

# High-Speed Digital Signal Processing for Satellite Communications

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## Abstract

New digital signal processing (DSP) technology opportunities, wherein signals of large bandwidth can be digitally processed and complex digital signal processing algorithms can be implemented, make the use of new techniques that have heretofore been limited to audio/voice applications now feasible for space applications. Although many efforts are being devoted to the technology aspects of this work, ongoing work by the authors is concerned with the unified consideration of the theoretical and practical aspects of high-speed digital signal processing and their applications in space communications. In this context, this paper comprises a review of some basic high-speed digital signal processing techniques that are applicable in the satellite communications domain. Specifically, this paper treats the subjects of numerically stable algorithms for high-speed DSP, adaptive signal processing techniques for demodulation of multi-access communications, interference suppression in wideband communications, on-board signal processing for demultiplexing and transmultiplexing, and oversampling techniques applied to sigma-delta modulation and related schemes.

## 1 Introduction and Overview

The field of digital technology is dramatically evolving due to the development of higher integration capacities, faster analog-to-digital (A/D) and digital-to-analog (D/A) converters, and other high-speed digital technologies. The potential applications of this new technology have a direct impact on satellite communications. First, they allow one to consider digital solutions for functional blocks that were traditionally analog in existing satellite payload. For instance, quite promising in this context are digital solutions for a significant part of the radio-frequency (RF) front end and their associated functional blocks (routing, downconversion, and filtering). Secondly, new digital processing possibilities arise for future satellite payloads, such as beamforming for example. And finally, complex digital signal processing (DSP) algorithms for small ground stations (VSAT type or portable terminals) can now be implemented, thereby significantly improving ground-terminal performance. A very promising possibility in this context is the use of advanced digital signal processing for mobile communications terminals (e.g., multipath cancellation, fading countermeasures, multiple-access interference suppression, etc.).

These new technology opportunities, wherein signals of large bandwidth can be digitally processed and complex digital signal processing algorithms can be implemented make the use of new techniques that have heretofore been limited to audio/voice applications now feasible for space applications. Although many efforts are being devoted to the technology aspects of this work, the present work is concerned with the unified consideration of the theoretical and practical aspects of high-speed digital signal processing and their applications in space communications.

The purpose of this paper is to provide an overview of key fundamental issues that arise in the use of high-speed DSP technology for the processing of wideband communication signals.

We begin, in Section 2, by focusing on the fundamental issue of general signal processing organization for the development of numerically stable algorithms for high-speed communications applications. In most such applications, it is likely (or even necessary) that implementation will take place at the wide bandwidths of the front end. Since this front-end (i.e., RF) bandwidth is much larger than the information bandwidths underlying the modulated signals, some signal processing difficulties may arise in the implementation of the required processing functions. In particular, it is widely recognized that many conventional discrete-time signal processing formulations become ill-conditioned when applied to continuous-time processes sampled at very fast rates relative to the underlying continuous bandwidth. Recent research has shown that the above-noted difficulties can be overcome by transforming the conventional sampled-data problems, which use a *shift-operator* representation of data dynamics, into problems based on *divided-differenced* representation of data dynamics. Signal processing formulations based on this divided-difference calculus are better conditioned numerically and their solutions are numerically stable for in the high-speed processing regime. Section 2 will provide a review of this methodology, which is a cornerstone of the general techniques of interest in this paper.

In Section 3 and 4 we will provide broad-brush overviews of two specific examples of signal processing challenges that arise in wideband communication systems in general. The framework of this discussion will be paradigmatic, as opposed to being focused on providing a comprehensive list of processing sub-system functionalities. Specifically, we will address briefly two general signal processing problems: demultiplexing/demodulation of wideband non-orthogonal signaling formats (such as code-division multiple-access (CDMA)), and narrowband interference (NBI) suppression. Each of these problems serves as a paradigm for a general class of problems that require high-speed DSP functions. The first problem is an example of a general problem that arises in wideband demultiplexing problems - namely crosstalk, which can arise due to intentional non-orthogonality (as in CDMA) or due to unintentional nonideal effects arising in the implementation of wideband filter banks. The second problem serves as an example of general narrowband adaptive technology, which includes beamforming, among other problems.

Thus, in Section 3, we consider signal processing techniques for demultiplexing/ demodulation of multiuser communications. This problem is a central one in the processing of uplink satellite channels and cellular radio signals. The focus of Section 3 is on *adaptive* techniques for multi-user demultiplexing. Such techniques permit significant performance gains (and the attendant increases in system capacity) over the conventional methods currently in use. However, these methods are signal-processing intensive, and thus they are a natural framework within which high-speed DSP methods can be applied to produce considerable improvements in system performance. For example, the work presented in Section 3 serves as the basis for studying the applicability of high-speed signal processing methodologies for the cancellation of the multiple-access interference and the mitigation of the mobile channel and the multipath degradation effects. The adaptive multiuser framework described in Section 3 is thus a natural one for addressing these issues.

Similarly, in Section 4, we give a brief review of the problem of NBI mitigation in direct-sequence spread-spectrum (DSSS) signaling systems. A chief advantage of using a wideband format such as DSSS in radio networks is that such signals can share bandwidth with narrowband communication signals. In particular, the low spectral density of spread-spectrum signals does not interfere unduly with conventional narrowband communications; and, conversely, spread spectrum communications is inherently resistant to the NBI caused by co-existence with conventional communications. However, it has been demonstrated that the performance of spread-spectrum systems in the presence of narrowband signals can be enhanced significantly through

the use of active NBI suppression prior to despreading, and Section 4 reviews this methodology. As noted above, this area serves as a good example of the general area of narrowband adaptivity, which is of general interest in on-board processing functions such as beamforming and narrowband filtering. These functions are natural ones to implement with high-speed DSP technology.

Section 5 provides a very brief overview of some recent advances in the application of DSP techniques for bulk on-board wideband demultiplexing and transmultiplexing. These problems represent more specific examples of the kind of subsystems to which the methods of Section 2 are applicable. In particular, for such a digital demultiplexers there is an inherent oversampling factor in many parts of the circuitry if a per-block approach is implemented due to the discrepancy between the total bandwidth to be processed and the underlying single-user bandwidth. Thus, the framework of Section 2 can be exploited to devise effective processing architectures for these applications.

In the final section, Section 6, we address the general issue of oversampling techniques. Such techniques rely on the tradeoff between temporal resolution (i.e., sampling speed) and amplitude resolution (i.e., quantization), to achieve greater A/D resolution than that imposed by the matching tolerances of VLSI technologies. Since very high and accurate clock frequencies are straightforward to generate with current technology, this tradeoff is one that can be practically effected. The oversampling that is an *a fortiori* aspect of such methods makes the fundamental issues of Section 2 of particular interest in this framework. In Section 5, this methodology is reviewed in the contexts of  $\Sigma - \Delta$  modulation and data modulation.

## 2 Numerically Stable Algorithms for High-speed DSP

In the application of DSP in wideband space communications, it is likely (or even necessary) that implementation will take place at the wide bandwidths of the front end. Since this front-end (i.e., RF) bandwidth is much larger than the information bandwidths underlying the modulated and interfering signals, some signal processing difficulties may arise in the implementation of the required processing functions. In particular, it is widely recognized that many conventional discrete-time signal processing formulations become ill-conditioned when applied to continuous-time processes sampled at very fast rates relative to the underlying continuous bandwidth. For example, two algorithms in which this phenomenon arises are two of the most widely used signal processing techniques - namely, Kalman-Bucy filtering and autoregressive modeling/prediction [72]. Since these and other algorithms arise frequently in communications signal processing, this issue is of considerable concern in the design of algorithms for DSP in high-bandwidth systems.

In this section, we will focus on the fundamental issue of general signal processing architecture for the development of numerically stable algorithms for high-speed communications applications. In particular, recent research has shown that the above-noted difficulties can be overcome by transforming the conventional sampled-data problems, which use a *shift-operator* representation of data dynamics, into problems based on *divided-differenced* representation of data dynamics. Signal processing formulations based on this divided-difference calculus are better conditioned numerically and their solutions are numerically stable for in the high-speed processing regime. Thus, we provide a review of this methodology, which is a cornerstone of the general techniques of interest in this study.

In the standard formulation of digital signal processing problems, the dynamics of the data and the signal processing elements are described in terms of the classical forward shift operator  $q$ :

$$q\{x_k\} = \{x_{k+1}\}. \quad (2.1)$$

From a practical viewpoint, this operator is undesirable as a means of dynamical representation for rapidly sampled signals, because it becomes essentially an identity operator. This means that finite wordlength implementations of systems represented with the shift operator waste valuable bits of accuracy in representing dynamical behavior that is trivial. That is, in the high-speed regime, the data dynamics are better captured in the *deviation* from the identity, and this is where the numerical accuracy of finite wordlength processors should be concentrated. This objective can be formalized by replacing the shift operator with an incremental difference operator (or *delta operator*) defined by  $\delta = (q - 1)/\Delta$  where  $\Delta$  is the sampling interval. Thus,

$$\delta\{x_k\} = \left\{ \frac{x_{k+1} - x_k}{\Delta} \right\}. \quad (2.2)$$

Note that this operator is a numerical derivative, and as such it is an approximation to the basic dynamical element in continuous time - namely, the forward derivative. Thus, this is a natural framework within which to consider high-speed DSP problems. Of course, since the  $\delta$  and  $q$  operators are related by a one-to-one mapping, signal processing solutions for the  $\delta$  formulation are equivalent to those based on the traditional shift operator  $q$  from a theoretical viewpoint. However, as noted above,  $\delta$ -based algorithms are better conditioned numerically and (unlike shift-operator based algorithms) are numerically stable for high-speed processing envelopes.

The incremental difference operator approach to high-speed sampling problems has been studied for a number of algorithms in digital signal processing, and a survey of this methodology can be found in [27]. (For example, the two particular problems noted above - Kalman-Bucy filtering and autoregressive model fitting and prediction - have been studied in [87] and [116, 114], respectively.) In this section, we will review this methodology, focusing primarily on the specific problem of finite-length linear modeling and prediction. This problem - known as the *Levinson problem* - represents a ubiquitous problem in communications signal processing, and thus it serves well to illustrate the techniques of high-speed processing based on incremental difference operators.

Traditional fast algorithms for fitting finite-length linear models (i.e., autoregressions) to digital signals include the Levinson-Durbin [72] and Schur [125] algorithms. When these algorithms are used to solve the Levinson problem, large computational errors can occur due to the problems noted above. In this case, these problems can be traced to the ill-conditioning of the Toeplitz data covariance matrix (which these algorithms are trying to invert) in this high-speed regime. Unfortunately, most fast computational algorithms for digital signal processing are based specifically on the shift-operator formalism, which gives rise to this same Toeplitz covariance structure. Thus, the main challenge in applying the incremental difference operator formalism is to develop algorithms that are competitive with their shift-operator counterparts in terms of complexity. Fast divided-difference counterparts to each of the algorithms noted above - namely, the Levinson-Durbin algorithm and the Schur algorithm - have been developed, and these will be discussed in the sequel.

### *The Standard Levinson Problem*

The standard Levinson autoregressive modeling problem is concerned with choosing a parameter vector  $\underline{a}_n = [a_{n,0}, a_{n,1}, \dots, a_{n,n}]^T$  with  $a_{n,0} \equiv 1$  to minimize the mean-squared prediction error  $E\{\epsilon_n^2(t)\}$  where  $\epsilon_n(t)$  is the error in the model

$$Y_{t+1} + \sum_{k=1}^n a_{n,k} Y_{t+1-k} = \epsilon_n(t), \quad t \in \mathcal{Z}, \quad (2.3)$$

where  $\{Y_k\}_{k=-\infty}^{\infty}$  is an observed wide-sense-stationary signal. This formulation is the basis for a large number of DSP applications.

As is well known, the coefficients minimizing this mean-squared error are the solutions to the so-called *Yule-Walker equations*:

$$\mathbf{R}_n \underline{a}_n = \begin{pmatrix} \pi_n \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad (2.4)$$

where  $\mathbf{R}_n$  is the  $(n+1) \times (n+1)$  Toeplitz matrix whose  $i, j^{\text{th}}$  element is  $c_{|i-j|}$ , the  $|i-j|$ -lag correlation coefficient of the signal. The *Levinson-Durbin algorithm* is an algorithm for recursively solving the Yule-Walker equations of successively higher order. This algorithm is given by [72]:

$$\underline{a}_{n+1} = \begin{bmatrix} \mathbf{I}_{n+1} \\ 0 \dots 0 \end{bmatrix} \underline{a}_n - \gamma_{n+1} \begin{bmatrix} 0 \dots 0 \\ \mathbf{J}_{n+1} \end{bmatrix} \underline{a}_n, \quad n = 0, 1, 2, \dots, \quad (2.5a)$$

with initialization  $\underline{a}_0 = 1$ , where, for each positive integer  $k$ ,  $\mathbf{I}_k$  denotes the  $k \times k$  identity matrix and  $\mathbf{J}_k$  denotes the  $k \times k$  matrix that has all zero entries except for 1's in its anti-diagonal. The *reflection coefficients*  $\{\gamma_n\}$  are given by

$$\gamma_{n+1} = \alpha_n / \pi_n \quad (2.5b)$$

where

$$\alpha_n = [0, \dots, 0, 1] \mathbf{R}_{n+1} \begin{bmatrix} \underline{a}_n \\ 0 \end{bmatrix}, \quad (2.5c)$$

and

$$\pi_n = E\{\epsilon_n^2\} = [1, 0, \dots, 0] \mathbf{R}_n \underline{a}_n. \quad (2.5d)$$

The mean-squared error sequence  $\{\pi_n\}$  satisfies the recursion

$$\pi_{n+1} = \pi_n - \alpha_n^2 / \pi_n, \quad n = 0, 1, \dots \quad (2.5e)$$

with initialization  $\pi_0 = c_0$ .

The numerical stability of the Levinson-Durbin algorithm for solving (2.4) has been established by Cybenko in [16]. However, as pointed out in [16], in many cases of practical interest the matrix  $\mathbf{R}_n$  is ill-conditioned, which results in unacceptable errors when the algorithm is implemented. This ill-conditioning occurs when the prediction error  $\pi_n$  is very small, or, equivalently, when the reflection coefficients are close to  $\pm 1$ . An important case where ill-conditioning of this nature occurs is when the discrete-time signal of interest is obtained by sampling a continuous-time process at fairly rapid rates, since  $\lim_{\Delta \rightarrow 0} c_{|k-l|} = c_0$  for all  $k$  and  $l$ , if the underlying continuous-time process  $\{Y(t); t \in \mathcal{R}\}$  is sufficiently smooth (mean-square continuous). (Here, and in the sequel,  $\Delta$  denotes the sampling interval used to produce the discrete-time signal under study.) Since the problem is due to the poor conditioning of  $\mathbf{R}_n$ , it cannot be solved by using alternative algorithms, like the Schur algorithm, to solve (2.4), as noted in [16] and by Yagle and Levy in [125].

Moreover, in this sampled-data case, the following result can be proved.

**Proposition 2.1** (Vijayan, *et al.* [116]) Assume that the continuous-time process satisfies the following conditions:

- i.)  $\{Y(t); t \in \mathcal{R}\}$  has  $n-1$  mean square derivatives.

ii.) The random vector of derivatives  $[Y^{(n-1)}(0), \dots, Y^{(1)}(0), Y(0)]$  has a non-singular covariance matrix.

Then,

$$\lim_{\Delta \rightarrow 0} a_{n,j} = (-1)^j \binom{n}{j}, \quad j = 0, 1, \dots, n, \quad (2.6)$$

and

$$\lim_{\Delta \rightarrow 0} \gamma_n = (-1)^{n-1}. \quad (2.7)$$

Thus we see that, as  $\Delta \rightarrow 0$ , if the autocovariance function of the continuous-time process has sufficiently many derivatives, the coefficients obtained by the Levinson algorithm will converge to the binomial coefficients  $(-1)^j \binom{n}{j}$  independently of the underlying process. This points to a major difficulty with the standard Levinson formulation for finite-length linear modeling; namely, the parameters of this model contain no information about the statistics of the underlying process except in terms that are of higher-order in  $\Delta$ .

### *The High-speed Levinson Problem*

In order to correct the difficulties noted above, we consider the reformulation of the autoregressive modeling problem (2.3) in terms of the divided-difference, or *delta*, operator. To do so, we assume for the remainder of Section 2 that the signal  $\{Y_k\}_{k=-\infty}^{\infty}$  is obtained by uniformly sampling a continuous time process  $\{Y(t); t \in \mathcal{R}\}$  at interval  $\Delta$ .

A mainstay of discrete-time signal and system modeling is the representation of dynamics in terms of the *shift operator*  $q$ , defined in (2.1). The  $n^{\text{th}}$ -order autoregressive model (2.3) can be rewritten in terms of the shift operator as

$$A_n(q)Y_{t+1-n} = \epsilon_n(t), \quad t \in \mathcal{Z}, \quad (2.8)$$

with  $A_n(q) = \sum_{k=1}^n a_{n,k} q^{n-k}$ . (Here, of course,  $q^\ell$  denotes  $\ell$  repeated applications of  $q$ ; i.e.,  $q^\ell Y_k = Y_{k+\ell}$ .)

For high-speed processing, it has been demonstrated (see [116, 114, 115]) that the numerical problems plaguing such representations can often be ameliorated by considering alternative formulations based on the use of the delta operator as the fundamental dynamical element as noted above. Recall that the delta operator,  $\delta$ , for sampling interval  $\Delta$  is given by

$$\delta = \frac{q-1}{\Delta}, \quad (2.9)$$

as in (2.2). In this context, it is of interest to consider a model of the form

$$\left( \sum_{k=0}^n \beta_{n,k} \delta^{n-k} \right) Y_{t+1-n} = \nu_n(t), \quad t \in \mathcal{Z} \quad (2.10)$$

where  $\underline{\beta}_n = [\beta_{n,0}, \beta_{n,1}, \dots, \beta_{n,n}]^T$  with  $\beta_{n,0} \equiv 1$ , and  $\{\nu_n(t)\}_{t=-\infty}^{\infty}$  is the sequence of modeling errors in this  $n^{\text{th}}$ -order model. Note that the iteration of the delta operator is

$$\delta^\ell Y_k = \frac{\delta^{\ell-1} Y_{k+1} - \delta^{\ell-1} Y_k}{\Delta}, \quad (2.11)$$

so that

$$\delta^2 Y_k = \frac{Y_{k+2} - 2Y_{k+1} + Y_k}{\Delta^2}, \quad (2.12)$$

and so forth.

One motivation for considering the model (2.10) is its parallelism with the continuous time autoregressive model given by

$$dY^{(n-1)}(t) + \hat{a}_{n,1}Y^{(n-1)}(t)dt + \dots + \hat{a}_{n,n}Y(t)dt = dW(t)$$

where  $\{W(t) ; t \in \mathcal{R}\}$  is a Brownian motion. Another continuous-time model that has been used in [19, 41, 42] is the following, which is based on an integral operator:

$$dY(t) + \int_{t-T}^t a(T; T - (t - s))dY(s) = dW(s). \quad (2.13)$$

In [19], this model is approximated by using the standard discrete-time AR model with order  $n = T/\Delta$ . As  $\Delta \rightarrow 0$ ,  $n \rightarrow \infty$  and the limiting values of the discrete AR parameters  $a_{n,j}$  are related to the continuous AR function  $a(T; t)$ . The disadvantage of this approach is that, for small  $\Delta$ , the number of parameters in the model becomes very large (the number  $n$  of parameters should grow at a  $\Delta^{-1}$  rate). In comparison, (2.10) gives a parsimonious parametrization that also converges to a continuous-time model.

Note that the variables  $\delta^n Y_k, \delta^{n-1} Y_k, \dots, Y_k$  are obtained by linear transformation of the variables  $q^n Y_k, q^{n-1} Y_k, \dots, Y_k$ . Since  $\delta^k = (q-1)^k / \Delta^k$ , this transformation can be represented by  $\mathbf{T}_n$ , an  $(n+1) \times (n+1)$  matrix whose  $l, k^{\text{th}}$  element is given by

$$(\mathbf{T}_n)_{l,k} = \frac{(-1)^{l-k}}{\Delta^{n-k}} \binom{n-k}{l-k}, \quad 0 \leq l, k \leq n. \quad (2.14)$$

(Here, we follow the convention that the binomial coefficient  $\binom{n}{k} = 0$  for  $k < 0$  and  $k > n$ .) Thus,  $\mathbf{T}_n$  is an invertible lower triangular matrix whose  $n \times n$  right lower submatrix is  $\mathbf{T}_{n-1}$ , with  $\mathbf{T}_0 = 1$ . The inverse of this matrix is given by

$$(\mathbf{T}_n^{-1})_{l,k} = \Delta^{n-l} \binom{n-k}{l-k}, \quad 0 \leq l, k \leq n. \quad (2.15)$$

It is straightforward to see that the vector  $\underline{\beta}_n$  that solves

$$\min_{\underline{\beta}_n} E\{\nu_n^2(t)\} \quad (2.16)$$

is given by

$$\underline{\beta}_n = (\mathbf{T}_n)_{0,0} \mathbf{T}_n^{-1} \underline{a}_n = \Delta^{-n} \mathbf{T}_n^{-1} \underline{a}_n, \quad (2.17)$$

where  $\underline{a}_n$  solves the Yule-Walker equations (2.4).

As noted previously, it is the Toeplitz property of  $\mathbf{R}_n$ , the  $n^{\text{th}}$ -order covariance matrix of the signal, that makes it possible to solve (2.4) recursively using  $O(n^2)$  computations via (2.5). Since  $\mathbf{T}_n$  is triangular, if we knew  $\mathbf{R}_n$ , it would be possible to solve (2.16) using  $O(n^2)$  computations, by first solving (2.4) for  $\underline{a}_n$  using the Levinson-Durbin algorithm, and then using (2.17) to obtain  $\underline{\beta}_n$ . However, in this procedure, any numerical errors in calculating  $\underline{a}_n$  due to the ill-conditioning of  $\mathbf{R}_n$  would carry over to the calculation of  $\underline{\beta}_n$ . Also, this type of calculation is not recursive in  $n$ .

Exploiting the special structure of the matrix  $\mathbf{T}_n$ , Vijayan, *et al.* [116] have obtained an  $O(n^2)$  algorithm for solving (2.16), that only requires knowledge of the non-Toeplitz covariance matrix of  $(\delta^n Y_k, \delta^{n-1} Y_k, \dots, Y_k)$ . This algorithm has the added advantage of being recursive, like the Levinson algorithm. It is summarized in the following:

**Proposition 2.2** (Vijayan, *et al.* [116]) The argument solving (2.16) is given recursively (in  $n$ ) by

$$\underline{\beta}_{n+1} = \mathbf{C}_n \underline{\beta}_n + \frac{1}{\Delta^2} \frac{\tilde{\gamma}_{n+1}}{\tilde{\gamma}_n} \begin{bmatrix} 0 \\ \underline{\beta}_n \end{bmatrix} - \frac{\tilde{\alpha}_n}{\tilde{\alpha}_{n-1}} \begin{bmatrix} 0 \dots 0 \\ \mathbf{C}_{n-1} \end{bmatrix} \underline{\beta}_{n-1}, \quad n = 0, 1, \dots, \quad (2.18a)$$

with initialization  $\underline{\beta}_{-1} = 0$ ,  $\underline{\beta}_0 = 1$ , and  $\tilde{\gamma}_0 = -\Delta^{-2}$ . Here  $\mathbf{C}_n$  is the  $(n+2) \times (n+1)$  matrix defined by

$$\mathbf{C}_n = \frac{1}{\Delta} \begin{bmatrix} \Delta & 0 & 0 & \dots & 0 \\ 1 & \Delta & 0 & \dots & 0 \\ 0 & 1 & \Delta & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & \Delta \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}, \quad (2.18b)$$

$\tilde{\gamma}_n$  is defined by

$$\tilde{\gamma}_{n+1} = \tilde{\alpha}_n / \tilde{\pi}_n, \quad n = 1, 2, \dots \quad (2.18c)$$

with

$$\tilde{\alpha}_n = [0, \dots, 0, 1] \mathbf{Q}_{n+1} \begin{bmatrix} \underline{\beta}_n \\ 0 \end{bmatrix}, \quad n = 1, 2, \dots, \quad (2.18d)$$

$$\tilde{\alpha}_0 = E\{x(t)\delta x(t)\} + E\{x^2(t)\} / \Delta, \quad (2.18e)$$

and

$$\tilde{\pi}_n = E\{\nu_n^2(t)\} = [1, 0, \dots, 0] \mathbf{Q}_n \underline{\beta}_n, \quad n = 0, 1, 2, \dots \quad (2.18f)$$

with  $\mathbf{Q}_n$  the covariance matrix of  $(\delta^n Y_k, \dots, \delta Y_k, Y_k)$ . ( $\mathbf{Q}_n$  does not depend on  $k$  due to the assumed stationarity of the signal.)

We now note that (2.12) can also be written as

$$\beta_{n+1,j} = \beta_{n,j} + \frac{1}{\Delta^2} \left( \Delta + \frac{\tilde{\gamma}_{n+1}}{\tilde{\gamma}_n} \right) \beta_{n,j-1} - \frac{\tilde{\alpha}_n}{\tilde{\alpha}_{n-1}} \left( \beta_{n-1,j-1} + \frac{1}{\Delta} \beta_{n-1,j-2} \right), \quad j = 0, \dots, n+1, \quad (2.19)$$

where we assume that  $\beta_{n,j} = 0$  for  $j < 0$  and  $j > n$ . On comparing (2.19) with its Levinson counterpart (2.5), we see that the complexity of the new algorithm is of the same order despite the fact that the algorithm inverts a non-Toeplitz matrix.

However, unlike the Levinson coefficients, it can be shown that (under sufficient smoothness) the delta-Levinson coefficients converge to meaningful statistical parameters of the underlying process - namely, the regression coefficients of the  $n^{\text{th}}$  mean-square derivative on those of lower orders. Moreover, in the limit (2.19) has the form

$$\beta_{n,j} = \beta_{n-1,j} + \frac{\tilde{\pi}_{n-1,0}}{\tilde{\pi}_{n-2,0}} \beta_{n-2,j-2}, \quad j = 0, \dots, n, \quad (2.20)$$

which is the same as a Levinson type recursion for continuous-time autoregressive models derived by Pham and le Breton in [70]. Thus, the recursion takes on a meaningful (and stable) limiting form.

These favorable limiting properties suggest that the delta-Levinson formulation will be more stable numerically for small  $\Delta$  than is the standard Levinson formulation. Numerical results using floating-point calculations reported in [116] support this supposition; and, in fact, the numerical performance of (2.18) for high sampling rates is considerably better than that of the classical Levinson algorithm.

This superior numerical stability of the delta model can be explained in terms of the limiting results discussed above. In particular, there is a one-to-one correspondence between the parameter vectors  $\underline{a}_n$  and  $\underline{\beta}_n$ . However, the limiting value of  $\underline{a}_n$  is independent of the statistics of the process. Hence, the useful information in the parameter vector is being "compressed". As a result of this, small perturbations in the coefficients can cause large variations in the modeling error. The delta coefficients do not suffer from this problem since their limiting values contain useful information about the process.

### *Discussion*

We see from the preceding discussion that the delta version of the Levinson problem offers a number of advantages over the standard one for high-speed processing. However, the Levinson formulation has several useful properties that are not obviously present in its delta counterpart. Such issues as lattice implementation for layered adaptivity [28], Schur realization for parallelizability [125], and direct forms for calculating reflection coefficients with covariance data [84], fall within this category of problems. Other interesting issues include estimation techniques for extracting the covariance matrix  $\mathbf{Q}_n$  from noisy data (a process almost certainly to require regularization).

Progress has been made on some of these issues within the delta context. For example, in [114] a parallelizable Schur-type delta algorithm has been developed by exploiting the Toeplitz-like structure of  $\mathbf{Q}_n$ , defined in the sense of Heinig and Rost [33]. This algorithm exhibits the parallelizability of the conventional Schur algorithm, while retaining the numerical advantages of the delta-Levinson algorithm. Lattice structures based on the divided-differences have also been developed recently, as described in [115]. Unlike the Schur algorithm, however, these lattices do not enjoy all of the favorable properties of the conventional shift-operator lattice. For example, the shift-operator lattice can be adapted in layers, since the first  $p$  stages of the lattice depend only on the first  $p$  lag covariances. However, this is not the case for the delta-based lattices described in [114]. The development of a layered lattice based on the delta operator is an open problem.

The ideas developed here for the Levinson problem can be used in many problems of DSP that arise in space communications. The following sections briefly describe some of these problems.

## **3 Adaptive Multiuser Detection**

The methods described in the preceding section are ideally suited for application in the signal processing functions of wideband communication systems. In this and the following section we give overviews of two specific examples of signal processing problems that arise in such wideband communication systems. Systems of this type, such as CDMA implemented with DSSS, are currently under consideration for use in several applications [93, 92, 71, 49, 117], notably land-mobile satellite systems for mobile telephony and personal communications [31, 34, 6, 3, 45, 122]. The advantages of DSSS for these services include superior operation in multipath environments, flexibility in the allocation of channels, the ability to operate asynchronously, privacy, increased capacity in bursty or fading channels, and the ability to co-exist with narrowband services without undue degradation of either service.

A useful framework, then, to motivate high-speed DSP problems in satellite communications, is that of demultiplexing and interference mitigation for DSSS systems. Since DSSS signals are commonly used for non-orthogonal multiplexing in co-existence with narrowband communication services, reception of spread-spectrum signals must often take place in the presence of both wideband multi-access interference (MAI) and narrowband interference in addition to the usual

ambient channel noises. Here, we will discuss signal processing techniques for mitigation of both of these types of interference. In particular, in the current section we discuss demultiplexing/demodulation of wideband non-orthogonal signaling formats (such as CDMA) to minimize the effects of MAI, and in the next section we discuss NBI suppression through adaptive filtering and co-channel demodulation methods. As noted previously, each of these problems serves as a paradigm for a general class of problems that require high-speed DSP functions. The first problem is an example of a general problem that arises in wideband demultiplexing problems - namely crosstalk, which can arise due to intentional non-orthogonality (as in CDMA) or due to unintentional nonideal effects in arising in the implementation of wideband filter banks. The approach to mitigation of the interference arising from such non-orthogonality is the same regardless of its source. The second problem, i.e., narrowband interference suppression, serves as an example of general narrowband adaptive technology, which includes beamforming, among other problems.

In uplink satellite channels and cellular radio systems, where it is desired to demultiplex CDMA signals, the discipline of *multiuser detection* has recently attracted considerable attention. Indeed it has grown from its origins more than ten years ago to a vibrant research and development activity in industry and academia. As the needs to increase capacity in multiuser radio channels become more pressing, it is safe to expect that the interest in the subject will grow in the near future. The basic philosophy of multiuser detection is the judicious use of signal processing at the demultiplexer can simplify the design of the CDMA transmitters. Adaptive signal processing tailored to Code Division Demultiplexing is of particular interest.

Conventional CDMA receivers treat multiaccess interference as noise. Acceptable performance requires ultra-fine power control and the use of signature waveforms with large spreading ratios. Over the last ten years, the discipline of multiuser detection has shown that signal processing at the receiver can achieve near-far resistant demodulation without the foregoing two requirements. Various approaches have been proposed to achieve this goal:

- Optimum Multiuser Detection [108]: A bank of matched filters followed by a Viterbi algorithm.
- Decorrelating Detector [54, 55]: A linear transformation that tunes out the multiuser interference.
- Successive Cancellation [20]: Successive demodulation/subtraction of each user's signal in decreasing received power level.
- Minimum Mean-Square-Error Demodulation [124, 56, 83, 60]: A compromise solution between the decorrelator and the conventional detector, which optimally takes into account the received power of each interferer.
- Multistage Receivers [104, 105, 67]: Iterative demodulation taking into account previous tentative decisions.
- Blind Multiuser Detector [35]: Adaptive linear transformation which converges to the MMSE solution knowing the desired user's signature waveform and its timing only.

The results of [108] showed that the near-far problem is not a flaw of CDMA, as widely believed, but of the inability of the conventional receiver to exploit the structure of the MAI. This feature of multiuser detection sidesteps the need for sophisticated high-precision power control in mobile communication systems. Thus, an increase in the complexity of the receiver enables a considerable reduction in the complexity of the mobile transmitters. Equally important to the near-far resistant property of optimum multiuser detection, is the performance gain that

in its demodulation. This technique has the disadvantage that it requires extremely accurate estimation of the received amplitudes, and unless the received amplitudes are all dissimilar its performance is actually worse than that of the decorrelating detector which requires no knowledge of the received amplitudes.

The requirement of training sequences in the adaptive multiuser detection [111] is a cumbersome one in multiuser communications. Since transmitters start and finish their transmissions asynchronously, the "birth" (or "death") of an interferer requires the recomputation of the adaptive receiver coefficients. Often, decision-directed operation of the adaptive detector is not robust enough to take care of those sudden changes, and the desired user must be asked to interrupt its data transmission so that a training sequence is transmitted.

A recent blind adaptive multiuser detector due to Honig, Madhow and Verdú [35] has the following features:

- it achieves optimal near-far resistance.
- (approximate) knowledge of the signature waveform of the desired user is required.
- the timing of the desired user must be acquired.
- the received amplitudes need not be known or estimated.
- the signature waveforms of the interferers need not be known.
- the timing of interfering users need not be acquired.
- Training sequences are not required for any user.

down to the original data bandwidth. So, after despreading, the situation is reversed between the original narrowband interferer (which is now wideband), and the original data signal (which is now narrowband). A bandpass filter can be employed so that only the interferer power that falls in the bandwidth of the despread signal causes any interference. This will be only a fraction ( $1/N$ , where  $N$  denotes the spreading ratio) of the original narrowband interference that could have been in that same bandwidth before despreading.

Thus, spread spectrum communications is inherently resistant to the NBI caused by co-existence with conventional communications. However, it has been demonstrated that the performance of spread-spectrum systems in the presence of narrowband signals can be enhanced significantly through the use of active NBI suppression prior to despreading [96, 64, 61]. Not only does active suppression improve error-rate performance [5], but it also leads to increased CDMA cellular system capacity [71] and improved acquisition capability [62].

Over the past two decades, a significant body of research has been concerned with the development of techniques for active NBI suppression in spread-spectrum systems. Much of this methodology based on signal processing regimes of adaptive transversal filtering, and Fourier-domain filtering. Briefly described, this development has focused on two basic types of technologies: *estimator-subtractor* methods that perform time-domain notch filtering; and *transform-domain* methods that operate to block (or suppress) narrowband energy in the frequency domain (see, e.g., [61],[73]).

The estimator-subtractor implementation essentially forms a replica of the NBI which can be subtracted from the received signal to enhance the wideband components. Systems of this type are described in [36, 44, 50, 51, 57, 37, 89, 90, 38, 59, 58, 75, 113, 25, 86, 119, 99]. One way of forming such a replica is to exploit the disparity in predictability of the NBI and the spread-spectrum signal. In particular, since the spread data signal has a nearly flat spectrum, it cannot be predicted accurately from its past values unless, of course, use is made of knowledge of the PN sequence. The interfering signal, being narrowband, can be predicted accurately on

suppress the narrowband interference through co-channel demodulation [85]. This technique is of interest when the NBI is comprised on narrowband digital communications signals.

In transform-domain NBI suppression techniques the principal approach is to take the Fourier transform of the received signal, to apply a mask in the frequency domain to notch out the NBI, and then to inverse transform the result back to the time domain for correlation with the PN code (see, e.g., [44, 63, 71, 62, 17]). A useful mask to consider is an adaptive one that excises those Fourier components whose energy levels exceed a set threshold [61]. Alternatively, a whitening mask can be used by first applying a nonparametric spectrum estimator to the received signal, from which such a mask can be derived [44]. Depending on the overall system bandwidth and on the consequent processing speed requirements, the Fourier transforms required by this techniques can be performed in hardware such as surface-acoustic-wave (SAW) technology, or in software using the fast Fourier transform (FFT).

Each of these two general types of NBI suppression techniques is effective in improving the performance of spread-spectrum systems in the presence of NBI. Neither is uniformly superior to the other, and which is best for a given application depends largely on implementation considerations, some of which are discussed in [44] and [61]. A hybrid system involving both transform-domain and time-adaptive filtering has been considered in [88]. In this work, an LMS adaptive filter is used to suppress frequency components that are correlated from FFT frame to FFT frame. This approach yields a lower complexity system for multiple narrowband interferers, when compared to purely time-domain systems with similar performance characteristics.

The reader interested in further discussion of NBI methodology for DSSS systems is referred the recent survey paper [73].

## 5 On Board Signal Processing for DMUX/TMUX

In this section, we provide a brief review of some extant DSP techniques for bulk on-board demultiplexing [52] and transmultiplexing [91] of satellite communication signals.

Demultiplexing and transmultiplexing are among the main on-board signal processing functions for satellite communications. For example, one might consider the demultiplexing of a frequency division multiplexing (FDM) system consisting of 625 channels of 30kHz each, which yields a total signal processing bandwidth of 18.750 MHz. As envisioned, this demultiplexing task would be in support of mobile personal communications services in which many small on-ground terminals will access a geostationary satellite via an agile multibeam antenna system featuring channel multiplexing/demultiplexing performed at the individual antenna ports. Because of the very wide processing bandwidth involved in this scenario, on-board high-speed signal processing is a central issue in the development of suitable DSP algorithms implementation of such a demultiplexer (DMUX).

Since transmultiplexers (TMUX's) are used for conversion between FDM and time-division multiplexing (TDM), the DSP technology issues involved in TMUX design are quite relevant to the DMUX problem noted above [94]. From a signal processing point of view, transmultiplexing essentially involves the use of filter banks with precisely controlled phase characteristics. Thus, carefully chosen DSP techniques are of critical importance in the design of TMUX's, and TMUX design for wideband signal groups is a further problem in which high-speed DSP methods must play a central role.

Several recent works have discussed general on-board signal processing requirements and issues for satellite communications, including [6, 3, 45, 4, 31, 43, 34, 2, 47, 11, 30, 122, 40, 123]. Specific DSP algorithms for demultiplexing and transmultiplexing can be found in a number of works, some of which are described briefly below.

### *On-board Demultiplexing*

Recent studies of on-board DMUX can be found in [94, 6, 7, 8, 77, 53, 14, 97]. Approaches that have been considered for this problem include fast-Fourier transform (FFT) based methods [6, 7, 8], reconfigurable binary tree structures [77], surface acoustic wave (SAW) based implementations [53], and DSP methods based on specific types of filters such as all-zero lattice filters [14].

For example, a promising class of digital processing solutions for wideband bulk DMUX problems such as that described above are based on the use of an FFT method for channel isolation [52]. One such implementation suitable for the demultiplexing of many narrowband carriers is described in [6, 7]. In this implementation, which can be programmed from the ground to change the profile of center frequencies, bandwidths, and modulation methods, the bulk signal is sampled at its Nyquist rate and then digitized. A suitable time-domain window (the length of which depends on the required frequency resolution) is applied to the data, the FFT is taken, and the channels are selected by appropriately weighting the FFT coefficients with those of a channel filter. The signal is then inverse transformed to produce a time-domain baseband signal from which the data can be recovered. Advantages of such a technique for wideband DMUX implementation are that it can handle large numbers of Fourier coefficients, and the above-noted 18.750 MHz can be accommodated with current technology.

### *Transmultiplexing*

As noted above, the key issue in TMUX design is the choice of the channelizing filter bank,

which must have a carefully controlled phase response. Because of the wide discrepancy between the processing bandwidth and the bandwidth of the individual communication channels, the introduction of crosstalk among channels is a common problem in TMUX design. Also, because of the tightly controlled phasing necessary for proper TMUX operation, issues such as sampling errors and other nonideal effects are of importance. Thus, much of the work in this area is focused on the filter design problem, and in fact this problem has become a key paradigm for filter-bank design in general because of the complexity of the filter design issues arising therein.

A number of approaches to the TMUX filter design problem have been considered. These include periodically time-varying filters (PTV's) [76], discrete sine/cosine transforms (DST/DCT's) [119, 82], wave digital filters (WDF's) [48], fast Hartley transforms (FHT's) [100], polyphase FFT's [24], quadrature mirror filters (QMF's) [69, 46, 78], modulated filter banks [79], and others [80, 15, 126, 12, 21, 13, 81].

Not all of the above approaches have been considered in very wideband applications such as that of interest here. However, an example of a digital processing approach that is suitable for such very wideband applications is the FFT approach of [24]. In particular, in [24] a reconfigurable TMUX that is capable of on-board demultiplexing of a varying number of single-channel-per-carrier FDMA channels with varying bit rates is presented. The algorithm selected for demultiplexing the FDMA channels is the polyphase FFT method, which requires a bank of filters followed by an FFT operation. A reconfigurable shared filter bank and reconfigurable pipelined FFT architecture are used to implement the bank of filters and FFT operations. This architecture is suitable for satellite on-board processing as it is reconfigurable and modular, and can perform its processing in real time without large buffers. This architecture is illustrated in [24] specifically for demultiplexing 800 channels at 64 kbps, or a mix of 400 channels at 64kbps and 12 channels at 2.048 Mbps, or 24 channels at 2.048 Mbps. Thus, the bulk bandwidths discussed above can easily fit within this technique.

Another interesting approach involves the use of quadrature mirror filters (QMF's) in the design of crosstalk-free TMUX (CF-TMUX). In [46] such filters are used to effect crosstalk cancellation. The authors present an analysis of the CF-TMUX based on the polyphase component matrices of the filter banks used in TDM to FDM and FDM to TDM conversions, respectively, and a necessary and sufficient condition for complete CC is thereby obtained. It is shown that the filters for a CF-TMUX are the same as the filters for an alias-free QMF bank. In addition, if the QMF bank satisfies the perfect reconstruction property [101], then the TMUX also satisfies this property. The relation between CF-TMUX filters and alias-free QMF banks is used to obtain a direct design procedure for CF-TMUX filters (both FIR and IIR). It is also shown that approximately crosstalk-free TMUX filters can be obtained from any approximately alias-free QMF bank. This approach allows the considerable methodology for design of QMF banks to be applied directly to TMUX design.

## 6 $\Sigma - \Delta$ Modulation

Oversampling relies on the fundamental principle of trading off temporal resolution with amplitude resolution. Since very high and accurate clock frequencies are easy to generate, this trade-off allows the analog-to-digital converter to achieve resolution beyond the limitations imposed by the matching tolerance of VLSI technology. In addition, it simplifies the overall system design by relaxing the front-end anti-aliasing requirement. On the minus side, oversampling converters require long decimation and/or interpolation filters, which demand significant silicon area and dissipate power.

Oversampled A/D converters achieve high resolution by shifting their quantization noise outside of the signal band and then removing it with digital filters. In a  $\Sigma - \Delta$  modulator the

difference signal between the input and its prediction is integrated and then quantized. Thus, the process that shapes the quantization noise in  $\Sigma - \Delta$  modulation can be explained as making a prediction of low frequency values of the noise and subtracting it from the signal. When the quantization noise is white, this is a sensible strategy which provides fine resolution if the sampling rate is high with respect to the Nyquist rate. Whereas  $\Sigma - \Delta$  modulation makes prediction of noise, conventional  $\Delta$  modulation predicts signal values, and has been shown to be a less robust technique. Equivalently, we can view  $\Sigma - \Delta$  modulation as  $\Delta$  modulation of the integrated input signal. This has the effect of robustifying the demodulated waveform to channel errors, which in  $\Delta$  modulation have lingering effects.

The "order" of the  $\Sigma - \Delta$  modulator refers to the order of the prediction filter; with first-order being a simple integrator. In practice, there are disadvantages to choosing too large an order: circuits become less tolerant to imperfections and there is increased danger of limit cycles. Very reliable modulators have been built with just 1-bit quantization and second-order prediction. The risk of limit cycles with high order predictors has been avoided by cascading several first and second order  $\Sigma - \Delta$  modulators in order to mimic a high-order  $\Sigma - \Delta$  modulator. In this case, the error signal of each stage is the difference between the input to that stage and the overall output, with quantization provided only at the output stage.

In oversampled D/A conversion, input words are applied to digital interpolation filters that raise the word rate by the oversampling ratio. The signal may then be applied to a digital oversampled D/A converter that shortens words typically to one bit. The Mean-Square-Error of oversampled D/A conversion is inversely proportional to  $R^{2n+1}$  where  $n$  is the order of the low-pass filter that processes the quantized signal, and  $R$  is the oversampling rate. A further gain of a factor of  $R$  by careful reconstruction filtering is reported in [98].

$\Sigma - \Delta$  demodulators typically consist of an average of  $N$  consecutive modulator outputs, where  $N$  is the oversampling ratio. This is usually taken to be a good approximation to a uniform scalar quantizer at the original sampling rate.

The theoretical analysis of  $\Sigma - \Delta$  modulators is usually extremely difficult, because of their inherent nonlinearity. However, simulators are usually very simple to implement. The basic theory and analysis of  $\Sigma - \Delta$  conversion can be found in [39, 10, 9, 29]. The latter reference by Gray gives the first rigorous analysis of  $\Sigma - \Delta$  modulation in its simplest possible form. It shows that when the input is constant, the state sequence of the integrator in the encoder loop can be modelled as a linear system. Two-stage  $\Sigma - \Delta$  modulators driven by DC and sinusoidal inputs are analyzed in [121]. It is shown that the binary quantizing error which appears at the output of the modulator is asymptotically white, uniformly distributed and uncorrelated with the DC input. For sinusoidal inputs, the spectrum is no longer white (unless the oversampling ratio is asymptotically large) even though the other properties are retained. A rigorous analysis of  $\Sigma - \Delta$  modulation with time-varying inputs is not easy even in a purely deterministic setting. Delchamps has made several important contributions using the ergodic theory of the torus which result in appealing closed-form solutions [18]. Further progress dealing with nonideal effects from the viewpoint of nonlinear dynamics has been made in [23].

Multiloops sigma-delta modulators are analyzed in [32] which shows that the long-term time averages of the error process are consistent with that of signal-independent additive white uniform noise. This result is shown for DC inputs as well as for sinusoidal inputs provided the modulator has three or more loops.

In applications where signals with very different bandwidth and dynamic range are bundled together, such as for example the data signals and the servocontrol signals in a Compact-Disk player,  $\Sigma - \Delta$  analog-to-digital conversion has been shown [66] to be a very convenient technique, which relaxes the requirements on the front-end anti-aliasing filters. Similar advantages could be found in satellite applications. Moderate oversampling rates are used in [26] in order to perform

digital demultiplexing of Frequency-division channels using various established methods, such as the single-stage, polyphase and multistage methods.

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