that $(n, s) \in C(\gamma - \gamma/2, 4\gamma + \gamma/2) \cap E_{Q'(J)}$. The difficulty here is that V can take two possible values, and therefore Proposition 1 cannot be used directly. If $\tau_J \ge J$, then from (30) $M_{t+k} \in C(\gamma, 5\gamma) \cap U_M$ for $0 \le k \le J$, so that $V(M_{t+k}) = \max\{X_{t+k}, (1 + 3\gamma)/3\gamma(X_{t+k} - S_{t+k})\}$. Therefore,

$$\begin{aligned}
&E[(V(M_{t+J})-V(M_t))I(\tau_J\ge J)\,|\,M_t=(n,s)]\\
&=E\left[\max\left\{X_{t+J},\frac{1+3\gamma}{3\gamma}(X_{t+J}-S_{t+J})\right\}\cdot I(\tau_J\ge J)\,|\,M_t=(n,s)\right]\\
&\quad -E\left[\max\left\{X_t,\frac{1+3\gamma}{3\gamma}(X_t-S_t)\right\}\cdot I(\tau_J\ge J)\,|\,M_t=(n,s)\right]\\
&\le E\left[\max\left\{X_{t+J}-X_t,\frac{1+3\gamma}{3\gamma}(-\tilde X_{t+J}+\tilde X_t)\right\}\cdot I(\tau_J\ge J)\,|\,M_t=(n,s)\right]
\end{aligned}$$

since $\max\{a, b\} - \max\{c, d\} \le \max\{a - c, b - d\}$. Then using the fact that $\max\{a, b\} \le \max\{0, a + f\} + \max\{0, b + f\} - f$ for $f \ge 0$, we get

$$\begin{aligned}
&E[(V(M_{t+J})-V(M_t))I(\tau_J\ge J)\,|\,M_t=(n,s)]\\
&\le E\left[\max\left\{0,X_{t+J}-X_t+\delta_1\frac{J}{2}\right\}\cdot I(\tau_J\ge J)\,|\,M_t=(n,s)\right]\\
&\quad +E\left[\max\left\{0,\frac{1+3\gamma}{3\gamma}(-\tilde X_{t+J}+\tilde X_t)+\delta_1\frac{J}{2}\right\}\cdot I(\tau_J\ge J)\,|\,M_t=(n,s)\right]\\
&\quad -E\left[\delta_1\frac{J}{2}I(\tau_J\ge J)\,|\,M_t=(n,s)\right]
\end{aligned} \tag{36}$$

where $\delta_1 = \min\{1, (1-3\gamma)/3\gamma\}$ has been defined in (22). We show that the first two terms on the right-hand side of (36) are bounded. Since (33) $\lim_{J\to\infty} -\delta_1 J/2P[\tau_J \ge J] = -\infty$, this will be sufficient to prove that $\lim_{J\to\infty} E[(V(M_{t+J}) - V(M_t))I(\tau_J \ge J)|M_t = (n, s)] = -\infty$. Define $W_k = X_{t+k} - X_t + k\gamma_1/2$ and $\tilde F_k = F_{t+k}$; where F_t is the sigma-field generated by $\{A_s, s \le t - 1; X_s, s \le t\}$, representing the history of the process $(M_t)_{t\ge 0}$ up to time t. To prove that the first term in (36) is bounded, we show that there exists $\phi > 0$ such that $(Y_k, \tilde F_k)$ is a supermartingale, with $Y_k = e^{\phi W_k}I(\tau_J \ge k)$. We need to show that $E[Y_{k+1}|\tilde F_k] \le Y_k$, which is equivalent to

$$E[e^{\phi(X_{t+k+1}-X_t+(k+1/2)\delta_1)}I(\tau_J\ge k+1)\,|\,F_{t+k}] \le e^{\phi(X_{t+k}-X_t+(k/2)\delta_1)}I(\tau_J\ge k)$$

since $I(\tau_J \ge k + 1) = I(\tau_J \ge k)I(T_J \ge k + 1)$, and $I(\tau_J \ge k)$ is measurable with respect to F_{t+k}

$$I(\tau_J\ge k)E[e^{\phi(X_{t+k+1}-X_{t+k}+\delta_1/2)}\,|\,F_{t+k}]\le I(\tau_J\ge k). \tag{37}$$

Now if $\tau_J \ge k$, then from (30), $M_{t+k} \in C(\gamma, 5\gamma) \cap U_M$. Lemma 2.2 in [11] states that if X is a random variable such that $|X|$ is stochastically dominated by an exponential type random variable Z, and if the expectation of X is strictly negative, $E|X| < -\epsilon$, then there exist two constants $\eta > 0$ and $\rho < 1$ such that $E[e^{\eta X}] < \rho < 1$. Hence, there exists $\phi > 0$ such that

for all $(n, s) \in C(-5\gamma, 5\gamma) \cap U_M$,

$$E[e^{\phi(X_{t+1}-X_t+\delta/2)}\,|\,M_t=(n,s)]<1 \tag{38a}$$

for all $(n, s) \in C(-\infty, -\gamma) \cap U_M$,

$$E[e^{\phi(\tilde X_{t+1}-\tilde X_t+\delta/2)}\,|\,M_t=(n,s)]<1 \tag{38b}$$

for all $(n, s) \in C(\gamma, \infty) \cap U_M$,

$$E[e^{\phi(-\tilde X_{t+1}+\tilde X_t+\delta/2)}\,|\,M_t=(n,s)]<1. \tag{38c}$$

It follows from (37) and (38a) that $(Y_k, \tilde F_k)$ is a supermartingale. Therefore,

$$E[Y_J\,|\,\tilde F_0]=E[e^{\phi W_J}I(\tau_J\ge J)\,|\,F_t]\le E[Y_0\,|\,\tilde F_0]=1. \tag{39}$$

Finally, considering that $\max\{0, x\} \le 1/\phi\, e^{\phi x}$, it follows from (39) that the first term in (36) is bounded. Using (30) and (38c), it can be shown with the same method that the second term in (36) is also bounded. Thus, threre exists a constant B_T independent of J

such that

$$E[(V(M_{t+J}-V(M_t))I(\tau_J\geq J)|M_t=(n,s)]\leq B_T-\frac{J}{2}\delta_1 P[\tau_J\geq J].$$

The case $(n, s) \in C(-4\gamma - \gamma/2, -2\gamma + \gamma/2) \cap U_{Q'(J)}$ can be dealt with in a similar way, using (38a) and (38b). Therefore, we have shown that there exist $\mu_2 > 0$ and $J_2 > 0$ such that for all $J \geq J_2$ and for all $(n, s) \in Y_{Q'(J)} \cap Z_P$

$$E[(V(M_{t+J})-V(M_t))I(\tau_J\geq J)|M_t=(n,s)]<-J\mu_2. \quad (40)$$

It is shown in [9] that there exist a constant $B > 0$, a function $\nu_3(J)$ with $\lim_{J\to\infty} \nu_3(J)$, and a nonnegative function $\nu_J(M)$ depending on J verifying $\lim_{M\to\infty} \nu_J(M) = 0$, such that for all $(n, s) \in U_{Q'(J)+M_1} \cap Z_P$,

$$E[(V(M_{t+J})-V(M_t))I(\tau_J<J)|M_t=(n,s)] < B+\nu_3(J)+\nu_J(M_1). \quad (41)$$

We are now ready to conclude the proof of Proposition 3. From (40) and (41), we have for all $(n, s) \in U_{Q'(J)+M_1} \cap Z_P$, $E[V(M_{t+J}) - V(M_t)|M_t = (n, s)] \leq B - J\mu_2 + \nu_3(J) + \nu_J(M_1)$. Fix an integer $J_f \geq \max\{J_1, J_2\}$ such that for all $J \geq J_f$, $B - J\mu_2 + \upsilon\nu_3(J) < -\mu_2$. Then for all $(n, s) \in U_{Q'(J_f)+M_1} \cap Z_P$, we have $E[V(M_{t+J_f}) - V(M_t)|M_t = (n, s)] \leq -\mu_2 + \nu_{J_f}(M_1)$. On the other hand, we also have from (44), for all $(n, s) \in U_{Q'(J_f)+M_1} \cap Z_P$

$$E[V(M_{t+J_f})-V(M_t)|M_t=(n,s)]\leq -\mu_1 J_f.$$

Now fix M_1 large enough so that $\nu_J(M_1) \leq \mu_2/2$. Then define $M_f = Q'(J_f) + M_1$, and $\rho = \min\{\mu_2/2, J_f\mu_1\}$. □

We can now conclude that $(M_t)_{t\geq 0}$ is geometrically ergodic for $\lambda < \eta_c$ by invoking the following result.

Theorem (Hajek [11]): Let $\{W_t\}$ be a sequence of random variables adapted to an increasing family of σ-fields $\{F_t\}$. Suppose that W_0 is deterministic, that $\{W_t, F_t\}$ is exponential type, and that for some $\epsilon > 0$ and $a > 0$ we have $E[(W_{t+1} - W_t - \epsilon) I(W_t > a) |F_t] \leq 0$ for all $t \geq 0$. Then for each value of W_0 the stopping time $\tau = \min\{t \geq 0;\ W_t \leq a\}$ is exponential type.

Define $W_t = V(M_{tJ_f})$ and $a = M_f \max\{1, (1 + 3\gamma)/3\gamma, (1 - 3\gamma)/3\gamma\}$. If $V(M_t) > a$, then $M_t \in U_{M_f}$. From (24) and C0 $(V(M_t), F_t)$ is exponential type since A_t is. From Proposition 3, we can apply Hajek's result to our system to conclude that $\tau = \min\{t \geq 0,\ V(M_{tJ_f}) \leq a\}$ is exponential type for any initial state. Since $V(M_t) \leq a$ implies that $X_t \leq a$ and $S_t \leq a/(1 - 3\gamma)$, it follows that $\tau' = \min\{t \geq 0,\ X_{tJ_f} \leq a$, and $S_{tJ_j} \leq a/(1 - 3\gamma)\}$ is also exponential type for any initial state, as well as $\tau'' = \min\{t \geq 0, X_t \leq a$, and $S_t \leq a/(1 - 3\gamma)\}$. Hence, it follows from [14] that (X_t, S_t) is geometrically ergodic, concluding the proof of Theorem 4. □

IV. Stability Proof Via Mikhailov's Theorem

Mikhailov [19, Theorem 3] has recently found a powerful sufficient condition to guarantee the stability of a Markov process taking values on $R^+ \times R^+$. This result can be used to weaken the sufficient conditions we imposed in Section III and obtain a much more simple proof of stability. However, the form of stability used by Mikhailov is weaker than the geometric ergodicity used in Section III.

Let M_t be a discrete-time Markov process taking values in $Y \subseteq R^n$, $U(r) = \{x \in R^n: \|x\| \leq r\}$, and $\tau_x(S) = \min\{t \geq 0: M_t \in S | M_0 = x\}$, i.e., $\tau_x(S)$ is the time it takes to reach the set S from x. Then we say that the process M_t is stable if there exist constants c_1 and c_2 such that $E[\tau_x(U(r))] \leq c_1 \|x\| + c_2$ for all $x \in Y$. Using this definition of stability we show the following result which is analogous to Theorem 4.

Theorem 5: Suppose that:

i) the number of new packet arrivals per slot has finite second moment $E[A_t^2] < +\infty$;

ii) there exists $A \in (0, +\infty)$ such that $t(A) = \sup_{x\geq 0} t(x)$;

iii) C0″: there exists $B < +\infty$ such that for all $n \geq 1$, $\Sigma_{k=1}^n k^2\epsilon_{nk} \leq B$.

Fix $\lambda < \eta_c$ and $\xi > 0$ such that $\lambda < t(A\xi)$. Choose $\alpha < 0$ and $\beta > 0$ such that

$$C1':\ \beta(e^{A\xi}-1)=\frac{\lambda-t(A\xi)}{\xi}e^{A\xi}-\alpha$$

$$C_2':\ \beta>m_\xi(\lambda)=\sup_{x>0,x\neq\xi}\frac{\lambda-t(Ax)-xe^{-A(x-\xi)}\dfrac{\lambda-t(A\xi)}{\xi}}{x-xe^{-A(x-\xi)}}.$$

Then the control algorithm

$$p_t=\frac{A}{S_t}$$

$$S_{t+1}=\max\{A,\ S_t+\alpha I(Z_t=0)+\beta I(Z_t=\bar{0})\}$$

is stable.

Proof of Theorem 5: Let us state first Mikhailov's Theorem (cf. [35] for an exposition of this result and its application in the decentralized control of the conventional collision channel).

Theorem (Mikhailov [19]): Let $M_t = (X_t, S_t)$ be a homogeneous Markov process on $R^+ \times R_0^+$, with drifts

$$(c(n,s),e(n,s))=E[M_{t+1}-M_t|M_t=(n,s)].$$

Suppose that:

i) there exists $B < +\infty$ such that for all $(n, s) \in R^+ \times R_0^+$, $E[\|M_{t+1} - M_t\|^2 | M_t = (n, s)] \leq B$;

ii) for all $\psi \in (0, +\infty)$, the drifts $(c(n, n/\psi), e(n, n/\psi))$ converge uniformly in ψ as n goes to infinity to $(c(\psi), e(\psi))$;

iii) the limit drifts $(c(\psi), e(\psi))$ are differentiable on $[0, +\infty)$, with $(e(0), e(0)) = \lim_{s\to\infty} (c(0, s), e(0, s))$;

iv) there exists $\epsilon > 0$ such that if $c(\psi_0) = \psi_0 e(\psi_0)$, then $c(\psi_0) < -\epsilon$.

Then M_t is stable.

Since both the new packet arrivals and the rows of the reception matrix have finite variance, it is easy to check that condition i) in Mikhailov's Theorem holds

$$E[\|M_{t+1}-M_t\|^2|M_t=(n,s)] = E[(X_{t+1}-X_t)^2+(S_{t+1}-S_t)^2|M_t=(n,s)].$$

Now $E[(S_{t+1} - S_t)^2|M_t = (n, s)] \leq \alpha^2 + \beta^2$, and from (2)

$$E[(X_{t+1}-X_t)^2|M_t=(n,s)]\leq E[A_t^2]+E[\Sigma_t^2|M_t=(n,s)].$$

From C0′ the variance of the number of successes is also bounded

$$E[\Sigma_t^2|M_t=(n,s)] = \sum_{k=1}^{n}k^2\sum_{j=k}^{n}\binom{n}{j}\left(\frac{A}{s}\right)^j\left(1-\frac{A}{s}\right)^{n-j}\epsilon_{jk}\leq B.$$

It follows directly from (16) and (17) that the limit drifts are given by

$$c(\psi)=\lambda-t(A\psi)$$

$$e(\psi)=\beta+(\alpha-\beta)e^{-A\psi},$$

respectively, for $\psi \in [0, +\infty)$. Uniform convergence to the limit drifts follows immediately from the results given for the perfect state information case (Property 4). Also it is clear the $t(x)$ is differentiable (see (6), where $0 \leq C_n \leq n$). Therefore, properties ii) and iii) in Mikhailov's Theorem are satisfied.

In order to check property iv) note that if $\psi_0 = \xi$, then it follows from C1′ that

$$c(\psi_0) = \psi_0 e(\psi_0).$$

But, at that point, $c(\psi_0) < 0$ because of the choice of ξ. There is no other root of the equation $c(\psi) = \psi e(\psi)$, and, therefore, property v) follows. To see this, note that because of C1′, $c(\psi) = \psi e(\psi)$ for $\psi \neq \xi$ is equivalent to

$$\beta = \frac{\frac{\lambda - t(A\psi)}{\psi} - e^{A(\xi-\psi)} \frac{\lambda - t(A\xi)}{\xi}}{1 - e^{A(\xi-\psi)}}$$

which is impossible if $\psi \neq \xi$ because of C2′. □

It can be shown [9] that $m_\xi(\lambda)$ is finite for all nonnegative λ and ξ, and therefore the set of control laws defined by C1′ and C2′ is nonempty. Actually, the set of control laws in Theorem 4 is a subset of those in Theorem 5 because in Theorem 5 we can choose $\xi = 1$, in which case C2 is equivalent to C1′ and C1 is more restrictive than C2′ because $\lambda \geq m_1(\lambda)$ [9]. □

V. Conclusion

In this paper we have investigated the properties of decentralized control algorithms for a random access channel with multipacket reception capability. By using the working hypothesis that the users are aware of the value of the backlog, we have determined the best throughput achievable by any such protocol, as well as a simple way to achieve it. The optimum throughput has been shown to be given by the maximum average number of successes per slot when the number of transmissions, per slot is Poisson distributed. In the imperfect state information case, we have shown that the same throughput achieved in the perfect state information case can be achieved by using in lieu of the true backlog, an estimate of the backlog computed at each station using binary feedback, and we have used this estimate to derive a control scheme which is optimal in the sense that it achieves the optimal throughput determined earlier. This is true provided the reception matrix verifies condition C0, which puts some restrictions on the number of successes per slot. By using Mikhailov's result, C0 can be replaced by the weaker condition C0′. In this case however, geometric ergodicity was not ensured. Note that the feedback empty/nonempty used in Sections III and IV may be less than the available feedback in many practical situations, but no further information is needed: a ternary feedback would not shorten the proof or achieve better throughput.

Finally, let us mention that one can easily modify the proof of Theorem 4 to show that a similar result holds with the IFT access rule. More precisely, under a hypothesis paralleling those of Theorem 4, one can build a control scheme based on a binary feedback empty/nonempty such that the Markov vector (X_t, S_t) is geometrically ergodic for $\lambda < \sup_{x \geq 0} T(x)$. Using Theorem 3, it can be seen that the maximum stable throughput is the same for both access rules when the new packet arrivals are Poisson distributed.

Appendix

Kaplan's Condition

Consider a Markov chain with denumerable state-space D, and one-step transition probability matrix $(P_{xy})_{(x,y) \in D}$. Let $V(x)$ by a Lyapunov function on D. Then the generalized Kaplan's condition holds if there exists a positive constant B such that for all $z \in [0, 1[$ and all $x \in D$

$$z^{V(x)} - \sum_{y \in D} P_{xy} z^{V(y)} \geq -B(1-z).$$

1) One-Dimensional Kaplan's Condition: Consider the model of Section II with a control scheme $p_n = F(X_n)$, and the Lyapunov function $V(x) = x$. To check Kaplan's condition, it is enough from [27] to show that the downward part of the drift $-D(i) = \Sigma_{k=1}^{i} kP_{i,i-k}$ is bounded below. For $i \geq 1$ and $1 \leq k \leq i$ we have

$$P_{i,i-k} = \sum_{n=0}^{i-k} \lambda_n \sum_{j=k+n}^{i} \binom{i}{j} F(i)^j (1-F(i))^{i-j} \epsilon_{j,k+n}.$$

After a change of variable, it follows that

$$D(i) = \sum_{j=1}^{i} \binom{i}{j} F(i)^j (1-F(i))^{i-j} \sum_{n=0}^{j-1} \lambda_n \sum_{k=n+1}^{j} (k-n)\epsilon_{j,k}. \tag{A-1}$$

If $(C_n)_{n \geq 1}$ is bounded, then Kaplan's condition holds independent of the retransmission policy. Denoting by B_c an upper bound for $(C_n)_{n \geq 1}$, (A-1) becomes

$$\begin{aligned} -D(i) &\geq -\sum_{j=1}^{i} \binom{i}{j} F(i)^j (1-F(i))^{i-j} \sum_{n=0}^{j-1} \lambda_n C_j \\ &\geq -\sum_{j=1}^{i} \binom{i}{j} F(i)^j (1-F(i))^{i-j} C_j \geq -B_c. \end{aligned} \tag{A-2}$$

2) Two-Dimensional Kaplan's Condition: Consider now the multipacket channel with a general control algorithm (1). Then (X_t, S_t) is the Markov chain of interest, and the relevant Lyapunov function is $V(n, s) = n$. We prove again that Kaplan's condition holds provided that $(C_n)_{n \geq 1}$ is bounded. From [27], it is enough also in this case to show that the downward part $T(x)$ of the generalized drift is bounded below, with $T(x) = \Sigma_{y/V(y) < V(x)} P_{xy} (V(y) - V(x))$. Given a state $x = (i, s)$, we have

$$\begin{aligned} T(x) &= -\sum_{r=1}^{i} r \sum_k P[X_{n+1} = i-r, S_{n+1} = k \mid X_n = i, S_n = s] \\ &= -\sum_{r=1}^{i} rP[X_{n+1} = i-r \mid x_n = i, S_n = s] \end{aligned}$$

which is, in the same way as before

$$\begin{aligned} T(x) &= -\sum_{r=1}^{i-k} r \sum_{n=0}^{i-r} \lambda_n \sum_{j=r+n}^{i} \binom{i}{j} (F(s))^j (1-F(s))^{i-j} \epsilon_{j,r+n} \\ &= -\sum_{j=1}^{i} \binom{i}{j} F(s)^j (1-F(s))^{i-j} \sum_{n=0}^{j-1} \lambda_n \sum_{r=n+1}^{j} (r-n)\epsilon_{jr} \end{aligned}$$

this expression is similar to (A-1), and the end of the proof is the same as in (A-2).

References

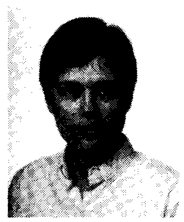
[1] N. Abramson, "The throughput of packet broadcasting channels," *IEEE Trans. Commun.*, vol. COM-25, pp. 117-128, 1977.

[2] J. C. Arnbak and W. van Blitterswijk, "Capacity of slotted Aloha in Rayleigh fading channels," *IEEE J. Select. Areas Commun.*, vol. SAC-5, no. 2, pp. 261-269, 1987.

[3] D. H. Davis and S. A. Gronemeyer, "Performance of slotted Aloha random access with delay capture and randomized time of arrival," *IEEE Trans. Commun.*, vol. COM-28, pp. 703-710, 1980.

[4] G. Fayolle, E. Gelenbe, and J. Labetoulle, "Stability and optimal control of the packet switching broadcast channel," *J. Ass. Comput. Mach.*, vol. 24, no. 3, pp. 375-386, 1977.

[5] F. G. Foster, "On the stochastic matrices associated with certain queuing processes," *Ann. Math. Stat.*, no. 24, pp. 355–360, 1953.
[6] S. Ghez, S. Verdú, and S. C. Schwartz, "Stability of multipacket Aloha," in *Proc. 21st Conf. ISS*, The Johns Hopkins Univ., Baltimore, MD, Mar. 1987.
[7] ——, "On decentralized control algorithms for multipacket Aloha," in *Proc. 25th Allerton Conf. Commun., Contr., Comput.*, Oct. 1987.
[8] ——, "Stability properties of slotted Aloha with multipacket reception capability," *IEEE Trans. Automat. Contr.*, vol. 33, July 1988.
[9] S. Ghez, "Random access communications for the multipacket channel," Ph.D. dissertation, Dept. Elec. Eng., Princeton Univ., 1989.
[10] D. J. Goodman and A. A. M. Saleh, "The near/far effect in local Aloha radio communications," *IEEE Trans. Vehic. Technol.*, vol. VT-36, no. 1, Feb. 1987.
[11] B. Hajek, "Hitting-time and occupation-time bounds implied by drift analysis with applications," *Adv. Appl. Prob.*, vol. 14, pp. 502–525, 1982.
[12] B. Hajek and T. Van Loon, "Decentralized dynamic control of a multiaccess broadcast channel," *IEEE Trans. Automat. Contr.*, vol. AC-27, pp. 559–569, 1982.
[13] M. Kaplan, "A sufficient condition for nonergodicty of a Markov chain," *IEEE Trans. Inform. Theory*, vol. IT-25, pp. 470–471, 1979.
[14] J. G. Kemeny, J. L. Snell, and A. W. Knapp, *Denumerable Markov Chains.* New York: Springer-Verlag, 1976.
[15] F. P. Kelly, "Stochastic models of computer communication systems," *J. R. Statist. Soc. B*, vol. 47, no. 3, pp. 379–395, 1985.
[16] C. C. Lee, "Random signal levels for channel access in packet radio networks," *IEEE J. Select. Areas Commun.*, vol. SAC-5, no. 6, pp. 1026–1034, 1987.
[17] N. Mehravari, "Collision resolution in random access systems with multiple reception," preprint, 1987.
[18] J. J. Metzner, "On improving utilization in Aloha networks," *IEEE Trans. Commun.*, vol. COM-24, pp. 447–448, Apr. 1976.
[19] V. A. Mikhailov, "Geometrical analysis of the stability of Markov chains in R_n^+ and its application to throughput evaluation of the adaptive random access algorithm," *Problemy Peredachi Informatsii*, vol. 24, no. 1, pp. 61–73, Jan.–Mar. 1988; translated in *Problems of Inform. Transmission*, July 1988.
[20] C. Namislo, "Analysis of mobile radio slotted Aloha networks," *IEEE J.Select. Areas Commun.*, vol. SAC-2, pp. 583–588, 1984.
[21] A. G. Pakes, "Some conditions for ergodicity and recurrence of Markov chains," *Operat. Res.*, vol. 17, pp. 1058–1061, 1969.
[22] A. Polydoros and J, Silvester, "Slotted random access spread-spectrum networks: An analytical framework," *IEEE J. Select. Areas Commun.*, vol. SAC-5, no. 6, pp. 989–1002, 1987.
[23] B. Ramamurthi, A. A. M. Saleh, and D. J. Goodman, "Aloha with perfect capture," preprint 1986.
[24] D. Raychaudhuri, "Performance analysis of random access packet-switched code division multiple access systems," *IEEE Trans. Commun.*, vol. COM-29, pp. 895–901, 1981.
[25] R. L. Rivest, "Network control by Bayesan broadcast," *IEEE Trans. Inform. Theory*, vol. IT-33, no. 3, pp. 323–328, 1987.
[26] L. G. Roberts, "Aloha packet system with and withpout slots and capture," *Comput. Commun. Rev.*, no. 5, pp. 28–42, 1975.
[27] L. I. Sennott, "Tests for the nonergodicity of multidimensional Markov chains," *Operat. Res.*, vol. 33, pp. 161–167, 1985.
[28] A. Schwartz and M Sidi, "Erasure capture and noise errors in controlled multiple access networks." in *Proc. 25th Conf. Decision Conf.*, Dec. 1986, pp. 1333–1334.
[29] M. K. Simon, J. K. Omura, R. A. Scholtz, and B. K. Levitt, *Spread-Spectrum Communications.* New York: Computer Science Press, 1985.
[30] J. N. Tsitsiklis, "Analysis of a multiaccess control scheme," *IEEE Trans. Automat. Contr.*, vol. AC-32, pp. 1017–1020, 1987.
[31] B. S. Tsybakov, V. A. Mikhailov, and N. B. Likhanov, "Bounds for packet transmission rate in a random multiple access system," *Problemy Peredachi Informatsii*, vol. 19, no. 1, pp. 61–81, 1983.
[32] R. L. Tweedie, "Sufficient conditions for ergodicity and recurrence of Markov chains on a general state space," *Stoch. Proc. Appl.*, no. 3, pp. 385–403, 1975.
[33] S. Verdú, "Minimum probability of error for asynchronous Gaussian multiple-access channels," *IEEE Trans. Inform. Theory*, vol. IT-32, pp. 85–96, 1986.
[34] J. E. Wieselthier, A. Ephremides, and L. A. Michaels, "An exact analysis and performance evaluation of framed Aloha with capture," preprint, 1987.
[35] A. Ephremides and S. Verdú, "Control and optimization methods in communication network problems," *IEEE Trans. Automat. Contr.*, vol. 34, pp. 930–942, Sept. 1989.



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