Impact of Out-of-Cell Interference on Strongest-Users-Only CDMA Detectors

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Abstract — **A randomly spread direct sequence code division multiple access (DS-CDMA) multi-cell system operating in flat fading is considered. The model adheres to Wyner's (1994) infinite linear cell-array setting, where only adjacent-cell interference is present, and characterized by a *single* parameter $0 \le \alpha \le 1$. Users are assumed to employ equal rates and transmit powers. The receiver ranks the intra-cell users according to their received powers, and decodes only the subset that can be reliably decoded. Confining the discussion to asymptotic analysis (in terms of number of users and processing gain), we study four multiuser detection strategies, differing in the amount and type of information available to the receiver with respect to (both intra-cell and out-of-cell) interfering users. The total capacities under outage constraint, derived as functions of the fraction of users that *cannot* be reliably decoded (equivalent to the "outage probability"), are analyzed and compared.[1]**

I. Introduction

Focusing on the limiting scenario, where both the number of users and the processing gain go to infinity, while their ratio goes to some *finite* constant, the *maximum* achievable throughput, or "spectral efficiency", of single-cell randomly spread direct sequence code division multiple access (DS-CDMA) systems with flat-fading channels has been thoroughly analyzed in [1]. These results were extended in [2] to a simple multi-cell model (to be described in the following).

Tacit in the spectral efficiency analysis is the assumption that all active users are decoded regardless of their received powers, and that the users adjust their rates (and possibly their powers) as a function of the channel fading level they experience (via feedback from the receiver). Unfortunately, in practice, such feedback and ideal tuning of the users' transmissions can hardly be accomplished if the fading varies too fast. Motivated by *practical* considerations and excluding any form of feedback from the receiving cell-site, but an ACK/NACK type, it is assumed in this paper that all users transmit at *equal rates* and *powers* regardless of the individual fade levels. Therefore, due to fading, the receiver at the cell-site can no longer guarantee reliable decoding of all active users (only of users with high enough received powers).

The above strongest-users decoding scheme was first considered in [3] for a single-cell setup in which no spreading is employed (and all bandwidth is available for coding). It was then extended to randomly spread DS-CDMA systems (in the limiting regime) in [4] and [5]. In this paper, it is applied to a simple *multi-cell* model (as in [2]) based on the infinite linear cell-array model suggested by Wyner in [6] (see also [7]). Accordingly, a *fully synchronous* cellular system is assumed, where the cells compose an infinite *linear* array, and where the received signal at each cell-site is the sum of the faded signals received from intra-cell users, plus a factor α ($0 \le \alpha \le 1$) times the sum of the faded signals generated by users in the two adjacent cells. Non-adjacent-cell users are assumed to produce no interference. The received signal is embedded in ambient Gaussian noise. The multi-cell effect on performance is thus specified by a *single* parameter (α). Independent flat-fading channels are assumed, as in [1], and system performance is analyzed while employing *optimally* coded *randomly* spread DS-CDMA with multiuser detection. The limiting scenario is considered in which the number of active intra-cell users K, and the spreading factor N (processing gain), both go to infinity, while $\frac{K}{N} \to \beta < \infty$ (referred to as the "system load").

Considering *single cell-site processing*, it is assumed that each cell-site receiver ranks all active intra-cell users by their received powers, and then only decodes the transmissions of a *subset* of these users. The number of decoded intra-cell users is chosen as the largest integer for which decoding is successful (this number depends on the realizations of both the fade levels and the additive background noise). The underlying assumption of our analysis is that the system designer sets the transmission rate of all users in order to achieve a target maximum fraction of *undecodable* users (FUU) per cell, which is equivalent to the notion of outage probability. Since different users experience independent fades, as the number of users grows, the percentage of undecodable users converges to a deterministic constant. Specifically, denoting by J the number of *decoded* users per cell, it is assumed that at the limiting scenario $\frac{J}{K} \to 1 - Q$, where $Q \in [0, 1)$ denotes the limiting system FUU. Optimization of the overall

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throughput in this regime, by choosing the optimum Q, is also considered. The rationale behind such optimization is the tradeoff between the (fixed and equal) rate to be employed by each individual user, and the FUU. As shown in the following, the above tradeoff does not lead to trivial results for the optimum Q.

Four types of multiuser detection strategies are considered. The first is the *"conventional" matched-filter detector* that treats *all* (either intra-cell or inter-cell) interference as additive white Gaussian noise (AWGN). The second is the *linear MMSE detector* that knows the signature sequences of all interfering users (both intra-cell and in adjacent cells) and mitigates their interference by means of a linear MMSE filter. The third is a detector that *"optimally"* decodes the transmissions of the maximum decodable subset of intra-cell users, while taking into account the structure of the multiuser interference generated by all remaining undecodable *intra-cell* users (assuming their signatures are known at the receiver). *Out-of-cell interference* is treated as AWGN. This detector is referred to henceforth as the "single-cell optimum (SCO) detector". Finally, an "optimum" detector is considered (analogous to the SCO detector) that takes into account the structure of *all* undecodable multiuser interference (*both intra-cell and out-of-cell*), assuming all signatures are known at the receiver. This detector is referred to henceforth as the "single cell-site processing optimum (SCPO) detector".

It is emphasized that neither the linear MMSE detector, nor the SCPO detector, try to *decode* the transmissions of adjacent-cell users (which might be prohibitive if α is small), and that these detectors are only aware of the signature sequences of such users. In addition to the above, it is also assumed that all detectors are provided with the required knowledge regarding the received powers of the interfering signals.

These multiuser detection strategies are analyzed and compared in terms of their total achievable rate sum over all *decodable* users, to be referred to henceforth as the *"outage constrained capacity"*. In this context, it is assumed that the system designer may in general modify the target FUU as a function of the signal-to-noise ratio (SNR), or $\frac{E_b}{N_0}$, to optimize the outage constrained capacity. It is noted however that in some scenarios, practical considerations such as the receiver's complexity, or its available resources (translating to a maximum number of simultaneously decodable users), may restrict the minimum FUU. System performance can be further optimized if the system load β is a degree of freedom.

II. System Model

Using the standard discrete time equivalent channel representation, the signal vector received at an arbitrary cell-site, at the discrete time related to the transmission of the ith symbol, is given by

$$y_i = S_i H_i x_i + \alpha S_i^- H_i^- x_i^- + \alpha S_i^+ H_i^+ x_i^+ + n_i \,. \tag{2-1}$$

The vector $x_i = [x_{1,i}, \ldots, x_{K,i}]^T$ in (2-1) comprises the K code symbols transmitted by intra-cell users at the ith discrete time. The vectors $x_i^\pm = \left[x_{1,i}^\pm, \ldots, x_{K,i}^\pm\right]^T$ denote the vectors of code symbols originated from users operating in adjacent cells. These symbols are assumed to be i.i.d., circularly symmetric complex Gaussian random variables (which conforms with the capacity achieving statistics), with $E\{x_{k,j}\} = 0$ and $E\{|x_{k,j}|^2\} = \bar{P}\ \forall k, j$, where $\bar{P}$ is the equal transmit power of all users. This model is justified by assuming that the codebooks of all users are chosen randomly, governed by an underlying i.i.d. Gaussian distribution per symbol, and *independently* for each message transmission (see [7]).

The columns of the $N \times K$ matrices S_i and $S_i^\pm$ are the spreading sequences (signatures) of the K users in the considered cell and in its adjacent cells, respectively. The entries of the above matrices are treated as i.i.d. zero mean random variables, with variance $1/N$. The vector n_i represents a zero mean white circularly symmetric complex Gaussian noise vector, with $E\{n_i n_i^\dagger\} = I,\ \forall i$. Without loss of generality all received powers are thus normalized with respect to the noise spectral level, and represent in fact the signal to noise ratios (SNRs) at the input to the multiuser detectors.

Finally, $H_i \triangleq \mathrm{diag}\,(h_{1,i}, \ldots, h_{K,i})$ and $H_i^\pm \triangleq \mathrm{diag}\left(h_{1,i}^\pm, \ldots, h_{K,i}^\pm\right)$, where $\{h_{k,i}\}_{k=1}^K$ and $\left\{h_{k,i}^\pm\right\}_{k=1}^K$ designate the i.i.d., zero mean, channel fading gains associated with the signals of the different users, at the ith discrete time. It is assumed henceforth that as the system size becomes large ($N, K \to \infty$, $\frac{K}{N} \to \beta < \infty$), the empirical distribution of the channel fading (power) levels, $\nu_{k,i}^{(\pm)} \triangleq \left|h_{k,i}^{(\pm)}\right|^2$, converges almost surely (a.s.) to a distribution $\mathcal{F}_\nu$. The fading levels are assumed throughout to be normalized so that $E_{\mathcal{F}_\nu}\{\nu\} = 1$ (where ν denotes some arbitrary fading level). It is noted that the analysis to follow applies for a general fading distribution, however Rayleigh fading is assumed whenever explicit results are obtained.

III. Outage Constrained Capacity

The outage constrained capacity results of each of the four multiuser detection strategies, under the strongest-users decoding scheme, are shortly presented in the following (the reader is referred to [5] for more details on the derivation). The notation $(\cdot)_{\mathrm{mf}}$, $(\cdot)_{\mathrm{mmse}}$, $(\cdot)_{\mathrm{sco}}$ and $(\cdot)_{\mathrm{scpo}}$ is used to designate entries related to the matched-filter detector, the linear MMSE detector, the SCO detector and the SCPO detector, respectively. The outage constrained capacity of the linear detectors is most conveniently expressed in terms of their *multiuser efficiency* [8], defined as the ratio between the detector's output signal-to-interference-plus-noise ratio (SINR) and the SNR. The derivation of the outage constrained capacity of the SCO and SCPO detectors is more involved and only the final results are presented here due to space limitations.

The multiuser efficiency of the matched-filter and linear MMSE detectors is equal for all users and independent of the instantaneous fading realization experienced by any particular user at any time instant. Denoting the multiuser efficiency by η, and following central limit results showing that the interference at the output of each of the two linear detectors is well approximated by a Gaussian noise (see [1] and references therein), the kth strongest intra-cell user is decodable if the (equal) individual spectral efficiency (bits/sec/Hz) satisfies $\tilde{R} \leq \frac{1}{N}\log\left(1+\nu_{i_k}\eta\bar{P}\right)$, where i_k denotes the index of the kth strongest user at the receiver (i.e., $\nu_{i_1} \geq \nu_{i_2} \geq \cdots \geq \nu_{i_K}$). Hence, the outage constrained capacity for a given FUU of Q equals

$$\begin{aligned}\tilde{R}_T(Q) &= \lim_{\substack{K,N,J\to\infty,\\ \frac{K}{N}\to\beta,\frac{J}{K}\to 1-Q}} \frac{1}{N} J \log\left(1+\nu_{i_J}\eta\bar{P}\right) \\ &= \beta(1-Q)\log\left(1+\mathcal{F}_\nu^{-1}(Q)\eta\bar{P}\right) .\end{aligned} \tag{3-1}$$

It is noted that when different systems are compared (with possibly different spreading gains and data rates), it is useful to express the outage constrained capacity in terms of the system average $\frac{E_b}{N_0}$, which is done through the relation

$$\bar{P} = \frac{1}{\beta}\tilde{R}_T\frac{E_b}{N_0} . \tag{3-2}$$

For simplicity of notation, however, most of the equations to follow are expressed in terms of the transmit power $\bar{P}$, being in fact the SNR, following the normalization with respect to the noise spectral level.

Substituting (3-2) into (3-1), focusing on the low SNR regime $\bar{P} \to 0$, and observing the limiting behavior of the multiuser efficiency (see below), the minimum received $\frac{E_b}{N_0}$ that enables reliable communications is found to be

$$\frac{E_b}{N_0}_{\min} = \frac{\ln 2}{(1-Q)\mathcal{F}_\nu^{-1}(Q)} . \tag{3-3}$$

Since $\frac{E_b}{N_0}_{\min} = \ln 2 \approx -1.59$dB when the rates are such that *all* users are decoded [1], the penalty in $\frac{E_b}{N_0}_{\min}$ induced by the strongest-users decoding scheme is a factor of $\left[(1-Q)\mathcal{F}_\nu^{-1}(Q)\right]^{-1}$. Although less obvious, the same result can be verified (analytically) for the SCO and SCPO detectors as well. The above penalty is explained by the power "wasted" when users fail to be decoded.

For the particular case of Rayleigh fading ($\mathcal{F}_\nu^{-1}(x) = -\ln(1-x)$) the outage constrained capacity of linear detectors is given by

$$\tilde{R}_T(Q) = \beta(1-Q)\log\left(1-\ln(1-Q)\eta\bar{P}\right) . \tag{3-4}$$

Applying (3-3), it follows that for *all* four multiuser detection strategies

$$\frac{E_b}{N_0}_{\min} = -\frac{\ln 2}{(1-Q)\ln(1-Q)} , \tag{3-5}$$

the minimum of which is $\frac{E_b}{N_0}_{\min} = e\ln 2 \approx 2.75$dB, achieved for $Q^* = 1-1/e$ (being hence the *optimum FUU* in the low SNR regime, for Rayleigh fading channels). As can be observed, the strongest-users decoding scheme induces a *severe* penalty of $10\log_{10}e \approx 4.34$dB in Rayleigh fading channels.

The multiuser efficiency of the matched-filter detector, in the limiting scenario considered here, converges in probability to [2] $\eta_{mf} = 1/\left[1+\beta\left(1+2\alpha^2\right)\bar{P}\right]$. The multiuser efficiency of the linear MMSE detector converges a.s. as $K,N\to\infty$, $\frac{K}{N}\to\beta<\infty$, to a non-random limit given by the unique positive solution to the following implicit equation [2] (see also [9] and [1])

$$1 = \eta_{mm} + \beta E_{\mathcal{F}_\nu}\left\{\frac{\bar{P}\nu\eta_{mm}}{1+\bar{P}\nu\eta_{mm}} + \frac{2\alpha^2\bar{P}\nu\eta_{mm}}{1+\alpha^2\bar{P}\nu\eta_{mm}}\right\} . \tag{3-6}$$

The outage constrained capacity for both linear detectors, for a target FUU of Q, is evaluated by substituting the above results into (3-1) (or (3-4) for Rayleigh fading).

Turning to the SCO detector, it is noted that it becomes in fact the "optimum" detector, under the strongest-users decoding scheme, in the particular case in which $\alpha = 0$, corresponding to the single-cell setup (in such case the SCO and the SCPO detectors coincide). The outage constrained capacity of the detector in the single-cell setup was derived in [4]. In the multi-cell setting considered in this paper, adding a mild restriction that the additive adjacent-cell interference is ergodic in second moment, the SCO detector is equivalent to an "optimum" detector in a *single-cell* system, where the additive white Gaussian background noise process has a spectral level given by $1+2\beta\alpha^2\bar{P}$ (see [2] and references therein). The desired outage constrained capacity can then be obtained by substituting $\bar{P}_{eq} \triangleq \bar{P}/(1+2\beta\alpha^2\bar{P})$ instead of $\bar{P}$ into the corresponding single-cell setup expressions. Explicit results are omitted here for conciseness, however it is noted that all results can also be derived from Theorem III.1 to follow, substituting $\alpha = 0$.

Finally, the outage constrained capacity of the SCPO detector is established by the following theorem.

Theorem III.1 *Let* $\eta_{scpo}(\delta;\bar{P})$ *be the unique solution to the equation* $1-\beta(3-\delta) = \eta - \beta\Theta_{scpo}(\delta;\bar{P}\eta)$, *where* $\Theta_{scpo}(\omega;\varsigma) = \int_{\mathcal{F}_\nu^{-1}(0)}^{\mathcal{F}_\nu^{-1}(1-\omega)} 1/(1+\varsigma\phi)$

$d\mathcal{F}_\nu(\phi) + 2E_{\mathcal{F}_\nu}\left\{1/(1+\alpha^2\varsigma\nu)\right\}$. *Define* $\Gamma_{\rm scpo}(w;\varsigma) = \int_{\mathcal{F}_\nu^{-1}(0)}^{\mathcal{F}_\nu^{-1}(1-w)} \log(1+\varsigma\phi)\, d\mathcal{F}_\nu(\phi) + 2E_{\mathcal{F}_\nu}\left\{\log\left(1+\alpha^2\varsigma\nu\right)\right\}$, *and* $\mathcal{C}_{\rm scpo}(\delta;\bar{P}) = \beta\Gamma_{\rm scpo}\left(\delta;\bar{P}\eta_{\rm scpo}(\delta;\bar{P})\right) - \log\eta_{\rm scpo}(\delta;\bar{P}) + (\eta_{\rm scpo}(\delta;\bar{P})-1)\log e$. *Then* $\tilde{R}_{\rm scpo}(Q)$, *the outage constrained capacity of the SCPO detector, for a target FUU of* Q, *equals*

$$\inf_{0\le x<1} \frac{\mathcal{C}_{\rm scpo}\left[(1-Q)(1-x);\bar{P}\right] - \mathcal{C}_{\rm scpo}\left[(1-Q);\bar{P}\right]}{x}. \quad (3\text{-}7)$$

IV. Analysis and Numerical Results

Comparative outage-constrained capacity results in Rayleigh fading channels, for the four multiuser detectors, are presented in Fig. 1. An interference factor of $\alpha = \frac{1}{2}$ is assumed, to mimic the case in which the total out-of-cell interference power equals one half of the intra-cell interference power. The outage constrained capacity for all detectors was evaluated for the (numerically obtained unless otherwise stated) optimum values of both the system load β and the FUU. Both optimum parameters are in general functions of $\frac{E_b}{N_0}$. For the sake of comparison the *spectral efficiencies* of the four detectors (reproduced from [2]) are also included in Fig. 1. Fig. 2 shows the corresponding optimum FUU as a function of $\frac{E_b}{N_0}$.

A careful examination shows that the outage constrained capacity of the matched-filter detector monotonically increases with the system load β. It is therefore optimum to take $\beta \to \infty$, and it can be shown that the optimum FUU in Rayleigh fading channels, for *all* $\frac{E_b}{N_0}$ values, is $Q^* = 1 - 1/e$. The detector is obviously interference limited, as can also be seen from Fig. 1, and the outage constrained capacity reaches the limit of $\frac{\log e}{e(1+2\alpha^2)} = 0.3538$ bits/sec/Hz (for $\alpha = \frac{1}{2}$) as $\frac{E_b}{N_0} \to \infty$ (as opposed to $\frac{\log e}{1+2\alpha^2} = 0.9618$ when all users are decoded).

The SCO detector is also interference limited (note that $\bar{P}_{\rm eq} \to \frac{1}{2\beta\alpha^2}$ as $\frac{E_b}{N_0} \to \infty$). The results show that while optimizing with respect to *both* β and Q, it is optimum in terms of the outage constrained capacity to take $\beta \to \infty$ for all $\frac{E_b}{N_0}$ values. The optimum Q monotonically decreases with $\frac{E_b}{N_0}$ as can be observed in Fig. 2, eventually reaching a limit of $Q^* = 0.4795$, for $\alpha = \frac{1}{2}$, as $\frac{E_b}{N_0} \to \infty$. However, fixing the FUU, the optimum choice of system load β (as a function of $\frac{E_b}{N_0}$) depends to the value of the adjacent-cell interference factor α, and may take on finite values in the high $\frac{E_b}{N_0}$ region (decreasing from infinity beyond some critical $\frac{E_b}{N_0}$), provided that the factor α is low enough. This behavior is in agreement with the behavior of the detector in a *single-cell* setup [5] corresponding to $\alpha = 0$ (in which case it is the "optimum" strongest-users-only detector), where it was shown that the detector is interference limited when $\beta \to \infty$ for any *fixed* FUU, and therefore finite system loads are optimum beyond some critical $\frac{E_b}{N_0}$. For the particular case of $\alpha = \frac{1}{2}$, however, the results show that taking $\beta \to \infty$ is optimum for *all* $\frac{E_b}{N_0}$ values.

In contrast to the above two detectors, the linear MMSE detector is not interference limited, and its *optimized* outage constrained capacity grows without bound with $\frac{E_b}{N_0}$. For low $\frac{E_b}{N_0}$ values, it is optimum to take $\beta \to \infty$, in which case the outage constrained capacity of the linear MMSE detector coincides with that of the matched-filter detector. However, with the increase of $\frac{E_b}{N_0}$ the optimum system load β starts to decrease beyond a critical $\frac{E_b}{N_0} \triangleq \frac{E_b}{N_0}_{\rm cr}$, eventually becoming lower than $\frac{1}{3}$ (note that the detector processes the signals of users of *three cells*). As shown in [5], assuming $E_{\mathcal{F}_\nu}\{\nu^2\} < \infty$,

$$\frac{E_b}{N_0}_{\rm cr} = \frac{2(1+2\alpha^4)E_{\mathcal{F}_\nu}\{\nu^2\}\frac{E_b}{N_0}_{\rm min}}{2(1+2\alpha^4)E_{\mathcal{F}_\nu}\{\nu^2\} - (1+2\alpha^2)\mathcal{F}_\nu^{-1}(Q)} \underset{\rm Rayleigh}{=} \frac{4(1+2\alpha^4)\frac{E_b}{N_0}_{\rm min}}{4(1+2\alpha^4)+(1+2\alpha^2)\ln(1-Q)}; \quad (4\text{-}1)$$

where $\frac{E_b}{N_0}_{\rm min}$ is given by (3-3) (or (3-5) for Rayleigh fading). The linear MMSE detector outperforms the matched-filter detector for $\frac{E_b}{N_0} > \frac{E_b}{N_0}_{\rm cr}$. The optimum FUU also decreases with $\frac{E_b}{N_0}$ beyond $\frac{E_b}{N_0}_{\rm cr}$, below which it equals $Q^* = 1 - 1/e$ for Rayleigh fading channels (as observed in Fig. 2). For Rayleigh fading and $\alpha = \frac{1}{2}$, (4-1) yields $\frac{E_b}{N_0}_{\rm cr} = 1.5e\ln 2 \approx 4.51$dB.

The SCPO detector is also not interference limited, provided that *both* the system load β and the FUU are appropriately chosen. For low $\frac{E_b}{N_0}$ values it is optimum to take $\beta \to \infty$. However, beyond some critical $\frac{E_b}{N_0}$ the optimum system load starts to decrease monotonically with $\frac{E_b}{N_0}$, while the outage constrained capacity grows without bound. With $\beta \to \infty$ the outage constrained capacity of the SCPO detector coincides with that of the SCO detector, and the detector becomes interference limited, regardless of the FUU. This behavior is in sheer contrast to single-cell setup [5], in which $\beta \to \infty$ is optimum for *all* $\frac{E_b}{N_0}$ values if the optimization is preformed over *both* Q and β. In fact, in the single-cell setup, letting the FUU vanish with $\frac{E_b}{N_0}$ at an appropriate rate, the outage constrained capacity of the detector approaches the *optimum spectral efficiency* at the high $\frac{E_b}{N_0}$ region. As can be observed from Fig. 1, this is no longer the case when undecodable out-of-cell interference is introduced, and a clear performance degradation is observed as compared to the ultimate spectral efficiency of the detector. Examining the optimum FUU of the detector, as plotted in Fig. 2, it is observed that in the low $\frac{E_b}{N_0}$ region where it is optimum to take $\beta \to \infty$, the optimum FUU of the SCPO detector coincides with that of the SCO detector. However, beyond a critical $\frac{E_b}{N_0}$, which corresponds to the point beyond which the optimum system load decreases to finite values, the optimum FUU of the SCPO detector starts to decrease more

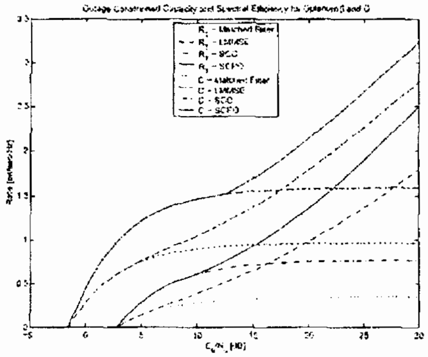

Figure 1: Spectral efficiency and outage constrained capacity for optimum choice of β and FUU (Rayleigh fading).

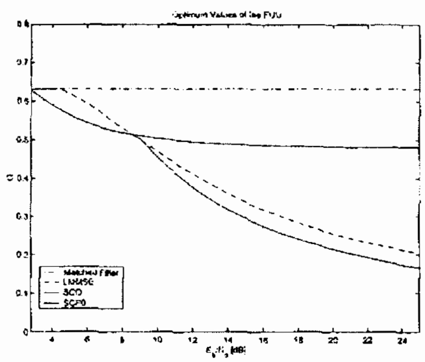

Figure 2: Optimum FUU (Rayleigh fading).

rapidly with $\frac{E_b}{N_0}$ as compared to that of the SCO detector, which goes to a limit.

Comparing the outage constrained capacity of all four multiuser detectors to their corresponding optimum spectral efficiency [2], the strongest-users decoding scheme is observed to induce a severe penalty in system performance, emphasizing the crucial role of rate-adjustment feedback in this regime.

V. Concluding Remarks

We addressed a practically appealing transmission and decoding strategy in randomly spread DS-CDMA systems. Accordingly, all users are assumed to employ equal and constant transmission powers and rates, the receiver ranks all users according to their *received* powers, and decodes only a subset of strongest users. The FUU is assumed to be a system design parameter.

Considering the outage constrained capacity of the matched-filter detector, the linear MMSE detector, the SCO detector and the SCPO detector, the strongest-users decoding scheme was shown to induce a penalty of at least $10\log_{10} e \approx 4.34$dB in the minimum $\frac{E_b}{N_0}$ allowing reliable communications, for Rayleigh fading channels. The above minimum penalty is attained by setting the FUU to $Q^* = 1 - 1/e$, the optimum choice at the low SNR regime.

All of the four multiuser detection strategies suffer a significant performance degradation for all $\frac{E_b}{N_0}$ values, as compared to their spectral efficiency (specifying the ultimate performance [1]). This performance degradation emphasizes the crucial role of *rate-adjustment feedback* for all $\frac{E_b}{N_0}$ values in the presence of undecodable out-of-cell interference. This comes in contrast to the single-cell setup [5] (corresponding to $\alpha = 0$), in which case both SCO and SCPO detectors coincide and become the "optimum" strongest-users-only detector, and their outage constrained capacity can be made to approach the ultimate spectral efficiency at the high SNR region (properly optimizing with respect to both the system load and FUU).

Finally, it is noted that the outage constrained capacities when decoding of adjacent-cell users is allowed, and with multiple cell-cite processing, is currently under investigation.

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