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INFORMATION THEORETIC ASPECTS OF SYSTEMATIC CODING

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ABSTRACT

The information theoretic aspects of systematic coding, where the raw information is transmitted on a channel as part of the coded message are investigated.

The achievable average distortion is characterized and the necessary and sufficient conditions under which systematic communication does not incur loss of optimality are found. This framework extends the application reported recently by Shamai & Verdu [1] of the Slepian-Wolf theorem to arbitrarily reliable communication with side information attained from the systematic part of the message. In the more general case treated here, the Wyner-Ziv rate distortion theorem plays a fundamental role.

The general result is applied to several scenarios where binary memoryless data is transmitted over a binary symmetric channel and over a Gaussian channel. It is shown that if nonnegligible bit error rate is tolerated, systematic encoding is strictly suboptimal. A structured random coding construction is introduced, which generalizes the linear coding interpretation of the Slepian-Wolf theorem, and which is directly used to demonstrate achievability of the optimal possible performance of systematic coding over a binary symmetric channel.

1 INTRODUCTION

Systematic codes, where the raw uncoded information is transmitted as part of the whole coded message, whether linear or not, are a standard frequently used element in coding theory [2],[3]. It is well known that binary linear systematic codes achieve capacity on a binary symmetric channel (BSC) [4], and in fact the performance of systematic linear coding measured in terms of message error probability is equivalent to any linear non-systematic code (with linearly independent codewords) on a binary input symmetric channel [2]. These optimal and capacity achieving features of systematic coding remain valid for extended coded symbols alphabets, other channels and general communication scenarios [1],[5]. In [1] the celebrated Slepian-Wolf Theorem [6] was used to

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consider more general communication settings where, for example the channels over which the systematic and non-systematic parts of the message are transmitted, may differ.

The ultimate potential and limitations of systematic coding became of major interest since the introduction of Turbo-Coding [7], demonstrating close-to-Shannon capacity performance. The classical construction of Turbo-Codes [7] comprises parallel concatenation of systematic component (block or recursive convolutional) codes, where the systematic structure plays a key role in the iterative decoding procedure in the parallel concatenated Turbo-Codes.

Message (Block) error characterization of systematic linear codes via the random coding error exponents in symmetric channels [2] exhibits no degradation as compared to the standard random coding error exponent, indicating thus that for high rate coding above the critical rate (in the region where the random coding exponent is tight) and on symmetric channels, optimal systematic codes demonstrate overall optimal performance in terms of message probability error exponents. This however does not imply optimality in terms of bit error probability or other bitwise quality distortion measures, where not necessarily the exponential behavior is pursued, and where in principle transmission above capacity rate is attempted giving rise to non-negligible distortions. It should be noted also in this respect, that equivalence in message error probabilities (not necessarily in error exponents only) as is the case in systematic as compared to general linear block codes [2] and systematic recursive convolutional codes as compared to general not necessarily systematic linear feedforward convolutional codes [7], *does not* imply equivalence in bit error probability. This is since the mapping from input bits to codewords may affect bit-wise performance criteria, while it does not alter the message-based performance measures, as the set of transmitted codewords is not affected.

In this paper we characterize the achievable performance of systematic coding given in terms of average information-bit distortion measures, emphasizing the special case of Hamming distortion which gives rise to the bit-error-

probability. We examine the problem within the general framework of rate distortion in the presence of noisy side information at the decoder. This formulation facilitates not only to examine a general setting where not necessarily identical channels are used to convey the systematic and non-systematic parts of the transmission, but also to directly apply the Wyner-Ziv [8] classical results which implies here the direct part of our coding theorem.

In the next section, we formulate the general setting of a communication system with systematic transmission and present the main results. The conditions under which systematic transmission does not incur loss of optimality are identified and shortly discussed. The important case of binary information sources and the bit-error-rate used as the distortion criterion is treated in Sections 3, and 4. Binary symmetric channels are studied in Section 3, while Section 4 discusses antipodal modulation in Gaussian channels. Systematic encoding is shown to be strictly suboptimal for both BSC and Gaussian channels. Wyner's [9] interpretation of Slepian-Wolf coding is extended in Section 3 by the proposed construction of Wyner-Ziv codes for the binary source/channel problem based on good channel error-correcting codes. This construction is based on structured random codes and it directly demonstrates optimal achievable performance of systematic codes on a BSC. A short conclusions section terminate this paper. Detailed proofs and extensions of the results to analogue sources and various application of practical interest are reported in [10].

2 SYSTEM MODEL AND THE CODING THEOREM

The communication system used to model general systematic coding methods is depicted in Figure 1.

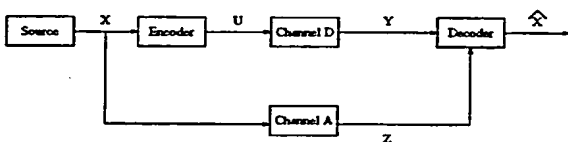


Fig. 1. Information Transmission by Systematic Coding

Consider the situation depicted in Figure 1 where a general memoryless information source X is transmitted to a decoder via two independent channels, only one of which (channel- D) is allowed to be preceded by an encoder. The objective is to reproduce the source by \hat{X} at the output of the decoder within some prescribed

distortion. This models a general systematic coding scheme where the systematic part is transmitted to the decoder via channel A , which may differ from channel D , through which the coded part of the message is conveyed. Though our interest here is focused on digital sources with finite alphabets, in this section we present the results in a general setting and allow for analogue as well as digital sources, where the notions 'systematic' stands for the raw data. For the sake of clarity and conciseness we limit our discussion to memoryless sources and channels as well as additive distortion measures.

Had we allowed an encoder preceding channel A , then the conventional separation theorem for lossy source/channel coding states that [11] distortion d is achievable/not achievable if the rate-distortion function of the source, $R(d)$, lies below/above the sum, $C_A + C_D$, of the capacities of the respective channels.

In the case of noiseless (arbitrary reliable) transmission, the separation theorem of [1] based on the Slepian-Wolf theorem [6], states that reliable transmission of the source in the setting of Figure 1, is possible/not possible if its entropy lies below/above $C_D + I(X; Z)$, where $I(X; Z)$ is the mutual information between the input and output of channel A . It follows that in the distortionless case the condition for the optimality of the systematic encoding is that the source maximizes the mutual information of channel A , i.e. $I(X; Z) = C_A$.

The theorem for lossy systematic coding proved in [10] applies to the system described in Figure 1.

Theorem [10] Consider the situation depicted in Figure 1 where the source and channels A and D are memoryless. For a joint distribution P_{VW} define the minimum possible estimation error of V given W by

$$\mathcal{E}(V|W) = \min_f E[D(V, f(W))] \quad (1)$$

where $D(\cdot, \cdot)$ is a properly defined distortion measure. Let the Wyner-Ziv rate-distortion function of X given that the decoder observes Z be [12]

$$\bar{R}_{X|Z}(d) = \inf_{\{W: Z-X-W, \mathcal{E}(X|W,Z) \leq d\}} I(X; W | Z) \quad (2)$$

where $Z - X - W$ denotes that Z and W are conditionally independent given X .

a) A sequence of blocklength- n encoders/decoders exists such that the source can be reproduced at the output for $n \rightarrow \infty$

with

$$\frac{1}{n} \sum_{k=1}^n E[D(X_k, \hat{X}_k)] \leq d \quad (3)$$

if

$$\bar{R}_{X|Z}(d) < C_D. \quad (4)$$

b) If $\bar{R}_{X|Z}(d) > C_D$, then no such coding scheme can exist.

Now we state the condition for optimality of systematic coding from Theorem 1. Suppose that distortion d^* is the best distortion achievable in a fully coded system (where channel A is preceded by an encoder). From the conventional separation theorem for lossy source/channel coding [11], d^* is the solution to

$$-R_X(d^*) + C_A + C_D = 0, \quad (5)$$

where

$$R_X(d) = \inf_{\{W: \mathcal{E}(X|W) \leq d\}} I(X; W) \quad (6)$$

is the standard rate distortion function [11]. To this end, we need the notion of the conditional rate distortion function $R_{X|Z}(d)$ [11], which can be formulated by:

$$R_{X|Z}(d) = \inf_{P_{XZ\hat{X}}:} I(X; \hat{X} | Z) \cdot \sum_c P_{XZ\hat{X}}(a, b, c) = P_{XZ}(a, b) \quad (7)$$

The necessary and sufficient condition on the source and channels for the optimality of systematic lossy encoding, i.e. achieving the performance of d^* of the optimal not necessarily systematic system, were derived in [10]. These conditions demand that in addition to satisfying with equality the theorem, that is:

- The channel D encoder is optimal, in the sense of reliably transmitting the necessary information:

$$\bar{R}_{X|Z}(d) = C_D, \quad (8)$$

the following three conditions must be satisfied:

A. The source maximizes the mutual information of the uncoded channel, i.e.,

$$I(X; Z) = C_A. \quad (9)$$

B. The output of the channel due to the uncoded source is not needed at the source encoder, i.e.,

$$R_{X|Z}(d) = \bar{R}_{X|Z}(d). \quad (10)$$

C. The output of the channel due to the uncoded source is maximally useful at the source decoder i.e.,

$$R_X(d) = R_{X|Z}(d) + I(X; Z). \quad (11)$$

Note that in general (11) is an inequality where 'smaller equal' replaces the 'equality' sign [13]. It is interesting to note that when condition B is not satisfied, $\bar{R}_{X|Z}(d) - R_{X|Z}(d)$ is typically small as shown in [12]. In the special case of zero distortion treated in [1] conditions B and C are always satisfied.

3 BERNOULLI SOURCE TRANSMITTED THROUGH BINARY SYMMETRIC CHANNEL WITH HAMMING DISTORTION

In this section we examine a special case of practical and theoretical interest of the general framework developed in Section 2 as described in Figure 1. The source is a Bernoulli symmetric source; the distortion criterion is Hamming, that is bit-error-rate; channel A is a BSC with crossover probability p_A ; and channel D is a BSC with crossover probability p_D .

The conventional source-channel separation theorem implies that the minimum bit error rate $d \leq \frac{1}{2}$ with full encoding of both channels, is given by (c.f. (5)),

$$R_X^b(d) = \min[1, C_b(p_A) + C_b(p_D)], \quad (12)$$

where $C_b(p) = 1 - h(p)$ is equal to the capacity of a BSC with crossover probability p and $R_X^b(d) = 1 - h(d)$ is the rate-distortion function of the Bernoulli symmetric source with Hamming distortion d and $h(x) = -x \log_2 x - (1-x) \log_2 (1-x)$ is the binary entropy.

According to the Theorem, when the systematic uncoded part is transmitted over channel A , the minimum achievable bit-error-rate d is determined by:

$$\bar{R}_{X|Z}^{bb}(d) = C_b(p_D), \quad (13)$$

where $\bar{R}_{X|Z}^{bb}(d)$ is the Wyner-Ziv rate distortion function for a Bernoulli source with a BSC (with crossover probability p_A) side information channel. $\bar{R}_{X|Z}^{bb}$ was obtained by Wyner and Ziv in [8, Section II] (referred to therein as the doubly symmetric binary source):

$$\bar{R}_{X|Z}^{bb}(d) = \left\{ \begin{array}{ll} g(d) & , 0 \leq d \leq d_c \\ g(d_c) \left(1 - \frac{(d-d_c)}{(p_A-d_c)}\right) & , d_c < d \leq p_A \end{array} \right\}, \quad (14a)$$

$$g(d) = \begin{cases} h(p_A * d) - h(d) & , \quad 0 \leq d < p_A \\ 0 & , \quad d = p_A \end{cases}, \quad (14b)$$

where $*$ stands for the binary convolution $a*b = a(1-b) + b(1-a)$, $0 \leq a \leq 1$, $0 \leq b \leq 1$, and d_c is the solution of the equation

$$\frac{g(d_c)}{d_c - p_A} = g'(d_c). \quad (14c)$$

A simple derivation of $\overline{R}_{X|Z}^{bb}(d)$ is given in [12] in terms of the *additive-noise rate-distortion function*.

The special case where both channels are identical $p_A = p_D = p$ is considered in Figure 2, which shows the achievable bit-error-rate for systematic and nonsystematic rate- $\frac{1}{2}$ coding as a function of the crossover probability of the channels. It is noted that for reliable communi-

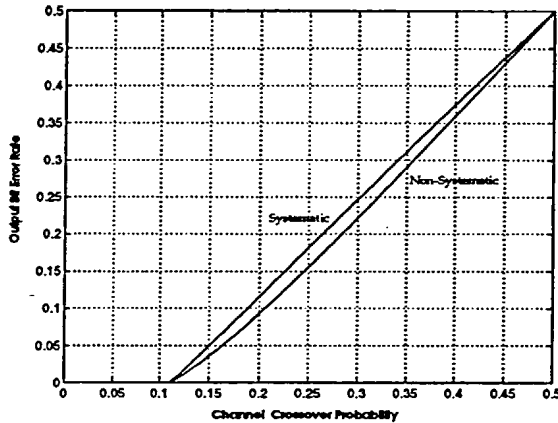


Fig. 2. Minimum bit error rate achievable at rate $\frac{1}{2}$ above the capacity of the binary symmetric channel

cation $d = 0$, no loss is incurred by systematic encoding as is already known [1],[5] and clearly this is also the case for the maximal distortion $d = 1/2$. However, we see in Figure 2, that for any bit-error-rate in $0 < d < p$ systematic encoding is strictly suboptimal. For any p_A and p_D (not necessarily equal) two out of the three required equalities, A and B (in Section 2) respectively, are satisfied:

$$I(X; Z) = C_A$$

and

$$R_{X|Z}^{bb}(d) = R_X^b(d) - I(X; Z);$$

only the fact that C (in Section 2) is not satisfied,

$$\overline{R}_{X|Z}^{bb}(d) > R_{X|Z}^{bb}(d)$$

for all $0 < d < p_A$, accounts for the suboptimality of systematic coding.

In an effort to characterize an explicit construction of optimal systematic code, which approaches the predicted performance in the Theorem, we adhere to the structure implied by the theorem that is an optimal Wyner-Ziv rate distortion encoder the output of which is encoded by a good, capacity approaching, channel code which operates reliably on channel D at a rate close to the capacity C_D . The second problem, i.e. a good capacity approaching channel codes on a BSC is standard and current practice [3],[7], demonstrates remarkable achievements. We therefore focus on the Wyner-Ziv rate distortion part and propose here a pseudo-constructive approach to achieve the Wyner-Ziv rate distortion function $\overline{R}_{X|Z}^{bb}$ in (14). This approach [10] is a substantive generalization of Wyner's construction of Slepian-Wolf codes from linear block channel codes [9]. We will assume that channel D is noiseless, i.e., $p_D = 0$, for otherwise, conventional, good channel codes can be employed to convey the output of the source encoder to the receiver with arbitrary reliability, as indicated above.

We will focus on bit error rates $0 \leq q \leq d_c$; for $d_c < q \leq p_A$ the strategy that follows should be time-shared with no coding. Choose two linear codes defined by parity-check matrices $(m_1 \times n)$ H_1 and $(m_2 \times n)$ H_2 , such that $\frac{m_1}{n} \simeq h(q)$, $\frac{m_2}{n} \simeq h(p_A * q)$, and Code 2 is a sub-code of Code 1. Thus, the parity check

$$\text{matrices } H_1 \text{ and } H_2 \text{ satisfy } H_2 = \begin{bmatrix} -H_1 \\ H_a \end{bmatrix}.$$

Every codeword of Code 1 $A = (a_1 \cdots a_n)$ satisfies $H_1 A^T = 0$, where the superscript T stands for the transpose operation. If, in addition, $H_a A^T = 0$, then A is also a codeword of Code 2. The decoders for these codes are defined by functions of the corresponding syndromes

$$f_1(H_1 \mathcal{Y}^T) = f_1(H_1 \mathcal{U}^T) \quad (15a)$$

$$f_2(H_2 \mathcal{Y}^T) = f_2(H_2 \mathcal{U}^T) = f_2(H_1 \mathcal{U}^T, H_a \mathcal{U}^T) \quad (15b)$$

where $\mathcal{Y} = A \oplus \mathcal{U}$, where \oplus denotes modulo 2 addition and where A in (15a) and (15b) is taken to be a codeword of codes 1 and 2 respectively. According to well-known properties of optimal linear codes, the codebooks and decoding functions of Codes 1 and 2 can be chosen so that

$$\begin{aligned} f_1(H_1 \mathcal{U}_1^T) &= \mathcal{U}_1 \\ f_2(H_2 \mathcal{U}_2^T) &= \mathcal{U}_2 \end{aligned} \quad (16)$$

for most of the realizations \mathcal{U}_1 of a Bernoulli(q) (probability of 1 = q) process, and for most of the realizations \mathcal{U}_2 of a Bernoulli($p_A * q$) process.

The Wyner-Ziv encoding of the binary source word consists of two steps.

(a) Among the codewords in Code 1 select X_q which is closest to X in Hamming distance. Let $E_q = X \oplus X_q$.

(b) Output the $m_2 - m_1$ vector $H_a X_q^T$.

Note that the output rate of the Wyner-Ziv encoder is

$$\frac{m_2 - m_1}{n} = h(p_A * q) - h(q) = g(q) = \bar{R}_{X|Z}^{bg}(q). \quad (17)$$

where $g(\cdot)$ is given by (14b).

The decoder receives $H_a X_q^T$ reliably through channel D (employing a good channel code) and the output of the BSC due to X which we denote by

$$Z = X \oplus N = X_q \oplus E_q \oplus N. \quad (18)$$

Since X_q is a codeword of Code 1, $H_1 X_q^T = 0$

and $H_2 X_q^T = \begin{bmatrix} 0 \\ -\frac{0}{H_a X_q^T} \end{bmatrix}$. The decoder outputs the n -codeword

$$Z \oplus f_2(H_2 X_q^T \oplus H_2 Z^T) = Z \oplus f_2(H_2(E_q^T \oplus N^T)). \quad (19)$$

If $E_q \oplus N$ were a Bernoulli($p_a * q$) process, the output of the decoder would yield with high probability

$$Z \oplus f_2(H_2(E_q^T \oplus N^T)) = Z \oplus E_q \oplus N = X_q, \quad (20)$$

in which case the decoder obtains a distorted version of the input X within distortion (bit error rate) q , as desired. By assumption, N is Bernoulli(p_A) and E_q is independent of N . Furthermore, for increasingly long blocklength, and for the appropriate choice, as described here, of Code 1, the distribution of E_q will resemble those of independent binary trials with parameter q . This can be expected by the backward channel interpretation of the standard rate-distortion function and the asymptotic rate-distortion optimality of linear block codes for the binary/Hamming case [2, Section 7.3] which imply that E_q resembles the noise process of the backward channel (a BSC in this case).

4 BERNOULLI SOURCE TRANSMITTED THROUGH GAUSSIAN CHANNEL, WITH HAMMING DISTORTION

In this section, Channel A is an additive Gaussian noise discrete-time channel (variance σ_A^2) with binary antipodal ($\pm\sqrt{P_A}$) inputs. Channel D is an additive Gaussian noise discrete-time channel (variance σ_D^2) with P_D -power-constrained (continuous) input, operating at a

rate of ρ_D channel uses per source information bit. The distortion measure remains, Hamming, that is the bit-error-rate.

The minimum bit-error-rate d achievable when channel A is also coded is given by the conventional source/channel separation theorem [11],

$$\bar{R}_X^b(d) = C_b(d) \leq C_{bg}(\text{SNR}_A) + \rho_D C_g(\text{SNR}_D), \quad (21)$$

where $\text{SNR}_A = P_A/\sigma_A^2$, $\text{SNR}_D = P_D/\sigma_D^2$ are respectively the signal-to-noise ratios of channels A and D , and $C_{bg}(s)$ and $C_g(s)$ are the capacities of the Gaussian channel with binary ($\pm\sqrt{s}$) inputs and average power-constrained s inputs, respectively:

$$C_{bg}(s) = 1 - \int_{-\infty}^{\infty} \frac{e^{-\tau^2/2}}{\sqrt{2\pi}} \log_2(1 + e^{-2\sqrt{s}\tau - 2s}) d\tau \quad (22)$$

$$C_g(s) = \frac{1}{2} \log(1 + s). \quad (23)$$

If channel A is connected directly to the source, then the achievable bit-error-rate is given by our Theorem

$$\bar{R}_{X|Z}^{bg}(d) \leq \rho_D C_g(\text{SNR}_D) = \frac{\rho_D}{2} \log(1 + \text{SNR}_D), \quad (24)$$

where the right-hand-side of the inequality is the capacity per source information bit of channel D . The function $\bar{R}_{X|Z}^{bg}(d)$ stands for the Wyner-Ziv rate distortion of a Bernoulli source where the side-information is given by the outputs of a binary-input Gaussian channel driven by the uncoded source with signal-to-noise ratio SNR_A . In [10], it is shown that $\bar{R}_{X|Z}^{bg}(d)$ is equal to the lower convex envelope (conv)

$$\bar{R}_{X|Z}^{bg}(d) = \text{conv}(F(d)) \quad (25)$$

of the function $F(d)$, $d \geq 0$ defined in a parametric form with the parameter $0 \leq q \leq 1/2$ via:

$$d = d(q) = qQ\left(\sqrt{\text{SNR}_A} - \frac{1}{2\sqrt{\text{SNR}_A}} \log \frac{1-q}{q}\right) + (1-q)Q\left(\sqrt{\text{SNR}_A} + \frac{1}{2\sqrt{\text{SNR}_A}} \log \frac{1-q}{q}\right), \quad (26a)$$

$$0 \leq q \leq \frac{1}{2},$$

$$F(d) = F(d(q)) =$$

$$\int_{-\infty}^{\infty} \frac{du}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} h\left(\frac{(1-q) + q e^{-2(u\sqrt{\text{SNR}_A} + \text{SNR}_A)}}{1 + e^{-2(u\sqrt{\text{SNR}_A} + \text{SNR}_A)}}\right) - h(q), \quad 0 \leq q \leq \frac{1}{2}. \quad (26b)$$

Note that the distortion achievable at zero rate, d_m , is the bit-error-rate of the uncoded channel,

p_u , and hence $F(d) \equiv 0$, $d \geq d_m = p_u \triangleq Q(\sqrt{\text{SNR}_A})$. In Figure 3, we illustrate (21) and (24) in the case $\text{SNR} = \text{SNR}_A = \text{SNR}_D$ and $\rho_D = 1$.

As we would expect, systematic coding is strictly suboptimal for $0 < d < d_m = Q(\sqrt{\text{SNR}_A})$. In this case, only one of the three required equalities is satisfied, i.e. $I(X; Z) = C_A$ (A, in Section 2), as equiprobable inputs maximize the capacity of the binary-input Gaussian channel. Thus, even in the hypothetical case in which the uncoded channel A outputs were available at the encoder, the performance would still be suboptimal since condition (C, in Section 2) is not satisfied.

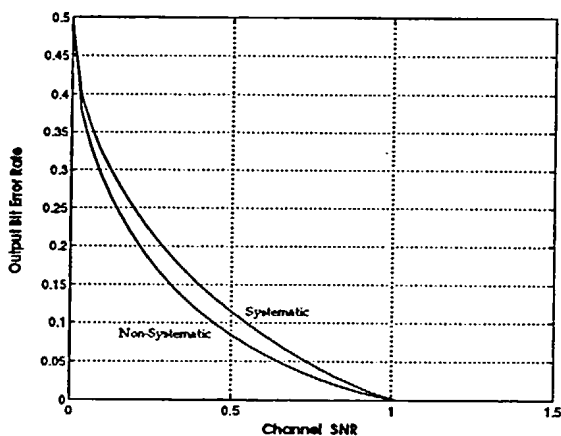


Fig. 3. Minimum Bit Error Rate for Transmission of a Bernoulli Source via a Gaussian Channel

5. CONCLUSIONS

We have presented an information theoretic approach to characterize the ultimately achievable performance of systematic codes, where the raw information is transmitted as part of the encoded message. Interpreting the noisy version of the systematic transmitted part as side information available at the receiver, provides a natural information theoretic setting for the problem and the achievable part of the coding theorem follows then as a corollary of the Wyner-Ziv source coding theorem [8]. The zero distortion case treated in [1] becomes then a special case where Slepian-Wolf [6] replaces the Wyner-Ziv [8] source coding mechanisms. The necessary and sufficient conditions for the optimality of systematic encoding are then identified and contrary to the zero-distortion case [1], it is found that typically systematic coding does incur loss of optimality. The general results were demonstrated for a Bernoulli source transmitted over either a binary symmetric or binary

input (with antipodal modulation) Gaussian channels and a bit error rate (Hamming) distortion measure. Systematic coding was shown to be suboptimal for nonnegligible or non-maximal distortion ($0 < d < 1/2$) in both cases and the associated distortion penalty was quantitatively assessed.

For the BSC (channels A and D in Figure 1), a pseudo-constructive method to approach optimal distortion performance with systematic coding was introduced. This scheme relies upon good (capacity achieving) channel codes with a special structure having also good sub-codes. Still the suggested structure is not fully constructive in terms of being based solely on given specific good channel codes, as is the case for zero distortion [1],[9]. Random coding arguments were harness to show the abundance of such structured codes and in this sense the approach is not constructive. It is of interest to obtain equivalent results based solely on the existence of specific families of good channel codes, scrapping thus the random coding arguments.

In [10], the general framework of Figure 1 was applied to investigate also analogue information sources as well as various digital settings in which channel A and D may cross-interfere with each other.

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REFERENCES

- [1] S. Shamai (Shitz) and S. Verdú, "Capacity of Channels with Uncoded Side Information," *European Trans. Telecommunications*, Vol. 6, No. 5, pp. 587-600, September-October 1995.
- [2] A.J. Viterbi and J.K. Omura, *Principles of Digital Communication and Coding*, McGraw-Hill, New York, 1979.
- [3] F.J. Mac Williams and N.J.A. Sloane, "The Theory of Error-Correcting Codes," North-Holland Publishing Company, Amsterdam, 1981.
- [4] P. Elias, "Coding for Noisy Channels," *IRE Convention Record*, Vol. 4, pp. 37-46, March 1955.
- [5] E. M. Gabidulin, "Limits for the Decoding Error Probability When Linear Codes are Used in Memoryless Channels," *Problemy Peredachi Informatsii*, Vol. 3, pp. 55-62, 1967.
- [6] J.D. Slepian and J.K. Wolf, "Noiseless Coding of Correlated Information Sources," *IEEE Trans. Inform. Theory*, Vol. IT-19, No. 4, pp. 471-480, July 1973.
- [7] C. Berrou and A. Glavieux, "Near Optimum Error Correcting Coding And Decoding: Turbo-Codes," *IEEE Trans. Commun.*, Vol. 44, No. 10, pp. 1261-1271, October 1996. See also, C. Berrou, A. Glavieux, P. Thitimajshima, "Near Shannon limit error correcting coding and decoding: turbo-codes," *Proc. International Conference on Communications*, pp. 1064-1070, Geneva, Switzerland, May 23-26, 1993.
- [8] A.D. Wyner and J. Ziv, "The Rate-Distortion Function for Source Coding with Side-Information at the Decoder," *IEEE Trans. Inform. Theory*, Vol. IT-22, No. 1, pp. 1-10, January 1976.

- [9] A.D. Wyner, "Recent Results in the Shannon Theory," *IEEE Trans. Inform. Theory*, Vol. IT-20, No. 1, pp. 2-10, January 1974.
- [10] S. Shamai (Shitz), S. Verdú and R. Zamir, "Systematic Lossy Source/Channel Coding," to appear in *IEEE Trans. Inform. Theory*.
- [11] T.M. Cover and J.A. Thomas, *Elements of Information Theory*, Wiley, New York, 1991.
- [12] R. Zamir, "The Rate Loss in the Wyner-Ziv Problem," *IEEE Trans. Inform. Theory*, Vol. 42, No. 6, pp. 2073-2084, November 1996.
- [13] R.M. Gray, "A New Class of Lower Bounds to Information Rates of Stationary Sources via conditional Rate-Distortion Functions," *IEEE Trans. Inform. Theory*, Vol. IT-19, No. 4, pp. 480-489, July 1973.