Improved Linear Receivers for BPSK-CDMA subject to Fading*

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Abstract

In this paper we design and analyze a new class of linear multiuser detectors, which can be used when the users employ BPSK modulation and the fading coefficients of the active users are known at the receiver (such as base-station demodulation). The tools of asymptotic distribution of the spectrum of large random matrices are used to show that relative to the classical MMSE receiver, the output SNR improves by halving the number of effective interferers and adding 3 dB to the input signal-to-noise ratio. We also propose sensible approximations to the proposed linear receivers so as to facilitate their use in CDMA systems that employ long codes.

1 Introduction

The classical linear multiuser detectors (decorrelating and MMSE receivers) exploit the geometry of the space spanned by the interference to mitigate the effect of multiaccess interference. However, unlike nonlinear multiuser detectors (maximum-likelihood, successive cancellation, etc.) those detectors do not exploit the structure of the modulation format of the desired user and the multiple-access interference. There is much to be gained by exploiting the structure of the modulation. For example, when an interferer is very strong a nonlinear receiver can essentially nullify its effect, whereas a decorrelator (or an MMSE receiver) would pay a penalty in performance due to the presence of that interferer. In this paper, we take a hybrid approach that considers linear receivers designed taking into account certain information available when the interferers use antipodal one-dimensional modulation (BPSK) and their channel fades are known at the receiver.

Unlike additive white Gaussian noise, when the users employ BPSK modulation, the baseband equivalent of the multiaccess interference process has a nonzero pseudo-autocovariance function when conditioned on the received fading coefficients. To see this, consider the contribution to the observable vector of an interferer with signature $\mathbf{s}_k$, symbol b_k and fading coefficient α_k. The pseudocovariance matrix [1] of that random vector is given by:

$$E[\alpha_k^2]E[b_k^2]\mathbf{s}_k\mathbf{s}_k^T$$

which is equal to 0 because the fading coefficient α_k is circularly symmetric. However, if α_k is known at the receiver, we are actually interested in the (generally complex valued) conditional pseudocovariance matrix

$$\alpha_k^2 E[b_k^2]\mathbf{s}_k\mathbf{s}_k^T.$$

If b_k were the result of a QPSK (or other such symmetric constellations in the complex plane), then $E[b_k^2] = 0$. However, in the case of BPSK $E[b_k^2] = 1$, and the pseudocovariance matrix of the multiaccess interferers whose fading values are known at the receiver is nonzero.

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[1] Unlike the covariance matrix which averages a vector times its Hermitian, the pseudo covariance matrix is the average of the vector times its transpose

Following up on work of the first author in [2, 1], in this paper we exploit the nonzero conditional pseudocovariance functions to obtain alternative decorrelator and MMSE receivers. Since these linear transformations operate on both the received observable vector and its conjugate we refer to them as *linear conjugate receivers.*

Our new performance results of the linear conjugate receivers use the tools developed in recent years [6, 5, 7] for the analysis of multiuser detectors in the wideband limit when the number of users goes to infinity while keeping the ratio β of users to spreading-gain constant. The performance gain afforded over the classical linear multiuser receivers by the linear conjugate receivers can be quite dramatic. For example, the near-far resistance of the decorrelator becomes $1 - \beta/2$ instead of $1 - \beta$ and the output SNR of the MMSE depends on β and the input SNR in the same way as in the classical MMSE receiver (Tse-Hanly formula [5]) except that the effective number of interferers is halved and 3 dB is added to the input SNR.

Another contribution of this paper is to propose low-complexity approximations to the original linear-conjugate receivers, which would be of interest when the CDMA system employs long codes. Matrix inverses are approximated by polynomials whose coefficients can be computed in various ways. A particularly promising way is to obtain them through an asymptotic design which, like the asymptotic SNR analysis, uses recent results on the asymptotic eigenvalue distribution of random matrices.

2 System Model

We consider a synchronous BPSK-modulated DS/CDMA system with K active users, employing long spreading codes and operating over a frequency-flat fading channel. The baseband equivalent of the received signal is:

$$r(t) = \sum_{\ell=-\infty}^{\infty} \sum_{k=1}^{K} \alpha_k(\ell)\sqrt{\mathcal{E}_k}b_k(\ell)s_k^\ell\,(t-\ell T_b)+n(t) \tag{1}$$

where

- $\mathcal{E}_k$ is the energy of the $k-$th user;
- $\{b_k(\ell)\}_{\ell=-\infty}^{+\infty}$ represents the bit stream of the $k-$th user, modeled as a sequence of independent and identically distributed binary variables, each taking on values in the set $\{-1,1\}$;
- T_b is the bit interval duration;
- $s_k^\ell(t)$ is the *signature* waveform assigned to the $k-$th user in the $\ell-$th signaling interval, expressed as
$$s_k^\ell(t) = \sum_{n=0}^{N-1} s_{k,n}^\ell u_{T_c}(t-nT_c)$$
with N the processing gain, $\boldsymbol{s}_k^\ell = [s_{k,0}^\ell,\ldots,s_{k,N-1}^\ell]$ the $k-$th spreading code in the $\ell-$th signaling interval, T_c the chip interval, and $u_{T_c}(\cdot)$ is a chip waveform with zero autocorrelation at multiples of T_c and unit energy. The spreading code $\boldsymbol{s}_k^\ell$ is normalized to have unit norm and its elements[2] are $s_{k,n}^\ell \in \{-1/\sqrt{N}, 1/\sqrt{N}\}\quad n=0,\ldots,N-1$.
- $\alpha_k(\ell)$ is a complex channel gain. The complex-valued fading parameters $\alpha_k(\ell)$ are zero-mean independent from user to user and follow a common distribution (for all k and ℓ) such that $\mathrm{E}\left[\alpha_k(\ell)\alpha_n^*(\ell)\right] = \delta_{nk}$ and $\mathrm{E}\left[\alpha_k(\ell)\alpha_n(\ell)\right] = 0$ with $(\cdot)^*$ denoting conjugate.

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[2] Similar results can be obtained for non-binary sequences.

Let us assume that we want to demodulate user "1" , and that a decision about $b_1(\ell)$ is made by processing the observation in the interval $[\ell T_b, (\ell+1)T_b]$. Chip-matched filtering the received waveform in the interval $[\ell T_b, (\ell+1)T_b]$ we obtain the following N-dimensional vector sequence of observables:

$$\begin{aligned} \boldsymbol{r}(\ell) &= b_1(\ell)\sqrt{\mathcal{E}_1}\boldsymbol{s}_1^{\ell}\alpha_1(\ell) + \sum_{k=2}^{K} b_k(\ell)\sqrt{\mathcal{E}_k}\boldsymbol{s}_k^{\ell}\alpha_k(\ell) + \boldsymbol{n}(\ell) = \\ &= b_1(\ell)\sqrt{\mathcal{E}_1}\boldsymbol{s}_1^{\ell}\alpha_1(\ell) + \boldsymbol{S}_1^{\ell}\boldsymbol{A}_1\boldsymbol{B}_1(\ell)\boldsymbol{\alpha}_1(\ell) + \boldsymbol{n}(\ell) \end{aligned} \tag{2}$$

where $\boldsymbol{A}_1 = \operatorname{diag}\left(\sqrt{\mathcal{E}_2}, \ldots, \sqrt{\mathcal{E}_K}\right)$, $\boldsymbol{B}_1(\ell) = \operatorname{diag}\left(b_2(\ell), \ldots, b_K(\ell)\right)$, $\boldsymbol{S}_1^{\ell}$ is a $N \times (K-1)$ dimensional matrix whose columns are the signatures $\left\{\boldsymbol{s}_k^{\ell}\right\}_{k=2}^{K}$ and $\boldsymbol{\alpha}_1(\ell) = [\alpha_2(\ell), \ldots, \alpha_K(\ell)]$ is the $K-1$ dimensional fading coefficient vector. In (2) the first term on the right-hand-side (RHS) represents the contribution from the bit to be decoded, while the other terms represent the contributions from MAI ($\boldsymbol{z}(\ell)$) and the thermal noise ($\boldsymbol{n}(\ell)$), respectively. Finally, notice that $\boldsymbol{n}(\ell)$ is a white complex Gaussian vector with covariance matrix $2\mathcal{N}_0\boldsymbol{I}_N$, with $\boldsymbol{I}_N$ the identity matrix of order N. The covariance matrix of the observables conditioned on all fading variables has the following expression:

$$\boldsymbol{M}_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell) \triangleq \mathrm{E}\left[\boldsymbol{r}(\ell)\boldsymbol{r}^H(\ell)|\boldsymbol{\alpha}_1(\ell)\right] = \mathcal{E}_1|\alpha_1(\ell)|^2\boldsymbol{s}_1^{\ell}\boldsymbol{s}_1^{\ell H} + \sum_{k=2}^{K}\mathcal{E}_k|\alpha_k(\ell)|^2\boldsymbol{s}_k^{\ell}\boldsymbol{s}_k^{\ell H} + 2\mathcal{N}_0\boldsymbol{I}_N \tag{3}$$

with $(\cdot)^H$ denoting conjugate transpose. The pseudocovariance matrix conditioned on all the fading variables [3] is given by:

$$\boldsymbol{M}'_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell) \triangleq \mathrm{E}\left[\boldsymbol{r}(\ell)\boldsymbol{r}^T(\ell)|\boldsymbol{\alpha}_1(\ell)\right] = \mathcal{E}_1(\alpha_1(\ell))^2\boldsymbol{s}_1^{\ell}\boldsymbol{s}_1^{\ell T} + \sum_{k=2}^{K}\mathcal{E}_k(\alpha_k(\ell))^2\boldsymbol{s}_k^{\ell}\boldsymbol{s}_k^{\ell T} \tag{4}$$

with $(\cdot)^T$ denoting transpose. Since $\boldsymbol{M}'_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}$ is nonzero, the baseband equivalent of the CDMA signals is an improper random processes[3]. This is also a straightforward, although nontrivial, consequence of the fact that, since the radio-frequency CDMA signals, conditioned on the fading coefficients, are not wide-sense stationary (WSS), the correlation properties of the corresponding complex envelopes are specified by four real functions, or, equivalently, by two complex functions. Likewise, the projections of these complex envelopes onto an orthonormal system are specified by two nonzero complex matrices, $\boldsymbol{M}_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}$ and $\boldsymbol{M}'_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}$.

For future reference, we denote by $\mathcal{R}(\boldsymbol{A})$ the column space of $\boldsymbol{A}$, by $\mathcal{N}(\boldsymbol{A})$ the null space of $\boldsymbol{A}$, by $\mathcal{C}^N$ the space of $N-$tuples on the complex field $\mathcal{C}$ with the usual internal and external operations, and by $\mathcal{S}(\ell) \triangleq \mathcal{R}\left(\boldsymbol{S}_1^{\ell}\right)$ the *interference subspace* , namely the subspace of $\mathcal{C}^N$ spanned by the MAI (i.e. by $\{\boldsymbol{s}_k^{\ell}\}_{\ell,k}$ with $k = 2, \ldots, K$).

3 Detector Design

Any *linear* one-shot detector for user "1" implements a decision rule based on the projection of $\boldsymbol{r}(\ell)$ along a given vector:

$$\widehat{b}_1(\ell) = \operatorname{sgn}\left[\Re\left\{\boldsymbol{c}_1^H(\ell)\boldsymbol{r}(\ell)\right\}\right] \tag{5}$$

where $\operatorname{sgn}(\cdot)$ denotes the signum function, $\Re\{\cdot\}$ denotes real part, while the vector $\boldsymbol{c}_1(\ell) \in \mathcal{C}^N$ is a suitable direction, dictated by some optimality criterion, by the complexity constraints and by the information available at the receiver.

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[3] According to [3], a complex random process $n(t)$ is said to be proper if its pseudoautocorrelation function $R_n(t,u) = \mathrm{E}[n(t)n(u)]$ is zero $\forall t, u$, and it is said to be improper in the opposite case that $R_n(t,u)$ is non-zero.

The most popular optimization criteria lead to the decorrelator and the MMSE techniques. If the desired signal does not belong to $\mathcal{S}(\ell)$, the decorrelator can be obtained as the unique solution to the following constrained maximization problem [6]:

$$\tilde{\boldsymbol{c}}_1(\ell) = \arg \max_{s.t. \boldsymbol{c}_1^H \boldsymbol{z}(\ell)=0} \left\{ \frac{\Re\left\{\alpha_1(\ell)\boldsymbol{c}_1^H \boldsymbol{s}_1^\ell\right\}}{\|\boldsymbol{c}_1\|} \right\} \tag{6}$$

Likewise, the classical MMSE detector may be obtained as the solution to the problem [6]:

$$\tilde{\boldsymbol{c}}_1(\ell) = \arg\min \mathrm{E}\left[\left|b_1(\ell) - \boldsymbol{c}_1^H \boldsymbol{r}(\ell)\right|^2\right] \tag{7}$$

or, alternatively, to the following constrained minimization of the output interference plus thermal noise energy:

$$\tilde{\boldsymbol{c}}_1(\ell) = \arg \min_{s.t. \boldsymbol{c}_1^H \boldsymbol{s}_1^\ell=1} \mathrm{E}\left[\left|\boldsymbol{c}_1^H(\boldsymbol{z}(\ell) + \boldsymbol{n}(\ell))\right|^2\right] \tag{8}$$

In spite of their being usually treated separately, the decorrelator (6), and MMSE receiver (8), can be subsumed under a single framework. In fact, both of them are solutions to a single general non linear constrained-optimization problem, viz.,

$$\left\{ \begin{array}{c} \tilde{\boldsymbol{c}}_1(\ell) = \arg \min_{\boldsymbol{c}_1 \in \mathcal{D}(\ell)} \|\boldsymbol{c}_1\| \\ \mathcal{D}(\ell) \triangleq \left\{\boldsymbol{c}_1 \in \mathcal{C}^N : \ \mathrm{E}_{\chi(\ell)}\left[\Re^2\left\{\boldsymbol{c}_1^H \boldsymbol{w}(\ell)\right\}\right] = \min, \ \ s.t. \ \Re\left\{\alpha_1(\ell)\boldsymbol{c}_1^H \boldsymbol{s}_1^\ell\right\} = 1\right\} \end{array} \right. \tag{9}$$

for different choices of the vector $\boldsymbol{w}(\ell)$. In (9) $\chi(\ell)$ is the vector of all parameters in $\boldsymbol{w}(\ell)$ which are not known to the receiver, and with respect to which we take the expectation. Specifically, here we are interested in two choices:

1. $\boldsymbol{w}(\ell) = \boldsymbol{z}(\ell) \triangleq \boldsymbol{S}_1^\ell \boldsymbol{A}_1 \boldsymbol{B}_1(\ell) \boldsymbol{\alpha}_1(\ell)$
2. $\boldsymbol{w}(\ell) = \boldsymbol{z}(\ell) + \boldsymbol{n}(\ell)$

which yield, as the solution to (9), a family of decorrelators and a family of MMSE receivers, respectively. Since the expectation in (9) is with respect to all parameters in $\boldsymbol{w}(\ell)$ not known to the receiver, the solution to (9) depends on the prior information available at the receiver. To illustrate further, let us consider the following situations:

[**a**] The receiver has prior knowledge of the fading coefficients $\{\alpha_1(\ell)\}_\ell$, but not of the realizations of the fading coefficients of the other users which are modeled as random variables.

[**b**] The receiver has prior knowledge of the realizations of the fading coefficients of all users $\{\alpha_k(\ell)\}_{\ell,k}$.

In both situations we assume that in each signaling interval the receiver knows the signatures of all users. Finally notice that $\Re\left\{\boldsymbol{x}^H \boldsymbol{y}\right\}$ (with $\boldsymbol{x}, \boldsymbol{y} \in \mathcal{C}^N$) is not linear in x and therefore we cannot apply the ortogonality principle. Thus, unlike the conventional optimization criteria (6) and (8), the constrained minimization problem (9) is not linearly constrained, and hence its solution requires special attention.

3.1 Unconditional linear receivers

First, let us focus on the situation [a]. In this case to solve the constrained minimization problem (9) is equivalent to solving:

$$\tilde{\boldsymbol{c}}_1(\ell) = \arg \min_{s.t.\ \boldsymbol{c}_1 \in \mathcal{D}(\ell)} \|\boldsymbol{c}_1\| \tag{10}$$

with $\mathcal{D}(\ell) \equiv \left\{ \boldsymbol{c}_1 \in \mathcal{C}^N : \ \mathrm{E}_{\chi(\ell)} \left[\left| \boldsymbol{c}_1^H \boldsymbol{w}(\ell) \right|^2 \right] = \min, \ \ s.t. \ \Re\left\{ \alpha_1(\ell) \boldsymbol{c}_1^H \boldsymbol{s}_1^\ell \right\} = 1 \right\}$.

1. Assuming that $\boldsymbol{w}(\ell) = \boldsymbol{z}(\ell)$, the solution to (10) is given by the following expression:

$$\boldsymbol{c}_{\mathrm{UD}}(\ell) = \gamma_{UD} \alpha_1(\ell) \left(\boldsymbol{M}_{\boldsymbol{zz}}(\ell) + \mathcal{E}_1 |\alpha_1(\ell)|^2 \boldsymbol{s}_1^\ell \boldsymbol{s}_1^{\ell H} \right)^+ \boldsymbol{s}_1^\ell \tag{11}$$

   where $\boldsymbol{M}_{\boldsymbol{zz}}(\ell) = E_{\chi(\ell)}[\boldsymbol{z}(\ell)\boldsymbol{z}(\ell)^H] = \boldsymbol{S}_1^\ell \boldsymbol{A}_1^2 \boldsymbol{S}_1^{\ell H}$ is the covariance matrix of the interference vector $\boldsymbol{z}(\ell)$ with $\chi(\ell) = [\boldsymbol{B}_1(\ell), \boldsymbol{\alpha}_1(\ell)]$, $(\cdot)^+$ denotes pseudo-inverse [6] and $\gamma_{\mathrm{UD}} = 1/(\alpha_1(\ell)\boldsymbol{s}_1^{\ell H} \boldsymbol{M}_{\boldsymbol{zz}}^+(\ell) \boldsymbol{s}_1^\ell)$. Notice that when $\boldsymbol{s}_1^\ell \notin \mathcal{S}(\ell)$, $\min \mathrm{E}\left[\boldsymbol{c}_1^H \boldsymbol{z}(\ell) \right]$ is equal to zero. As a consequence to solve the problem (10) for $\boldsymbol{w}(\ell) = \boldsymbol{z}(\ell)$ is equivalent to solving (6); thus (11) coincides with the classical Decorrelator detector given by (6). If instead, $\boldsymbol{s}_1^\ell \in \mathcal{S}(\ell)$, then (11) depends also on the energy of the interferers. We refer to (11) to as the *Unconditional* Decorrelator (UD).

2. When, instead, $\boldsymbol{w}(\ell) = \boldsymbol{z}(\ell) + \boldsymbol{n}(\ell)$, problem (10) admits as its solution the classical *Unconditional* MMSE (UM) receiver (8):

$$\boldsymbol{c}_{\mathrm{UM}}(\ell) = \gamma_{\mathrm{UM}} \alpha_1(\ell) \boldsymbol{M}_{\boldsymbol{rr}}^{-1}(\ell) \boldsymbol{s}_1^\ell \tag{12}$$

   where $\boldsymbol{M}_{\boldsymbol{rr}}(\ell) = E_{\chi(\ell)}[\boldsymbol{r}(\ell)\boldsymbol{r}(\ell)^H]$ is the covariance matrix of the received vector $\boldsymbol{r}(\ell)$ with $\chi(\ell) = [\boldsymbol{B}_1(\ell), \boldsymbol{\alpha}_1(\ell), \boldsymbol{n}(\ell)]$ and $\gamma_{\mathrm{UM}} = 1/(\alpha_1(\ell)\boldsymbol{s}_1^{\ell H} \boldsymbol{M}_{\boldsymbol{rr}}^{-1} \boldsymbol{s}_1^\ell)$.

3.2 Conditional linear receivers

Let us now consider situation [b], wherein the receiver has prior knowledge of the *realizations* of all fading coefficients, rather than of their ensemble features only. This might be the case of a base station in a cellular network. Consider the following results [2, 1]:

Proposition 1 *Assuming that $\boldsymbol{w}(\ell) = \boldsymbol{z}(\ell)$, define the $N \times 2N-$dimensional matrices $\boldsymbol{F} = [\boldsymbol{I}_N\, \boldsymbol{0}]$ and $\boldsymbol{F}' = [\boldsymbol{0}\, \boldsymbol{I}_N]$; then the solution to the constrained problem (9) is written as:*

$$\boldsymbol{c}_{\mathrm{LCD}}(\ell) = \gamma_{\mathrm{LCD}} \boldsymbol{F} \left(\mathbf{Q}_{\boldsymbol{\alpha}_1}^a(\ell) \boldsymbol{F}^T \boldsymbol{s}_1^\ell \alpha_1(\ell) + \mathbf{Q}_{\boldsymbol{\alpha}_1}^{a*}(\ell) \boldsymbol{F}'^T \boldsymbol{s}_1^\ell \alpha_1^*(\ell) \right) \tag{13}$$

wherein $\mathbf{Q}_{\boldsymbol{\alpha}_1}^a(\ell)$ is the projector onto $\mathcal{N}\begin{pmatrix} \boldsymbol{M}_{\boldsymbol{zz}|\boldsymbol{\alpha}_1}(\ell) & \boldsymbol{M}'_{\boldsymbol{zz}|\boldsymbol{\alpha}_1}(\ell) \\ \boldsymbol{M}'^*_{\boldsymbol{zz}|\boldsymbol{\alpha}_1}(\ell) & \boldsymbol{M}^*_{\boldsymbol{zz}|\boldsymbol{\alpha}_1}(\ell) \end{pmatrix}$ and γ_{LCD} ensures that $\Re\left\{ \alpha_1(\ell) \boldsymbol{c}_{LCD}^H \boldsymbol{s}_1^\ell \right\} = 1$.

We refer to (13) as the Linear-Conjugate Decorrelator (LCD).

Proposition 2 *Assuming $\boldsymbol{w}(\ell) = \boldsymbol{z}(\ell) + \boldsymbol{n}(\ell)$, the solution to the constrained minimization problem (9) is given by:*

$$\boldsymbol{c}_{\mathrm{LCM}}(\ell) \ = \ \gamma_{\mathrm{LCM}} \alpha_1(\ell) \boldsymbol{H}_{\boldsymbol{r}}^{-1}(\ell) \boldsymbol{s}_1^\ell - \alpha_1^*(\ell) \boldsymbol{M}_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}^{-1}(\ell) \boldsymbol{M}'_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell) (\boldsymbol{H}_{\boldsymbol{r}}^*(\ell))^{-1} \boldsymbol{s}_1^\ell \tag{14}$$

with:

$$\boldsymbol{M}_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell) = \mathrm{E}_{\chi(\ell)}[\boldsymbol{r}(\ell)\boldsymbol{r}^H(\ell)|\boldsymbol{\alpha}_1(\ell)], \qquad \boldsymbol{M}'_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell) = \mathrm{E}_{\chi(\ell)}[\boldsymbol{r}(\ell)\boldsymbol{r}^T(\ell)|\boldsymbol{\alpha}_1(\ell)],$$

$$\boldsymbol{H}_{\boldsymbol{r}}(\ell) = \left(\boldsymbol{M}_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell) - \boldsymbol{M}'_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell)\left(\boldsymbol{M}^*_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell)\right)^{-1}\boldsymbol{M}'^*_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell)\right),$$

$\chi(\ell) = [\boldsymbol{B}_1(\ell), \boldsymbol{n}(\ell)]$ *and* γ_{LCM} *such that* $\Re\left\{\alpha_1(\ell)\boldsymbol{c}^H_{LCM}\boldsymbol{s}^\ell_1\right\} = 1.$

We refer to (14) as the Linear-Conjugate MMSE (LCM) receiver. Notice that, unlike the situation [a], the proposed LCM receiver (14) does not coincide with the classical Conditional MMSE receiver with the same prior knowledge. The expression of the latter, apart from an irrelevant positive factor, is given by:

$$\boldsymbol{c}_{\mathrm{CM}}(\ell) = \alpha_1(\ell)\boldsymbol{M}^{-1}_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell)\boldsymbol{s}^\ell_1 \tag{15}$$

and can be obtained by solving (8) where the expectation in (8) is with respect to $\chi(\ell) = [\boldsymbol{B}_1(\ell), \boldsymbol{n}(\ell)]$. We refer to (15) as the Conditional MMSE (CM) receiver.

The receivers defined by (13) and (14) outperform the Unconditional Decorrelator (11) and the Unconditional and Conditional MMSE receiver, (12) and (15) respectively; the reason is that (13) and (14) fully exploit the information about the fading coefficients resorting to Linear/Conjugate processing, wherein not only the received vector but also its conjugate are processed.

Before proceeding further in our discussion, we provide some intuition as to why Linear-Conjugate techniques cannot be worse than classical linear optimization techniques, (6) and (8). The key point is that both decorrelator and MMSE receivers are *linear* receivers, achieving interference elimination by projecting the observables onto suitable subspaces. On the other hand, if the fading vector is known, the interference vector $\boldsymbol{z}(\ell)$ and the received vector $\boldsymbol{r}(\ell)$ do *not* represent complex envelopes of stationary radio-frequency signals. As a consequence, their correlation properties are described by a pair of complex covariance matrices: such information is *not* contained in the conditional covariance matrix of the vectors themselves, but in the pair of conditional matrices: $\boldsymbol{M}_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell)$ and $\boldsymbol{M}'_{\boldsymbol{rr}|\boldsymbol{\alpha}_1}(\ell)$. Notice also that the unconditional pseudocovariance matrices $\mathrm{E}\left[\boldsymbol{z}(\ell)\boldsymbol{z}^T(\ell)\right]$ and $\mathrm{E}\left[\boldsymbol{r}(\ell)\boldsymbol{r}^T(\ell)\right]$ are zero (due to $\mathrm{E}\left[\alpha_k(\ell)\alpha_n(\ell)\right] = 0$), in keeping with the fact that, in the *ensemble* of the fading realizations, the vectors $\boldsymbol{z}(\ell)$ and $\boldsymbol{r}(\ell)$ *do* represent complex envelopes of stationary signals, whose correlation properties can thus be characterized through the conventional covariance matrix only. Thus, the conditional covariance matrix of the augmented observables contains additional information which is lost once an ensemble average is performed. It is worth noticing that the baseband solutions (13) and (14), (unlike the conventional (11) and (15)) achieve the same performance that radio-frequency (or intermediate-frequency) processing would be able to achieve.

3.2.1 Suboptimal receivers

Notice that both proposed receivers, (13) and (14), depend on the eigenvalues and eigenvectors of the covariance and pseudocovariance matrices of the interference vector $\boldsymbol{z}(\ell)$ conditioned on $\{\alpha_k(\ell)\}_{\ell,k}$. However, in long-code CDMA, the spreading codes as well as the corresponding matrices $\boldsymbol{M}_{\boldsymbol{zz}}(\ell)$, $\boldsymbol{M}_{\boldsymbol{zz}|\boldsymbol{\alpha}_1}(\ell)$ and $\boldsymbol{M}'_{\boldsymbol{zz}|\boldsymbol{\alpha}_1}(\ell)$, change in every symbol interval. This implies that the proposed optimal solutions, (13) and (14), have to be computed and updated symbol by symbol, thus leading to a prohibitive computational effort in most cases. As a consequence, a less complex approach is desirable. To understand how to simplify the optimum structure, let us define the subset of $\mathcal{C}^{2N}$:

$$\mathcal{S} \triangleq \left\{\boldsymbol{x}_a : \ \boldsymbol{x}_a = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{x}^* \end{pmatrix}, \boldsymbol{x} \in \mathcal{C}^N\right\}$$

It is easily seen that, with the following external operation (multiplication by the complex scalar γ):

$$\gamma \cdot \boldsymbol{x}_a = \begin{pmatrix} \gamma \boldsymbol{x} \\ \gamma^* \boldsymbol{x}^* \end{pmatrix}$$

and the usual component-wise vector sum as internal operation, $\mathcal{V} = \{\mathcal{S}, +, \cdot\}$ is a vector space on $\mathcal{C}$. Furthermore:

$$d^2(\boldsymbol{x}_a, \boldsymbol{y}_a) \triangleq \| \boldsymbol{x}_a - \boldsymbol{y}_a \|^2 = 2d^2(\boldsymbol{x}, \boldsymbol{y}) \qquad 2\Re\left\{\boldsymbol{x}^H \boldsymbol{y}\right\} = \boldsymbol{x}^H \boldsymbol{y} + \boldsymbol{x}^T \boldsymbol{y}^* = \boldsymbol{x}_a^H \boldsymbol{y}_a \tag{16}$$

then, by considering the new vector space $\mathcal{V}$ in lieu of $\mathcal{C}^N$ in (9), we can work with a linear constraint. As a consequence, define:

$$\mathbf{u}_{1a}^{\ell} = \begin{bmatrix} \alpha_1 \boldsymbol{s}_1^{\ell} \\ \alpha_1^* \boldsymbol{s}_1^{\ell} \end{bmatrix}, \quad \boldsymbol{U}_{1a}^{\ell} = \begin{bmatrix} \boldsymbol{S}_1^{\ell}\ \mathrm{diag}\left(\boldsymbol{\alpha}_1(\ell)\right) \\ \boldsymbol{S}_1^{\ell}\ \mathrm{diag}\left(\boldsymbol{\alpha}_1^*(\ell)\right) \end{bmatrix}, \tag{17}$$

$$\boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a|\boldsymbol{\alpha}_1}(\ell) = \mathrm{E}_{\chi(\ell)}\left[\boldsymbol{z}_a(\ell)\boldsymbol{z}_a^H(\ell)|\boldsymbol{\alpha}_1(\ell)\right] = \begin{pmatrix} \boldsymbol{M}_{\boldsymbol{z}\boldsymbol{z}|\boldsymbol{\alpha}_1}(\ell) & \boldsymbol{M}'_{\boldsymbol{z}\boldsymbol{z}|\boldsymbol{\alpha}_1}(\ell) \\ \boldsymbol{M}'^*_{\boldsymbol{z}\boldsymbol{z}|\boldsymbol{\alpha}_1}(\ell) & \boldsymbol{M}^*_{\boldsymbol{z}\boldsymbol{z}|\boldsymbol{\alpha}_1}(\ell) \end{pmatrix} \tag{18}$$

with $\chi = [\boldsymbol{B}_1(\ell), \boldsymbol{n}(\ell)]$ and denote by $\boldsymbol{U}_J^{\ell}$ the $2N \times J$ matrix whose columns form a linearly independent subset spanning the column space of matrix $\boldsymbol{U}_{1a}^{\ell}$. Then, with the definition $P_1(\ell) \triangleq \mathcal{E}_1|\alpha_1(\ell)|^2$, the solutions (13) and (14) to the constrained optimization problem (9) can be rewritten, except for an irrelevant positive factor, as:

$$\boldsymbol{c}_{\mathrm{LCD}} = \boldsymbol{F}\Big(P_1(\ell)\mathbf{u}_{1a}^{\ell}\mathbf{u}_{1a}^{\ell H} + \boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a|\boldsymbol{\alpha}_1}(\ell)\Big)^{\dagger} \mathbf{u}_{1a}^{\ell} = \boldsymbol{F}\left(\boldsymbol{I}_{2N} - \boldsymbol{U}_J^{\ell}(\boldsymbol{U}_J^{\ell H}\boldsymbol{U}_J^{\ell})^{-1}\boldsymbol{U}_J^{\ell H}\right)\boldsymbol{s}_{1a}^{\ell} \tag{19}$$

$$\boldsymbol{c}_{\mathrm{LCM}} = \boldsymbol{F}\Big(\boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a|\boldsymbol{\alpha}_1}(\ell) + 2\mathcal{N}_0\boldsymbol{I}_{2N}\Big)^{-1}\mathbf{u}_{1a}^{\ell} \tag{20}$$

where $\boldsymbol{I}_{2N}$ is the $2N$-dimensional identity matrix. Based on (19) and (20), the $\boldsymbol{c}_{LCD}$ and the $\boldsymbol{c}_{LCM}$ vectors (13) and (14) can be expanded in an infinite power series of the conditional covariance matrix of the $2N-$dimensional vector $\boldsymbol{z}_a(\ell)$. As a consequence, the suboptimum approximation that we advocate consists of choosing $\widetilde{\mathbf{c}}_1(\ell)$ as corresponding to the minimum of (9) over a feasible constrained set. In this case after choosing the integer D depending on the allowable complexity, we obtain the parametric solutions:

$$\boldsymbol{c}_{\mathrm{SCD}}(\ell) = \sum_{m=0}^{D-1} w_{\mathrm{SCD}}^{m}(\ell)\boldsymbol{U}_J^{\ell}\left(\boldsymbol{U}_J^{\ell H}\boldsymbol{U}_J^{\ell}\right)^m \boldsymbol{U}_J^{\ell H}\mathbf{u}_{1a}^{\ell} \tag{21}$$

$$\boldsymbol{c}_{\mathrm{SCM}}(\ell) = \sum_{m=0}^{D-1} w_{\mathrm{SCM}}^{m}(\ell)\left(\boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a|\boldsymbol{\alpha}_1}(\ell) + 2\mathcal{N}_0\boldsymbol{I}_{2N}\right)^m \mathbf{u}_{1a}^{\ell} \tag{22}$$

where the vector weights, $\boldsymbol{w}_{(\cdot)}(\ell) = [w_{(\cdot)}^{0}(\ell), \ldots, w_{(\cdot)}^{D-1}(\ell)]$, are obtained by solving the constrained minimization problem (9). The optimal weight vectors are given by:

$$\boldsymbol{w}_{\mathrm{SCM}}(\ell) = \begin{pmatrix} \mathcal{H}_1^{\ell}, & \ldots, & \mathcal{H}_D^{\ell} \\ \vdots, & \ldots, & \vdots \\ \mathcal{H}_D^{\ell}, & \ldots, & \mathcal{H}_{(2D-1)}^{\ell} \end{pmatrix}^{-1} \begin{pmatrix} \mathcal{H}_1^{\ell} \\ \vdots \\ \mathcal{H}_D^{\ell} \end{pmatrix}, \quad \boldsymbol{w}_{\mathrm{SCD}}(\ell) = \begin{pmatrix} \mathcal{Q}_1^{\ell}, & \ldots, & \mathcal{Q}_D^{\ell} \\ \vdots, & \ldots, & \vdots \\ \mathcal{Q}_D^{\ell}, & \ldots, & \mathcal{Q}_{(2D-1)}^{\ell} \end{pmatrix}^{-1} \begin{pmatrix} \mathcal{Q}_1^{\ell} \\ \vdots \\ \mathcal{Q}_D^{\ell} \end{pmatrix} \tag{23}$$

where

$$\mathcal{H}_m^{\ell} \triangleq \mathbf{u}_{1a}^{\ell H}\left(\boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a|\boldsymbol{\alpha}_1}(\ell) + 2\mathcal{N}_0\boldsymbol{I}_{2N}\right)^m \mathbf{u}_{1a}^{\ell}, \qquad \mathcal{Q}_m^{\ell} \triangleq \mathbf{u}_{1a}^{\ell H}\left(\boldsymbol{U}_J^{\ell}\boldsymbol{U}_J^{\ell H}\right)^{m+2}\mathbf{u}_{1a}^{\ell} \tag{24}$$

We refer to (21) and (22) as Suboptimal Linear-Conjugate Decorrelator (SCD) and Suboptimal Linear-Conjugate MMSE detector (SCM) respectively. Eq. (23) implies that in order to compute the optimal weights (i.e. those solving the constrained minimization problem (9)) we have to invert a $D \times D$ Hankel matrix. The computational effort for on line inversion of the matrix in (23) is feasible when D is very small (this is equivalent, in order to obtain good performance, to saying that the number of users is not too large). If D is not too small the weight optimization approach cannot be adopted but we can resort to the design guidelines proposed at the end of this paper.

4 Performance Analysis

4.1 Nonasymptotic Analysis

Because of space limitation we omit our analytical results on nonasymptotic BER, Near-Far Resistance and signal-to-noise ratio performance. Our numerical results focus on a CDMA system employing Gold codes with spreading length $N = 31$ operating in Rayleigh fading. In Figure 1 we have represented the error probability versus the average received signal-to-noise ratio SNR$= (\mathcal{E}_1)/(2\mathcal{N}_0)$, for the conditional MMSE receiver (15), labeled as "CM", for the newly proposed MMSE receiver (14), labeled as "LCM", and for the suboptimal linear-conjugate MMSE detector (22), labeled as "SCM" with $D = 10$. We assume 24 interfering users with a power level of 5dB above the desired signal. The results clearly show the superiority of the new strategy, which largely outperforms the conditional MMSE receiver (15). For comparison purposes, we also report the error probability corresponding to an uncoded BPSK transmission over a MAI-free flat-fading channel. Figure 2 contrasts the classical linear detection structures to the newly proposed receivers in terms of near-far resistance, which is represented versus K. Again, it is seen that the new approach allows a noticeable performance improvement.

4.2 Asymptotic Analysis

In the case of long sequences (and in other cases, e.g., with sequences distorted by random multipath, or in a network with random access) it is reasonable to assume that the spreading sequences are randomly and independently chosen. In this case, the performance measures of the receivers can be modeled as random variables, since they are a function of the spreading sequences. In the following we average the performance measures with respect to the random sequences and we also show that in the limiting regime $K \to \infty$, the random performance measures converge to deterministic quantities. The elements of the signatures $s_{k,n}^{\ell} \in \{-1/\sqrt{N}, 1/\sqrt{N}\} \quad n = 0, \dots, N-1$ are chosen equally likely and independently for all (k, n, ℓ). Notice that similar results can be obtained for nonbinary sequences whose elements are independent and identically distributed (i.i.d.), with zero mean and variance $1/N$. The normalization ensures that $E[||s_k^{\ell}||] = 1$; however in this more general case we need also the assumption $E[|s_{k,n}^{\ell}|^4] < \infty$. The ratio of the number of users to the number of dimensions is denoted by $\beta \triangleq K/N$.

It is well known [6, 7] that the Near-Far resistance $\eta_1^C(\ell)$ of detectors (11), (12) and (15) converges almost surely as $K \to \infty$ to:

$$\lim_{K \to \infty} \eta_1^C(\ell) = [1 - \beta]^+ \tag{25}$$

Based on some tools from free-probability theory [8] as applied to random matrices, we prove the following:

Proposition 3 *The average of $\eta_1(\ell)$ w.r.t. the signatures of all users is lower bounded by:*

$$E[\eta_1(\ell)] \geq \left[1 - \frac{K-1}{2N}\right]^+$$

Furthermore, the Near-Far resistance $\eta_1(\ell)$ for any $0 \leq \beta \leq \infty$ converges almost surely to:

$$\eta_1^{\infty} = \lim_{K \to \infty} \eta_1(\ell) = \left[1 - \frac{\beta}{2}\right]^{+} \tag{26}$$

Let us focus now on the (finite SNR) multiuser efficiency [6, 7] of the proposed receivers. Define a nonnegative random variable $|A|^2$ whose distribution is the limit distribution of $\left\{\frac{\mathcal{E}_k}{\mathcal{E}_1}|\alpha_k|^2,\ k = 1, \ldots K\right\}$. For the classical linear detection structures (11), (12) and (15), the multiuser efficiency for $K \to \infty$ converges almost surely to the solution $\eta_{CM}^{\infty}(\mathsf{SNR})$ of [5, 4]:

$$\eta + \beta \mathrm{E}\left[\frac{|A|^2\ \mathsf{SNR}\,\eta}{1 + |A|^2\ \mathsf{SNR}\,\eta}\right] = 1 \tag{27}$$

One of our main results is:

Proposition 4 *The multiuser efficiency of the Linear-Conjugate receivers (13) and (14) converges almost surely to the solution $\eta_{LCM}^{\infty}(\mathsf{SNR})$ of:*

$$\eta + \frac{\beta}{2} E\left[\frac{2\,\mathsf{SNR}\,|A|^2\ \eta}{1 + 2\,\mathsf{SNR}\,|A|^2\ \eta}\right] = 1 \tag{28}$$

The main conclusions from Propositions 3 and 4 are (i) the linear conjugate filters half the number of effective interferers and (ii) they add 3 dB to the input SNR. Effect (i) can be explained by noticing that the conventional decorrelator combats each interfering user as if it were quadrature-modulated which has the same noise-enhancement effect as having two BPSK-modulated users (for high SNR). To explain effect (ii) it is best to consider the single-user case; whereas the conventional MMSE filter minimizes the averaged magnitude squared at its output, the linear conjugate MMSE detector targets its resources to minimize the averaged real part squared (to exploit the BPSK modulation format) which has the same effect as turning off half of the input noise power.

In Figure 3 we show the asymptotic multiuser efficiency of the Linear-Conjugate MMSE receiver versus the ratio $\beta = K/N$ for a fixed average received signal-to noise ratio $\mathsf{SNR} = 10dB$ and for equal powers. For comparison purposes the spectral efficiency of the conditional MMSE receiver (CM) for equal powers [4] is also given.

So far we have only analyzed the unconstrained-complexity solutions to (9). Let us focus on the suboptimal structure receivers (21) and (22) that we obtain as solution to (9) when complexity constraints are imposed. As we have already noticed, the optimal weights (23) can be computationally intensive. Instead we propose an asymptotic weighting that assumes to operate in a limiting regime (i.e. the number of users and processing gain are large with fixed ratio and random spreading sequences). Using some recent results on the asymptotic eigenvalues distribution of random matrices we have:

$$\lim_{K \to \infty} \boldsymbol{s}_1^{\ell H} \left(\boldsymbol{M}_{\boldsymbol{z}_a \boldsymbol{z}_a | \boldsymbol{\alpha}_1}(\ell) + 2\mathcal{N}_0 \boldsymbol{I}_{2N}\right)^m \boldsymbol{s}_1^{\ell} \overset{a.s.}{=} E\left[\mathrm{trace}\left(\boldsymbol{M}_{\boldsymbol{z}_a \boldsymbol{z}_a | \boldsymbol{\alpha}_1} + 2\mathcal{N}_0 \boldsymbol{I}_{2N}\right)^m\right]$$

$$\lim_{K \to \infty} \boldsymbol{s}_1^{\ell H} \left(\boldsymbol{M}_{\boldsymbol{z}_a \boldsymbol{z}_a | \boldsymbol{\alpha}}(\ell)\right)^m \boldsymbol{s}_1^{\ell} \overset{a.s.}{=} E\left[\mathrm{trace}\left(\boldsymbol{M}_{\boldsymbol{z}_a \boldsymbol{z}_a | \boldsymbol{\alpha}_1}\right)^m\right]$$

It follows that:

$$\boldsymbol{w}_{\mathrm{SCD}}^{\infty} = \mathrm{E}\begin{bmatrix} \Lambda^3 & \Lambda^4, & \ldots, & \Lambda^{D+2} \\ \Lambda^4 & \Lambda^5, & \ldots, & \Lambda^{D+3} \\ \vdots & \vdots, & \ldots, & \vdots \\ \Lambda^{D+2} & \Lambda^{D+1}, & \ldots, & \Lambda^{2D+1} \end{bmatrix}^{-1} \mathrm{E}\begin{pmatrix} \Lambda^3 \\ \Lambda^4 \\ \vdots \\ \Lambda^{D+2} \end{pmatrix} \tag{29}$$

$$\boldsymbol{w}_{\mathrm{SCM}}^{\infty} = \mathrm{E}\begin{bmatrix} (\Lambda+2\mathcal{N}_0) & (\Lambda+2\mathcal{N}_0)^2, & \ldots, & (\Lambda+2\mathcal{N}_0)^D \\ (\Lambda+2\mathcal{N}_0)^2 & (\Lambda+2\mathcal{N}_0)^3, & \ldots, & (\Lambda+2\mathcal{N}_0)^{D+1} \\ \vdots & \vdots, & \ldots, & \vdots \\ (\Lambda+2\mathcal{N}_0)^D & (\Lambda+2\mathcal{N}_0)^{D+1}, & \ldots, & (\Lambda+2\mathcal{N}_0)^{2D-1} \end{bmatrix}^{-1} \mathrm{E}\begin{pmatrix} (\Lambda+2\mathcal{N}_0) \\ (\Lambda+2\mathcal{N}_0)^2 \\ \vdots \\ (\Lambda+2\mathcal{N}_0)^D \end{pmatrix} \tag{30}$$

where the expectations are with respect to Λ, whose distribution is the asymptotic eigenvalue distribution of $\boldsymbol{M}_{\boldsymbol{z}_a\boldsymbol{z}_a|\boldsymbol{\alpha}_1}$. The entries of the matrices given in (29) and (30) can be computed by applying some tools of combinatorial theory [9]. In fact it can be shown that:

$$E\left[\Lambda^m\right] = \sum_{n=1}^{m} \beta^n \sum_{(j_1,\ldots,j_n)\in Z_n^{(m)}} \mathcal{E}_1^n c(j_1,\ldots,j_n)\mathrm{E}\left[(|A|^2)^{j_1}\right]\cdots\mathrm{E}\left[(|A|^2)^{j_n}\right]\cdot\beta^n$$

where the sum is over all possible $j_1,\ldots,j_n > 0$ such that $j_1 + \cdots j_n = n$ and for all $1 \leq n \leq m$ [9]. We refer to (21) and (22) (where the weights are given by (29) and (30)) as the Asymptotic Suboptimal Linear-Conjugate Decorrelator (ASD) and Asymptotic Suboptimal Linear-Conjugate MMSE detector (ASM). In Figure (3) we have represented the error probability versus the average received signal-to-noise ratio SNR= $(\mathcal{E}_1)/(2\mathcal{N}_0)$, for the newly proposed MMSE receiver (14), labeled as "LCM", and for the asymptotic suboptimal linear-conjugate MMSE detector (22), labeled as "ASM" with $D = 15$. We assume $N = 150$ and $K = 30$ and interfering users with equal powers. For comparison purposes, we also report the error probability corresponding to an uncoded BPSK transmission over a MAI-free flat-fading channel.

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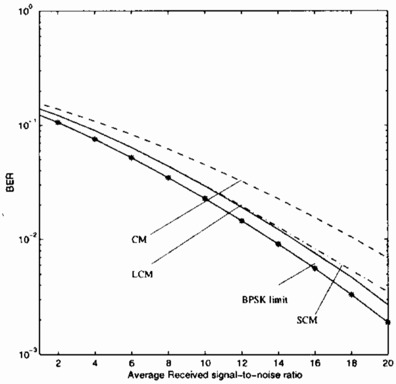

Figure 1: BER of the classical and of the newly proposed MMSE detectors, LCM and SCM, for $K = 25$.

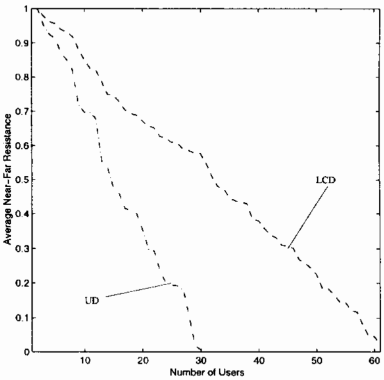

Figure 2: Near-Far resistance of the classical decorrelator and of the newly proposed Linear-Conjugate decorrelator.

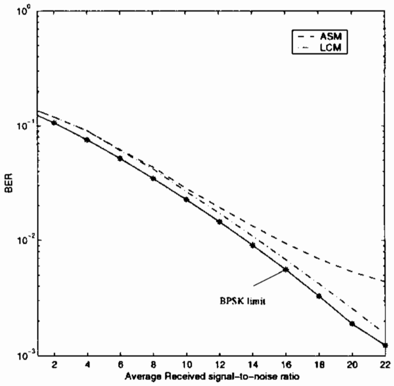

Figure 3: BER of the classical and of the newly proposed MMSE detectors, LCM and ASM, for $K = 30$ and $N = 150$.

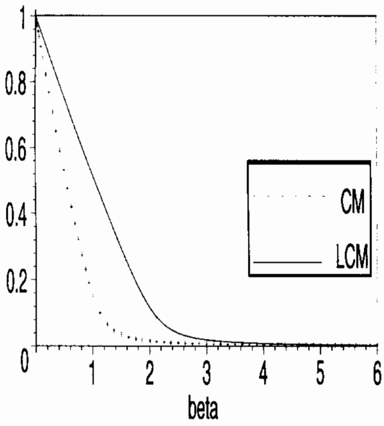

Figure 4: LCM multiuser efficiency for SNR = 10dB.