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Multicasting in Large Random Wireless Networks: Bounds on the Minimum Energy per Bit

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***Abstract*—We consider scaling laws for maximal energy efficiency of communicating a message to all the nodes in a random wireless network, as the number of nodes in the network becomes large. Two cases of large wireless networks are studied — dense random networks and constant density (extended) random networks.**

We first establish an information-theoretic lower bound on the minimum energy per bit for multicasting that holds for arbitrary wireless networks when the channel state information is not available at the transmitters. These lower bounds are then evaluated for two cases of random networks. Upper bounds are also obtained by constructing a simple flooding scheme that requires no information at the receivers about the channel states or the locations and identities of the nodes. The gap between the upper and lower bounds is only a constant factor for dense random networks and differs by a poly-logarithmic factor for extended random networks. Furthermore, the proposed upper and lower bounds hold almost surely in the node locations as the number of nodes approaches infinity.

I. Introduction

A. Prior Work

Determining energy efficiency of a point-to-point channel is a fundamental information-theoretic problem. This problem is considerably more complicated for networks. Even when just one helper (*relay*) node is added to the two terminal AWGN channel, the minimum energy per bit is still unknown despite many efforts ([4], [11] and references therein). As the number of relays k in a network grows, we can ask whether the energy efficiency improves and at what rate. It was shown in [3] that a two hop *distributed beamforming* scheme gives very good energy efficiency in dense random networks with the energy requirement falling as $\Theta(1/\sqrt{k})$. It is not clear, however, how to extend this idea to noncoherent or to multicasting scenarios.

Cooperation between nodes (also known as *cooperative diversity*) leads to capacity or reliability gains even with simple schemes. A simple cooperation idea in a multicast setting involves letting many nodes transmit the same signal (at lower power levels), so that each receiver can combine several low reliability signals to construct progressively better estimates. The works of [6], [7], [5] presented such multi-stage decode and forward schemes to reduce the transmission energy. In [7], achievability schemes were presented for dense networks, whereas our interest is in order of growth of energy requirement for simpler power allocation (uniform) for both dense and extended networks. A major difference in our setup from the previous works is our emphasis on minimal network and channel state information. This implies, among other things, that no centrally optimized transmission or power policies can be implemented.

The problem of communicating the same message to a set of nodes (multicasting) in a network with minimum energy consumption has drawn a lot of research interest. For wireless networks, there is an inherent *wireless multicast advantage* [10] that allows all the nodes within the coverage range to receive the message at no additional cost. Even the nodes out of the coverage range can overhear the transmissions made over the wireless medium. Such an advantage has been termed *Cooperative Wireless Advantage* (CWA) in [5]. A more fundamental approach to the modeling and analysis of wireless networks may yield better results based on exploiting the broadcast nature of wireless communications.

B. Summary of Results

In this work, our aim is to determine the maximum possible energy efficiency for multicasting in wireless relay networks. Besides developing converse bounds, we also show how cooperative communication is instrumental to approach them.

We first present, in Theorem 1, a lower bound on the energy requirement for multicasting in arbitrary wireless networks when there is no constraint on the available bandwidth. This lower bound is inversely proportional to the *effective radius* of the network, which is a fundamental property of the network and depends on the channel gains between the nodes. This bound is applicable when channel state information is not available at the transmitters, regardless of whether the channel state information is available at the receivers. For the achievability part, we propose a simple wideband flooding algorithm that does not require knowledge of the node locations, identities or channel states. These converse and achievability bounds are then evaluated for two cases of large random networks when the same message needs to be communicated to all the nodes. Following recent trends, our focus is on the order of scaling of the energy efficiency. For

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the cases that we consider, the upper and lower bounds hold almost surely in the placements of nodes as the number of nodes $k \to \infty$ and exhibit similar orders of scaling.

The physical channel is modeled as a fading channel subject to Gaussian noise. We operate in the wideband regime, which is essential to maximize the energy efficiency in a point-to-point Gaussian channel. For random networks, the power or channel gain between any two nodes separated by distance r is modeled as $r^{-\alpha}$ for $\alpha > 2$, with a near-field correction that prevents the gain from becoming arbitrarily large.

We consider two different kinds of random networks. In both cases, the network area (of size A_k) is assumed to be a square with diagonal coordinates at $(0,0)$ and $(\sqrt{A_k}, \sqrt{A_k})$. The source node is fixed at the origin and the remaining $k-1$ nodes are placed randomly independently and uniformly over the network area. In *dense random networks*, the area A_k of the network grows as $o(k/\log k)$. This implies that the density of nodes grows unbounded with k. In Theorem 2, we show that the minimum energy per bit of dense networks scales linearly with area and does not depend on k. In *extended random networks*, the density of nodes is constant, i.e., A_k grows linearly with k. In Theorem 3, we show that the minimum energy per bit is $\Omega(k)$, with the constant depending on the node density. A flooding algorithm is shown to come within a poly-logarithmic factor (in k) of the lower bound.

In Section II, we introduce the system model. In Section III, we prove a general result about the minimum energy requirement for multicasting in a wireless network. The flooding algorithm is described in Section IV. In Section V, the path loss model is covered, and the dense and extended random networks are studied in Section VI.

II. System Model

A. Channel Model

We deal with a discrete-time complex additive Gaussian noise channel with fading. Suppose that there are k nodes in the network, with node 1 being the source node. Let node $i \in \{1, ..., k\}$ transmit $x_{i,t} \in \mathbb{C}$ at time t, and let $y_{j,t} \in \mathbb{C}$ be the received signal at any other node $j \in \{1, ..., i-1, i+1, ..., k\}$. The relation between $x_{i,t}$ and $y_{j,t}$ is given by

$$y_{j,t} = \sum_{i=1}^{k} h_{ij,t} x_{i,t} + z_{j,t} \tag{1}$$

where $z_{j,t}$ is circularly symmetric complex additive Gaussian noise at the receiver j, distributed according to $\mathcal{N}_{\mathbb{C}}(0, N_0)$. The noise terms are independent for different receivers as well as for different times. The fading between any two distinct nodes i and j is modeled by complex-valued circularly symmetric random variables $h_{ij,t}$ which are i.i.d. for different times. We assume that $h_{ii,t} = 0$ for all nodes i and times t. Also, for all $(i,j) \neq (l,m)$, the pair $h_{ij,t}$ and $h_{lm,t}$ is independent for all times t. *Absence of channel state information at a transmitter* i implies that $x_{i,t}$ is independent of the channel state realization vector $(h_{i1,t}, h_{i2,t}, ..., h_{ik,t})^T$ for all time t. The quantity $\mathbb{E}[|h_{ij}|^2]$ is referred to as the *channel gain* between nodes i and j.

B. Problem Setup

All the nodes in the network are identical and are assumed to have receiving, processing and transmitting capabilities. The nodes can also act as relays to help out with the task of communicating a message to the whole network.

To define a multicast relay network, we extend the simple three terminal setting of a *relay channel* to include multiple relays and multiple destination nodes by providing a relay and a decoding function at each node. The relay function at a relay node i decides the channel input symbol $x_{i,t}$ at time t based on the previous $t-1$ channel outputs at the node. The decoding function at node i decodes a suitable message $\hat{m}_i$ from the message set $\mathcal{M}$ containing M messages, once all the n channel outputs at the node are received. Suppose that only a subset $\mathcal{R} \subseteq \{2, ..., k\}$ (also called the *destination set*) of the nodes is interested in receiving the message from the source node (node 1). An error occurs when any of the nodes in $\mathcal{R}$ fails to decode the correct message transmitted by the source. The probability of error of the code is defined as

$$P_e \triangleq \frac{1}{M} \sum_{m \in \mathcal{M}} P[\exists i \in \mathcal{R} : \hat{m}_i \neq m | m \text{ is the message}] \tag{2}$$

Define the expected total energy expenditure (for all nodes) of the code to be

$$E_{\text{total}} \triangleq \sum_{i=1}^{k} \sum_{t=1}^{n} \mathbb{E}\left[|x_{i,t}|^2\right] \tag{3}$$

where the expectation is over the message, noise and fading. The energy per bit of the code is defined as

$$E_b \triangleq \frac{E_{\text{total}}}{\log_2 M} \tag{4}$$

An $(n, M, E_{\text{total}}, \epsilon)$ code is a code over n channel uses, with M messages at the source node, expected total energy consumption at most E_{total} and probability of error at most $0 \leq \epsilon < 1$.

In [9], *channel capacity per unit cost* was defined for a channel without restrictions on the number of channel uses. Here, we are interested in the reciprocal of this quantity.

Definition: Given $0 \leq \epsilon < 1$, $E_b \in \mathbb{R}_+$ is an ϵ-achievable energy per bit if for every $\delta > 0$, there exists an $E_0 \in \mathbb{R}_+$ such that for every $E_{\text{total}} \geq E_0$ an $(n, M, E_{\text{total}}, \epsilon)$ code can be found such that

$$\frac{E_{\text{total}}}{\log_2 M} < E_b + \delta \tag{5}$$

E_b is an achievable energy per bit if it is ϵ-achievable energy per bit for all $0 < \epsilon < 1$, and the *minimum energy per bit* $E_{b\min}$ is the infimum of all achievable energy per bit values.

Minimal information framework: Our aim is to achieve low energy consumption per bit using no information at the nodes about the actual network realization (i.e., node locations). In addition, we also assume that the nodes have no information about the channel states. All the non-source nodes have the same relay and decoding functions. On the other hand, our converse results allow coding schemes to rely on any such information except for channel state information at the transmitters.

III. A LOWER BOUND ON THE MINIMUM ENERGY PER BIT

Theorem 1. *In a network with k nodes, where node 1 is the source node and the destination set is $\mathcal{R} \subset \{2, ..., k\}$, the required minimum energy per bit satisfies*

$$\frac{E_b}{N_0}_{min}\left(\mathcal{R}\right) \geq \frac{\log_e 2}{G(\mathcal{R})} \tag{6}$$

where G is the effective network radius *defined as*

$$G(\mathcal{R}) \triangleq \frac{1}{|\mathcal{R}|}\left(\max_{i\in\{1,\ldots,k\}} \sum_{j\in\mathcal{R}\setminus\{i\}} \mathbb{E}[|h_{ij}|^2]\right) \tag{7}$$

Crucial to the proof of Theorem 1, we state without proof Lemma 1 that provides a converse relating the minimum energy per bit to the channel capacity.

Dropping the time indices, the channel equation (1) for the received symbol y_j at node j can be rewritten as

$$y_j = \mathbf{h}_j^T \mathbf{x} + z_j \tag{8}$$

where $\mathbf{x} = (x_1, ..., x_k)^T$ is the transmission symbol vector and $\mathbf{h}_j = (h_{1j}, h_{2j}, ..., h_{kj})^T$ is the vector representing the fading where h_{jj} is set to 0.

Lemma 1. *For the destination set $\mathcal{R}$, the minimum energy per bit for the network satisfies*

$$E_{b\min}(\mathcal{R}) \geq \inf_{\substack{P_1,\ldots,P_k\geq 0:\\ \sum_{i=1}^k P_i>0}} \max_{j\in\mathcal{R}} \frac{\sum_{i=1}^k P_i}{\sup\limits_{\substack{P_{\mathbf{x}}:\\ \mathbb{E}[|x_i|^2]\leq P_i \text{ for } i=1,\ldots,k}} I(\mathbf{x}; y_j|\mathbf{h}_j)} \tag{9}$$

In lieu of the proof, we offer a brief rationale for Lemma 1. For the given constraints $P_1, P_2, ..., P_k$ on the transmission power, pick any node j belonging to the destination set $\mathcal{R}$. Consider the channel from the set of nodes $\{1, ..., j-1, j+1, ..., k\}$ to node j. By the *max-flow min-cut* bound [1, Theorem 4], [2, Theorem 15.10.1], the rate of reliable communication to node j by the rest of the nodes cannot exceed

$$C_j(P_1, ..., P_k) \triangleq \sup_{\substack{P_{\mathbf{x}}:\\ \mathbb{E}[|x_i|^2]\leq P_i \text{ for } i=1,\ldots,k}} I(\mathbf{x}; y_j|\mathbf{h}_j) \tag{10}$$

bits per channel use. Therefore, the number of channel uses per bit is at least $1/C_j(P_1, P_2, ..., P_k)$, which implies that the total energy spent per bit in communicating to node j is at least $\sum_{i=1}^k P_i/C_j$. Since the minimum energy required to communicate to node j does not exceed the minimum energy required to communicate to all the nodes in $\mathcal{R}$, we can lower bound $E_{b\min}$ by the energy spent communicating to any of the nodes in $\mathcal{R}$. A complete proof of Lemma 1 uses ideas from [4, Theorem 1], [9, Theorem 2] along with the *max-flow min-cut* bound.

Proof sketch of Theorem 1: We can lower bound the minimum energy per bit by

$$E_{b\min}(\mathcal{R}) \geq \inf_{\substack{\boldsymbol{w}\in\mathbb{R}_+^k:\\ w_i\geq 0,\\ \sum_{i=1}^k w_i=1}} \max_{j\in\mathcal{R}} \inf_{P>0} \frac{P}{\sup\limits_{\substack{P_{\mathbf{x}}:\\ \mathbb{E}[|x_i|^2]\leq w_i P}} I(\mathbf{x}; y_j|\mathbf{h}_j)} \tag{11}$$

$$\geq \min_{\substack{\boldsymbol{w}\in\mathbb{R}_+^k:\\ w_i\geq 0,\\ \sum_{i=1}^k w_i=1}} \max_{j\in\mathcal{R}} \frac{N_0 \log_e 2}{\sum_{i=1}^k \mathbb{E}[|h_{ij}|^2] w_i} \tag{12}$$

where (11) follows from Lemma 1 by rewriting (9) in terms of total power P and by using the fact that min-max is greater than or equal to max-min. Inequality (12) is obtained by upper-bounding, under the given power constraints, the mutual information term in (11) for the Gaussian channel where the channel state information is not known at the transmitters and the channel coefficients are i.i.d. with zero mean. We have also used the fact that $\log_e(1 + x) \leq x$ for all $x \geq 0$.

We note that

$$\max_{\substack{\boldsymbol{w}\in\mathbb{R}_+^k:\\ w_i\geq 0,\\ \sum_{i=1}^k w_i=1}} \min_{j\in\mathcal{R}} \sum_{i=1}^k \mathbb{E}[|h_{ij}|^2] w_i$$

$$\leq \max_{\substack{\boldsymbol{w}\in\mathbb{R}_+^k:\\ w_i\geq 0,\\ \sum_{i=1}^k w_i=1}} \frac{1}{|\mathcal{R}|} \sum_{j\in\mathcal{R}} \sum_{i=1}^k \mathbb{E}[|h_{ij}|^2] w_i \tag{13}$$

$$= G(\mathcal{R}) \tag{14}$$

which yields the final result through (12). Step (14) is obtained by interchanging the summations in (13) and maximizing the resulting expression over all possible values of $\boldsymbol{w}$. ■

Remark 1: Examples of networks can be constructed for which the effective network radius does not always decrease with the size of destination set. So, it is useful to maximize the right hand side of (6) by considering all non-empty subsets of the destination set $\mathcal{R}$. This gives the following tighter bound

$$\frac{E_b}{N_0}_{\min}\left(\mathcal{R}\right) \geq \max_{\substack{\mathcal{R}'\subset\mathcal{R}:\\ \mathcal{R}'\neq\phi}} \frac{\log_e 2}{G(\mathcal{R}')} \tag{15}$$

Remark 2: For the point-to-point Gaussian channel, the effective network radius is simply the channel gain from the source to the destination node. The bound is tight in this case [8, Theorem 1].

IV. FLOODING ALGORITHM

We now propose a version of flooding algorithm that, with suitable parameter values, is used to achieve energy-efficient multicasting for the networks considered later.

Since we operate in the wideband regime, we can assign each transmitter its own wide frequency band. In this regime, for the point-to-point case, the knowledge of the channel states at the receiver does not decrease the minimum energy per bit [8]. Furthermore, a necessary condition for reliable decoding is that the received energy per bit be greater than $N_0 \log_e 2$.

FLOOD(E_{b1}, E_{b2})

1) The source node transmits only in the 1^{st} time slot with energy per bit E_{b1}.
2) At the beginning of time slot $t = 2, ..., T$, each node (except the source node) executes the following
   - If the node was able to decode a message for the first time in the previous time slot, then it retransmits the same message in the current time slot with energy per bit E_{b2}.
   - Else, keep quiet.

Fig. 1. The Flooding Algorithm: FLOOD(E_{b1}, E_{b2})

Various wideband communication schemes can be constructed which let the receivers reliably decode a message if the total received energy per bit exceeds $N_0 \log_e 2$ [8]. We use one such repetition-code based scheme for our setup.

1) Description of the algorithm: The flooding algorithm consists of two parts: an outer algorithm and an inner coding scheme. The outer algorithm FLOOD(E_{b1}, E_{b2}) is the description at the *time slot* level using the *decoding* and *encoding* functionalities provided by the inner scheme. See Fig. 1 for the description of the outer algorithm.

Time is divided into slots: $1, 2, ..., T$, each slot consisting of enough time to let a node transmit one codeword. Multiple nodes can transmit simultaneously in a slot, albeit in their own mutually orthogonal frequency bands. The total number of slots T in the algorithm is a design parameter which depends on the size of the network.

The decoding process and the determination of the codeword to be transmitted is handled by the inner coding scheme which is a repetition code. The *transmit* operation in FLOOD(E_{b1}, E_{b2}) uses identical codebooks for all nodes. The task for each decoder is to observe transmissions over multiple time slots and frequency bands. Using these observations, it forms a reliable estimate of the source message. At the end of each slot it determines whether it has enough information to decode the message. If not, it keeps quiet and waits for more transmissions. If it is able to decode a message, it re-encodes the decoded message and transmits it in the next slot for the benefit of its peers, and remains quiet after that.

2) Energy consumption: In a network with k nodes, the source node transmits with energy per bit E_{b1} and each of the remaining $k-1$ non-source nodes transmit either never or once with energy per bit E_{b2}. Therefore, the total energy $E_{b\text{flood}}$ consumed per information bit by FLOOD(E_{b1}, E_{b2}) satisfies

$$E_{b\text{flood}} \leq E_{b1} + (k-1)E_{b2} \tag{16}$$

V. Path loss model

Before analyzing random networks, we still need to define the channel gains between all the node pairs. We assume that the channel gain of the link between nodes i and j is determined only by their distance r_{ij} through a monotonically decreasing *power gain* or *path loss* function $g(r) : \mathbb{R}_+ \mapsto \mathbb{R}_+$, i.e., $\mathbb{E}[|h_{ij}|^2] = g(r_{ij})$, where for all $r \geq r_0$

$$g(r) = r^{-\alpha} \tag{17}$$

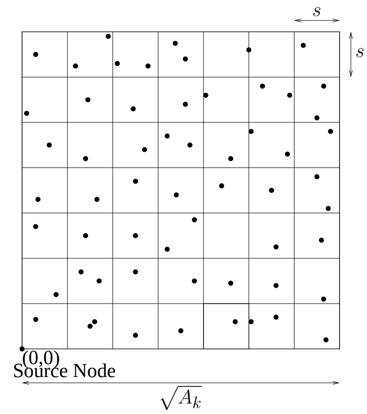

Fig. 2. Dense Random Network

Here, $r_0 > 0$ and $\alpha > 2$ are constants of the model. To deal with the near-field case, we also put an upper bound on the gain function, i.e., there is a constant $\bar{g} > 0$ such that

$$g(0) \leq \bar{g} \tag{18}$$

VI. Large Random Networks

This section is devoted to the analysis of dense and extended random networks, when the number of nodes k goes to infinity. The destination set is the set of all the non-source nodes.

A. Dense Random Networks

A dense random network with $k \geq 2$ nodes consists of a source node at the origin and $k-1$ non-source nodes distributed independently and uniformly over a square of area

$$A_k = o\left(k/\log k\right) \tag{19}$$

In addition, we also assume that $r_0^2 \leq 8A_k$ for all $k \geq 2$.

Theorem 2. *With probability 1, the node placement is such that the following hold*

$$c_1 \leq \frac{1}{A_k}\frac{E_b}{N_0}_{\text{min}} \tag{20}$$

$$\frac{1}{A_k}\frac{E_b}{N_0}_{\textit{flood}} \leq c_2 \tag{21}$$

for all but a finite number of k, where $c_1, c_2 > 0$ are constants depending only on the parameters of the path loss model.

Proof sketch: Partition the area A_k into square cells with side length $0 < s \leq r_0/\sqrt{8}$. For simplicity, we assume that $\sqrt{A_k}$ is a multiple of s. (See Fig. 2). For any $\delta > 0$, define a *good placement* event $\mathcal{D}_k$ as the collection of node placement realizations for which all the cells contain at least $(1-\delta)(k-1)s^2/A_k$ nodes and less than $((1+\delta)(k-1)s^2/A_k)+1$ nodes. It can be shown that the event $\mathcal{D}_k$ occurs almost surely as $k \to \infty$. Therefore, we only need to prove the statements (20) and (21) conditioned on $\mathcal{D}_k$.

First, the converse. The first step is to evaluate $\sum_{j \in \{2,...,k\} \setminus \{i\}} \mathbb{E}[|h_{ij}|^2]$ for any node i. This quantity can be bounded by considering a sum indexed over the cells rather than the nodes. Care needs to be exercised, treating those cells falling in the near-field separately from those in the far-field. After some manipulations, $\sum_j \mathbb{E}[|h_{ij}|^2]$ can be shown to grow as $O(k/A_k)$ for all nodes i. Hence, the effective network radius grows as $O(1/A_k)$ which immediately implies (from Theorem 1) that the minimum energy per bit grows as $\Omega(A_k)$.

For the achievability part, set $s = r_0/\sqrt{8}$ and consider the algorithm

$$\texttt{FLOOD}\left(\frac{N_0 \log_e 2}{g(\sqrt{8}s)} + \epsilon_1, \frac{(1+\epsilon_2)A_k N_0 \log_e 2}{s^2(k-1)g(\sqrt{8}s)}\right) \quad (22)$$

for any $\epsilon_1, \epsilon_2 > 0$.

Suppose that a non-source node i belongs to cell C. For any cell C, there is a sequence $(C_1, C_2, ..., C_T)$ of $T \leq \sqrt{A_k}/s$ horizontally, vertically or diagonally adjacent cells such that C_1 is the cell containing the source node and $C_T = C$. From (22), the initial transmission by the source node is received with enough energy ($> N_0 \log_e 2$) by all the nodes in cells C_1 and C_2. Thus, the nodes in C_1 and C_2 are able to decode the message with arbitrarily small probability of error [8, Theorem 1]. Suppose that, by the end of slot $t-1$, all the nodes in C_t decode the message successfully and thus, transmit the message by the end of slot t (possibly earlier). Since the gain from any node in cell C_t to any node in cell C_{t+1} is at least $g(\sqrt{8}s) = r_0^{-\alpha}$ and, conditioned on the event $\mathcal{D}_k$, there are at least $(1-\delta)(k-1)s^2/A_k$ nodes in C_t, the total received energy per bit at any node in C_{t+1} is greater than $N_0 \log_e 2$ for any $\delta < \epsilon_2/(1+\epsilon_2)$. This allows for successful decoding of the message at all the nodes in C_{t+1} by the end of time slot t. By induction, all the cells in the network are served by our scheme. Finally, the total energy per bit of the scheme (22) grows as $O(A_k)$ with k. ∎

Remark 1: Consider a *single shot transmission* scheme in which the source tries to broadcast the message in a single transmission. Its energy requirement is proportional to $A_k^{\alpha/2}$.

Remark 2: If $\alpha = 2$, the converse bound becomes

$$\frac{E_b}{N_0}_{\min} = \Omega\left(\frac{A_k}{\log A_k}\right) \quad (23)$$

This order of growth was shown achievable in [7] in a different setting (as mentioned in Section I-A).

B. Extended Random Networks

The extended random network case differs from the dense case because the density of the nodes is now a constant:

$$\lambda = k/A_k \quad (24)$$

in nodes/m^2.

Theorem 3. *With probability 1, the node placement is such that the following hold*

$$c_1 \leq \frac{1}{k}\frac{E_b}{N_0}_{\min} \quad (25)$$

$$\frac{1}{k(\log k)^{\alpha/2}}\frac{E_b}{N_0}_{\mathit{flood}} \leq c_2 \quad (26)$$

for all but a finite number of k, where $c_1, c_2 > 0$ are constants depending only on the path loss model and λ.

Proof sketch: Partition the network area into square cells with side length $\lambda^{-1/2}$. Next, right at the center of each cell, consider a small square *window* of side length $\beta\lambda^{-1/2}$, where $0 < \beta \leq 1$ is a constant. Define a non-origin cell to be *good* if it contains exactly one node (a *good node*) within its window and no nodes outside the window. We take the set of good nodes $\mathcal{R}_1 \subset \{2, ..., k\}$ of cardinality k_1, to be our destination set. When λ is large enough ($\geq 1/(9r_0^2)$), we set $\beta = 1$. The calculation of the effective network is now similar to that in the proof of Theorem 2. When $\lambda < 1/(9r_0^2)$, set $\beta < 1$ in which case any good node is at least $(1-\beta)\lambda^{-1/2}/2 > 0$ distance away from any other node. The calculation of the effective network radius is now simplified since we only need to deal with the distances in far-field. In either case, the effective network radius grows as $O(k_1^{-1})$, where the constant depends on λ. It can also be shown that $k_1 = \Theta(k)$ for both cases, almost surely as $k \to \infty$. Using Theorem 1, this directly implies the converse part of Theorem 3.

The main idea of the achievability part is similar to that of Theorem 2. Dividing the network into square cells of area $(2+\delta)\lambda^{-1}\log_e A_k$ for any $\delta > 0$, it can be shown that no cell is empty almost surely as $k \to \infty$. Thus, the point-to-point multihopping scheme

$$\texttt{FLOOD}\left(\frac{N_0 \log_e 2}{g(\sqrt{8}s_k)} + \frac{\epsilon}{k}, \frac{N_0 \log_e 2}{g(\sqrt{8}s_k)} + \frac{\epsilon}{k}\right) \quad (27)$$

reaches all the nodes with $E_{b\,\mathrm{flood}} = O\left(k(\log k)^{\alpha/2}\right)$, where s_k is the side length of the cells and $\epsilon > 0$ is arbitrary. ∎

Remark: Note that the flooding algorithm is inherently fair in the sense that all the non-source nodes expend the same amount of energy. Moreover, in both (22) and (27), the source node spends at least as much energy as each of the other nodes.

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