INFORMATION THEORETIC ASPECTS OF CODED RANDOM DIRECT-SEQUENCE SPREAD-SPECTRUM

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ABSTRACT

Information theoretic aspects of Code Division Multiple Access (CDMA) random direct-sequence spread-spectrum (DSSS) are investigated. The CDMA-DSSS channel with randomly and independently chosen spreading sequences accurately models the situation where pseudo-noise sequences span many symbol periods. Its information theoretic analysis provides a comparison baseline for CDMA channels with carefully designed signature waveforms that span one bit period on one hand and optimal multiple-user coded systems on the other. We analyze the spectral efficiency (total capacity per chip) as a function of the number of users, spreading gain and signal-to-noise ratio, and we quantify the loss in efficiency relative to an optimally chosen set of signature sequences and to an optimal multiaccess system without spreading. White Gaussian background noise and equal-power synchronous users are assumed. The analysis comprises the following receivers: a) optimal joint processing, b) single-user matched filtering; c) decorrelation and d) minimum mean square error linear processing. Some implications due to fading are also addressed.

1. INTRODUCTION

Coded direct-Sequence Spread-Spectrum (DSSS) Code Division Multiple Access (CDMA) has well-known advantages over time/frequency division multiple access: dynamic channel sharing, robustness to channel impairments, graceful degradation, ease of cellular planning, etc. These advantages result from the assignment of "signature waveforms" with large time-bandwidth products to every potential user of the system. The coded DSSS K-user model we focus on is as in [1], [2]

$$y(t) = \sum_{k=1}^{K} \sum_{l} b_k(l)\, s_k(t - lT - \tau_k) + n(t) ,$$

where $y(t)$ is the received signal. The signature waveform of the k-th user is designated by $s_k(t)$, the l-th coded information symbol of the k-th user is $b_k(l)$. τ_k stands for the delay of the k-th user. The additive white Gaussian noise with a two sided power spectral density of σ^2 is designated by $n(t)$. In the sequel we confine our attention to piecewise constant bipolar valued signatures composed on N bipolar chips each assuming the values $\left(\pm\frac{1}{\sqrt{N}}\right)$. We further assume full synchronism $\tau_k = 0$ for all users. This gives rise to a discrete time model where each signature can be viewed as a unit-norm vector in an N-dimensional signal space, where N is the spreading gain or number of chips per symbol. In the model considered in this paper, K users linearly modulate their signatures with the outputs of respective autonomous encoders which map information bits to channel symbols. The central question addressed in this paper is the capacity loss incurred by the imposition of such a structure on the transmitted signals, and by the imposition of several suboptimal but practically appealing receiver structures for which single-user decoding are sufficient. Our analysis considers a white Gaussian channel with users constrained to have identical average powers. We shall focus on randomly selected signatures modeling, among other scenarios, the long signature sequences used in some practical systems such as the IS–95 [3] standard.

1.1. Spectral Efficiency

The fundamental figure of merit is the *spectral efficiency*, C, defined as the total number of information bits per chip that can be transmitted arbitrarily reliably. Since the bandwidth of the CDMA system is (roughly) equal to the reciprocal of the chip duration, the spectral efficiency can be viewed as the bits/s/Hz supported by the system. Note that if the code rates (bits/symbol) employed by each individual user are identical and denoted by R, then the spectral efficiency is equal to the product:

$$\mathsf{C} = \frac{K}{N} R . \tag{1}$$

In a system where no spreading is imposed, the encoders are able to control the symbols modulating each chip independently. Therefore, assuming chip-synchronism, the Cover-Wyner capacity region of the conventional Gaussian multiaccess channel [4] applies to this case and the spectral efficiency in the absence of spreading is given by

$$\mathsf{C}^* = \frac{1}{2}\log(1 + \frac{K}{N}\,\mathsf{SNR}) . \tag{2}$$

where, for consistency with the results below, SNR denotes the energy per transmitted N chips divided by the Gaussian

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noise spectral level, σ^2. This means that the energy per bit divided by $N_0 = 2\sigma^2$ is

$$\frac{E_b}{N_0} = \frac{\mathsf{SNR}}{2R}. \qquad (3)$$

Once the spectral efficiency is determined, it is possible to obtain the minimum bandwidth necessary to transmit a predetermined information rate or the maximum information rate that can be supported by a given bandwidth. In order to compare different systems (with possibly different spreading gains and data rates), the spectral efficiency must be given as a function of $\frac{E_b}{N_0}$. According to (1) and (3), if the system operates at full capacity, then SNR in (2) can be substituted by

$$\mathsf{SNR} = \frac{2N}{K}\frac{E_b}{N_0}\mathsf{C}^*$$

so the maximum spectral efficiency $\mathsf{C}^*\left(\frac{E_b}{N_0}\right)$ in the absence of spreading is the solution to

$$\mathsf{C}^*\left(\frac{E_b}{N_0}\right) = \frac{1}{2}\log\left(1 + 2\mathsf{C}^*\left(\frac{E_b}{N_0}\right)\frac{E_b}{N_0}\right) \qquad (4)$$

or equivalently

$$\frac{4^{\mathsf{C}^*\left(\frac{E_b}{N_0}\right)} - 1}{2\mathsf{C}^*\left(\frac{E_b}{N_0}\right)} = \frac{E_b}{N_0}.$$

The solution is well-known to satisfy $\mathsf{C}^*\left(\frac{E_b}{N_0}\right) > 0$ if and only if $\frac{E_b}{N_0} > \log_e 2 = -1.6$ dB, and $\lim_{\frac{E_b}{N_0}\to\infty} \frac{\mathsf{C}^*\left(\frac{E_b}{N_0}\right)}{10\log_{10}\frac{E_b}{N_0}} = \frac{\log 10}{20} = 0.166$ bits/dB. Since (4) does not depend on K, when the transmitted signals are not constrained to the spread-spectrum format, the spectral efficiency is the same as in a single-user system.

The total capacity (sum-rate) of the power-constrained synchronous CDMA channel is equal to [1]

$$\frac{1}{2}\log\left(\det\left[\mathbf{I} + \sigma^{-2}\mathbf{ARA}\right]\right), \qquad (5)$$

where $\mathbf{A} = \mathrm{diag}\{A_1, \ldots, A_K\}$, and $\mathbf{R}$ is the matrix of normalized crosscorrelations of the bipolar signature sequences.

If the users have equal power, then $A_k = A$, $\mathsf{SNR} = A^2/\sigma^2$, and the optimum spectral efficiency is equal to

$$\mathsf{C}^{\mathsf{opt}}(\mathsf{SNR}, \mathbf{R}, K, N) = \frac{1}{2N}\log\left(\det\left[\mathbf{I} + \mathsf{SNR}\mathbf{R}\right]\right). \qquad (6)$$

For example, in the case of orthogonal sequences the spectral efficiency is equal to

$$\mathsf{C}^{\mathsf{orth}} = \frac{K}{2N}\log(1 + \mathsf{SNR}), \quad \text{if} \quad K \le N. \qquad (7)$$

Substituting $\mathsf{SNR} = \mathsf{C}^{\mathsf{orth}}\frac{2N}{K}\frac{E_b}{N_0}$, we obtain that if $K \le N$, then

$$\mathsf{C}^{\mathsf{orth}}\left(\frac{K}{N}, \frac{E_b}{N_0}\right) = \frac{K}{N}\mathsf{C}^*\left(\frac{E_b}{N_0}\right). \qquad (8)$$

The equality of $\mathsf{C}^{\mathsf{orth}}$ and C^* for $K = N$ is a consequence of the well-known fact [4] that for equal-rate equal-power users orthogonal multiple access incurs no loss in capacity relative to unconstrained multiple access. It is also known [5] that even if $K > N$, there exist spreading codes (*Welch-Bound-Equality*) which incur no loss in capacity.

1.2. Scope of the Study

Optimal spectral efficiency in non-orthogonal CDMA requires joint processing and decoding of the users. As advocated in a number of recent works, see [6, and references therein], it is sensible in terms of complexity-performance tradeoff to adopt as a front-end a (soft-output) *multiuser detector* [7] followed by autonomous single-user error-control decoders. In our analysis of spectral efficiency we consider, in addition to optimal decoding, some popular linear multiuser detector front-ends: Single-user matched filter, Decorrelator and Linear Minimum Mean-Square-Error (MMSE).

Our purpose is to evaluate the spectral efficiency of CDMA systems where *signature waveforms are assigned at random.* Denote (in the discrete time framework) the unit-norm signature of the kth user by $[c_{k1}, \cdots c_{kN}]$, and assume that $c_{kj} \in \{-1/\sqrt{N}, +1/\sqrt{N}\}$ are chosen equally likely and independently for all (k, j). The rationale for averaging capacity with respect to random signature waveforms is twofold. As indicated, it accurately models CDMA systems where pseudo-noise sequences span many symbol periods. Further, the spectral efficiency averaged with respect to the choice of signatures provides a lower bound to the optimum spectral efficiency achievable with a deterministic choice of signature waveforms.

In our asymptotic (in K) analysis we do not just average spectral efficiency with respect to the spreading sequences, but we show convergence of the spectral efficiencies to deterministic quantities.

Most analyses of multiuser detectors have focused on the bit-error-rate of uncoded communication [7]. The results found on this paper give the best achievable performance with error-control coding assuming random signature waveforms (Figure 1). The corpus of works dealing with coded DSSS systems with either deterministic and random signature sequences is surveyed in [6].

2. RESULTS

We summarize here the main conclusions found in this work on the capacity of spread-spectrum systems with random spreading. For further details and proofs, see the full version of this work [6]. Since the spectral efficiency depends on the spreading sequences, it is a random variable itself. In our asymptotic (in K) analysis we do not just average spectral efficiency with respect to the spreading sequences, but we show convergence of the (random) spectral efficiencies to deterministic quantities.

For an optimum receiver, the gain in spectral efficiency achievable by dynamic power allocation allocating instantaneous power as a function of the spreading waveforms, is small enough [6] not to warrant the required increase in complexity, and therefore not considered.

a. Asymptotic Optimum Spectral Efficiency

If we let $\beta = \frac{K}{N}$, then the optimum spectral efficiency for $0 < \beta$ converges almost surely as $K \to \infty$ to

$$\lim_{K\to\infty} \mathsf{C}^{\mathsf{opt}} = \frac{1}{4\pi}\int_{a(\beta)}^{b(\beta)} \log(1+\mathsf{SNR}x)\sqrt{(b(\beta)-x)(x-a(\beta))}\frac{dx}{x},$$

where $a(\beta) = (\sqrt{\beta}-1)^2$, and $b(\beta) = (\sqrt{\beta}+1)^2$.

Loss in Spectral Efficiency.

When $K = N = 2$, binary random sequences achieve 75% of the spectral efficiency of orthogonal sequences [6]. When K is large, the loss in spectral efficiency as a function of E_b/N_0 due to a random choice of sequences (as opposed to optimal) vanishes as $E_b/N_0 \to \infty$ or as $\beta \to \infty$. The maximum loss is 50% and occurs at $K = N$, $E_b/N_0 \downarrow \log_e 2$.

b. Matched Filter Spectral Efficiency.

The spectral efficiency of the single-user matched filter [7] converges almost surely as $K \to \infty$ to

$$\lim_{K\to\infty} \mathsf{C}^{\mathsf{sumf}} = \frac{\beta}{2}\log\left(1+\frac{\mathsf{SNR}}{1+\mathsf{SNR}\beta}\right).$$

The maximum (over K/N) spectral efficiency of the single-user matched filter receiver is

$$\lim_{\beta\to\infty} \mathsf{C}^{\mathsf{sumf}}(\beta, \frac{E_b}{N_0}) = \frac{\log_2 e}{2} - \frac{1}{2}\frac{N_0}{E_b} \qquad \frac{E_b}{N_0} > \log_e 2.$$

The use of random signatures as opposed to optimally chosen sequences brings about substantial losses in spectral efficiency for the single-user matched filter, unless $\frac{E_b}{N_0}$ is relatively low and K/N is high.

c. Decorrelator Spectral Efficiency.

If $\beta \leq 1$, the spectral efficiency of the decorrelator converges in mean-square as $K \to \infty$ to

$$\lim_{K\to\infty} \mathsf{C}^{\mathsf{deco}} = \frac{\beta}{2}\log\left(1+\mathsf{SNR}(1-\beta)\right),$$

which yields

$$\mathsf{C}^{\mathsf{deco}}\left(\beta, \frac{E_b}{N_0}\right) = \beta\mathsf{C}^*\left((1-\beta)\frac{E_b}{N_0}\right). \qquad (9)$$

d. MMSE Spectral Efficiency.

If $\beta > 0$, the spectral efficiency of the linear MMSE transformation converges in mean-square sense as $K \to \infty$ to

$$\lim_{K\to\infty} \mathsf{C}^{\mathsf{mmse}} = \frac{\beta}{2}\log\left(1+\mathsf{SNR}-\frac{1}{4}\mathcal{F}(\mathsf{SNR},\beta)\right)$$

where

$$\mathcal{F}(x,z) \stackrel{\text{def}}{=} \left(\sqrt{x(1+\sqrt{z})^2+1}-\sqrt{x(1-\sqrt{z})^2+1}\right)^2.$$

e. Optimum Coding-Spreading Tradeoff.

When the spreading gain, N, is a free design parameter, it is of course interesting to solve for the N that optimizes the spectral efficiency with random spreading. The answer, as we can see in Figure 1, depends heavily on the type of receiver. For either optimum processing or matched-filtering followed by single-user decoding, spectral efficiency is maximized by letting $K/N \to \infty$. Thus for those receivers, the coding-spreading tradeoff favors coding: it is best to use error-correcting codes with very low rates (cf. (1)) and a negligible spreading gain with respect to the number of users. This conclusion was known to hold for the single-user matched filter [8] (although it may not extend to non-coherent demodulation models [9]). Note, however, that the behavior of optimum processing and the conventional single-user matched filter at $K/N \to \infty$ are quite different: the optimal spectral efficiency grows without bound with $\frac{E_b}{N_0}$, whereas the matched filter efficiency approaches 0.72 bit/sec/Hz monotonically as $\frac{E_b}{N_0} \to \infty$.

For large K, the optimum choice of K/N for the decorrelator ranges from 0 for $\frac{E_b}{N_0} \downarrow -1.6\ dB$ to 1 for $\frac{E_b}{N_0} \to \infty$ (cf. Figure 3). The optimum coding-spreading tradeoff of the decorrelator dictates using codes whose rates (bits/symbol) lie between 0 ($\frac{E_b}{N_0} \downarrow -1.6\ dB$) to $\mathsf{C}^{\mathsf{deco}}$ ($\frac{E_b}{N_0} \to \infty$). With an optimum choice of spreading gain, the decorrelator spectral efficiency with random signature waveforms is better than that of the single-user matched filter for $\frac{E_b}{N_0} > 5.2$ dB (Figure 2). Unlike the single-user matched filter, the spectral efficiency of the decorrelator grows without bound as $\frac{E_b}{N_0} \to \infty$.

As far as the optimum coding-spreading tradeoff for the MMSE receiver, for low $\frac{E_b}{N_0}$ it favors making K/N very large in which case the MMSE receiver achieves essentially the same spectral efficiency as the single-user matched filter (Figure 2). The optimum K/N reaches 1 at $\frac{E_b}{N_0} = 4$ dB, and a minimum of 0.75 at $\frac{E_b}{N_0} = 10$ dB (cf. Figure 3).

3. DISCUSSION AND CONCLUSIONS

A misconception that has arisen in the last few years [10] claims that in CDMA systems with large number of users, error-control-coding, perfect power control and long codes, little can be gained by exploiting the structure of the multiaccess interference at the receiver. Our results have shown that exactly the opposite conclusion is true. Because of the deleterious effects of imperfect power control on the single-user matched filter, we would expect that the spectral inefficiency of that receiver to be even greater in that situation. Another misconception [10] predicts that multiuser detectors suffer from high sensitivity to the actual signature waveforms. On the contrary, our convergence results have shown that, as the number of users grows, the variability in achievable signal-to-noise ratio and spectral efficiency due to the choice of signature waveforms vanishes.

With large K/N, random CDMA incurs in negligible spectral efficiency loss relative to no-spreading if an optimum receiver is used. However, we have shown that linear multiuser detection is distinctly suboptimal for large K/N. This warrants the study of non-linear suboptimal multiuser detection, such as iterative decoding procedures, which have already demonstrated very competitive performance with limited complexity [11]. The optimum coding-spreading tradeoff favors negligible spreading (with respect to the number of users) for either optimum or single-user matched filter processing. In contrast, non-negligible spreading is optimum for linear multiuser detectors such as the decorrelator and the MMSE receiver. With an optimal choice of spreading factor, the spectral efficiencies of the decorrelator and MMSE receivers grow without bound as $\frac{E_b}{N_0}$ increases, in contrast to the single-user matched filter for which large signal-to-noise ratios offer little incentive (Figure 2). For large K/N, even if the signal-to-noise ratio is very low, the spectral efficiency of the single-user matched filter is a fraction of the optimum one: even though the background noise is dominant it pays to exploit the structure of the multiaccess interference because there are several users per degree of freedom.

We have focused exclusively on power-constrained inputs. If the channel symbols modulating the signature waveforms are restricted to be binary, then existing results on the capacity of single-user binary input Gaussian channels can be used to deal with the decorrelator, MMSE and single-user matched filter. However, optimal spectral efficiency under such constraint is unknown, except when K/N is large in which case the symbol SNR is low and binary inputs are almost as good as Gaussian [1].

For low K/N systems (such as state-of-the-art CDMA), either the decorrelator or the MMSE are excellent choices and little inefficiency results from random rather than orthogonal signatures.

Our analysis has focused on symbol-synchronous CDMA channels. The generalization to symbol-asynchronous CDMA is non-trivial (cf. [2]), but highly interesting for many applications. Other extensions of our analysis that we expect to be easier than asynchronous systems deal with the cases of unequal powers and non-baseband models.

Effect of Fading:

Fading can be incorporated in the analysis, replacing in the introduction $b_k(i)$ by $\tilde{b}_k(i) = \alpha_{ki}\, b_k(i)$, where α_{ki} are iid random variables known to the receiver but not to the transmitter (we assume that the receiver has full channel state information). This framework can be used to model either a classical fading effect (independent from symbol to symbol because of interleaving) or to account for non ideal power control fluctuations and also to treat unequal powers assigned to the users. Our asymptotic-in-K results can be generalized to this setting and to non-equal deterministic received powers, using recent results on the spectral distribution of random matrices [12], giving the sought for distribution in terms of a Stieltjes transform.

The coding-spreading tradeoff considered in this paper is not limited to Direct-Sequence Spread Spectrum systems; it can be interpreted in a general way, where degrees of freedom in time/frequency are used for coding and spreading purposes. For example, multicarrier CDMA [13] can be considered a dual (in frequency) to the direct-sequence format (in time) [7]. In this case, there are N orthogonal frequency slots and the chip waveforms are orthogonal complex exponential functions. Within this setting, consider fading which is assumed to be identically distributed, independent in different frequency slots. The formulation of the problem is as before but now the chips of the users $\tilde{c}_{kl}$ are modified and given by $\tilde{c}_{kl} = \alpha_{kl}\, c_{kl}$, where $\{\alpha_{kl}\}$ are the iid nonnegative random variables standing for fading assumed to be perfectly known to the receiver, but unavailable to the transmitters. We normalize the fading power $E(\alpha_{kl}^2) = 1$. Thus this problem referred to as homogeneous fading in [7], is fully accounted by replacing the distribution of c_{kl} by that of $\tilde{c}_{kl}$. For example, the results for the optimal processing as discussed in section IIa are in fact insensitive to the distribution of c_{kl} under mild conditions [12]. Actually, the difference between optimized spreading sequence sets and random sequences diminishes because even if $\{c_{kl}\}$ are given and optimally chosen $\{\tilde{c}_{kl}\}$ are still iid random variables which satisfy the conditions for a limiting distribution [12].

In the case of $K = N \to \infty$, it is interesting to compare the performance of such a frequency division spreading scheme, to that where classical frequency division (Orthogonal Frequency Division Multiplex—OFDM) is used as an orthogonal channel accessing technique. The latter gives rise to the spectral efficiency (in bits per frequency slot)

$$C^{\mathsf{orth}}_{\mathsf{fdm}} = \frac{\beta}{2}\, E\, \log\Big(1 + \alpha^2\, \mathsf{SNR}_f\Big)$$

, where the expectation is taken with respect to the fading power random variable α^2 and where SNR_f is the individual signal-to-noise ratio (identical over all frequency slots). This is to be compared to the results of section Ia. Certainly for no fading $C^{\mathsf{opt}}_{\mathsf{fdm}}$ is advantageous, being equivalent to the optimal accessing technique [4]. This advantage is also maintained for Rayleigh fading where α^2 is exponentially distributed [14]

$$C^{\mathsf{opt}}_{\mathsf{fdm}} \to \frac{1}{2}\, \log\left(\frac{\mathsf{SNR}_f}{e^{\mathcal{C}}}\right) \quad , \quad \mathsf{SNR}_f \to \infty$$

, where $\mathcal{C} = 0.57721$ is the Euler constant, answering in part an open problem posed in [14] referring to the relative advantage of CDMA vs. TDMA (or other equivalents as OFDM here) as in a single cell fading channel.

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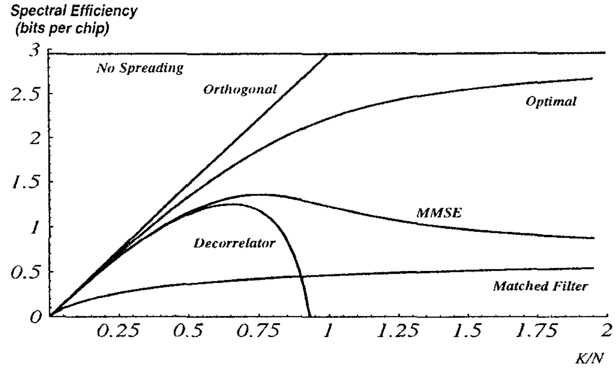

Figure 1: Large–K Spectral Efficiencies for $\frac{E_b}{N_0} = 10$ dB.

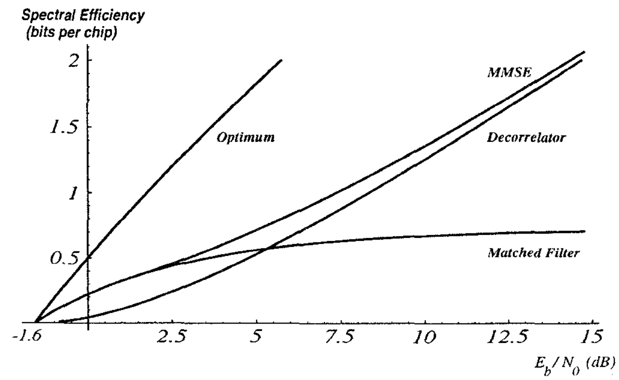

Figure 2: Large–K Spectral Efficiencies with Optimum K/N .

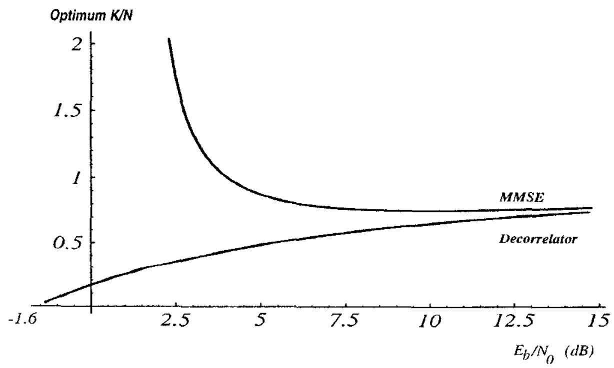

Figure 3: Optimum K/N for large K.