Is TDMA Optimal in the Low Power Regime?

Sergio Verdú
Princeton University
Princeton, NJ 08544 USA
e-mail: verdu@ee.princeton.edu

Giuseppe Caire and Daniela Tuninetti
Institut Eurécom
06904 Sophia-Antipolis, France
e-mail: firstname.name@eurecom.fr

We consider two-user additive Gaussian noise *multiple-access* and *broadcast* channels. Although the set of rate pairs achievable by time-division multiple-access (TDMA) is not equal to the capacity region (achieved by superposition), as the power decreases, the TDMA achievable region converges to the capacity region. Furthermore, TDMA achieves the same minimum energy per bit as superposition.

Despite those features of TDMA, this paper answers the question in the title negatively except in two special cases: multiaccess channels where the users' energy per bit are constrained to be identical and broadcast channels where the receivers have identical signal-to-noise ratios.

The most common practical embodiment of superposition multiaccess strategies is CDMA. Thus, in practice, superposition is particularly relevant in the wideband low-power regime where the received energy per information bit may not be far from its minimum value. Therefore, it is of considerable practical interest to compare the capabilities of TDMA to the capabilities of superposition in the low-power regime. To make the comparison as crisp as possible and in order not to incorporate features such as communication in a cellular environment or in the presence of fading, which may tilt the comparison in favor of superposition strategies, we limit our analysis to additive white Gaussian noise channels not subject to fading.

We consider the complex-valued multiple-access channel

$$Y = X_1 + X_2 + N$$

where N is Gaussian with independent real and imaginary components and $E[|N|^2] = \sigma^2$, $E[|X_1|^2] \leq P_1$ and $E[|X_2|^2] \leq P_2$. The (received) energy per information bit relative to the noise spectral level of user $i = 1, 2$ is $E_i/N_0 = P_i/R_i\sigma^2$. It can be shown that the minimum values of E_1 and E_2 achieved by TDMA are optimum and equal to $\log_e 2 = -1.59dB$. The minimum values of energy per bit are obtained in the limit of infinite bandwidth and therefore imply zero spectral efficiency, and give no indication about the bandwidth-power tradeoff in the channel. As explained in the recent work [1], the key performance measure in the wideband regime is the slope $\mathcal{S}_0$ of the spectral efficiency vs $\frac{E_b}{N_0}$ curve (b/s/Hz/3 dB) at $\frac{E_b}{N_0}_{\min}$. Whereas the conventional capacity region supplies the tradeoff of rates for fixed powers, we can define a corresponding "slope region" that gives the tradeoff of individual user slopes for a fixed ratio with which the individual rates vanish.

Theorem 1 *Let the rates vanish while keeping $R_1/R_2 = \theta$. The multiaccess slope region achieved by TDMA is:*

$$\{(\mathcal{S}_1, \mathcal{S}_2):\ 0 \leq \mathcal{S}_1,\ 0 \leq \mathcal{S}_2,\ \mathcal{S}_1 + \mathcal{S}_2 \leq 2\}.$$

whereas the optimum multiaccess slope region (achieved by superposition) is:

$$\mathbf{S}(\theta) = \{(\mathcal{S}_1, \mathcal{S}_2)\ :\ 0 \leq \mathcal{S}_1 \leq 2,\ 0 \leq \mathcal{S}_2 \leq 2,\ \frac{1}{2} \leq \left(\frac{\theta}{1+\theta}\right)^2 \frac{1}{\mathcal{S}_1} + \left(\frac{1}{1+\theta}\right)^2 \frac{1}{\mathcal{S}_2}\}.$$

Furthermore,

$$\textit{closure}\left\{\bigcup_{\theta>0} \mathbf{S}(\theta)\right\} = \{(\mathcal{S}_1, \mathcal{S}_2):\ 0 \leq \mathcal{S}_1 \leq 2,\ 0 \leq \mathcal{S}_2 \leq 2\}.$$

Theorem 1 shows that *both* users can achieve slopes that are arbitrarily close to the single-user slopes provided they use superposition, optimum decoding, and their powers are sufficiently unbalanced. Therefore, even in the simple setting of the two-user additive Gaussian multiaccess channel the low-power capabilities of TDMA are markedly suboptimal, unless the users are constrained to have identical received energies per bit in which case TDMA achieves optimum slopes.

We now turn our attention to the simple complex-valued two-user broadcast Gaussian channel where users 1 and 2 receive the same signal from the transmitter embedded in independent Gaussian noise with different powers:

$$Y_1 = X + N_1, \qquad Y_2 = X + N_2$$

where $E[|X|^2] \leq P$, $E[|N_i|^2] \leq \sigma_i^2$. We will assume $\sigma_1^2 < \sigma_2^2$ as in the case $\sigma_1^2 = \sigma_2^2$ TDMA is trivially optimal. Define $E_i/N_0 = P/(R_i\sigma_i^2)$. It can be shown that TDMA achieves the minimum energies per bit. Indeed, if there is no interest is conserving bandwidth [2, 3] conclude that TDMA is asymptotically optimum in the wideband regime. However Theorem 2 implies that except in the trivial case $\sigma_1^2 = \sigma_2^2$, TDMA can be quite wasteful of bandwidth in the low-power regime:

Theorem 2 *Let the rates vanish while keeping $R_1/R_2 = \theta$. The broadcast slope region achieved by TDMA is:*

$$\{(\mathcal{S}_1, \mathcal{S}_2):\ 0 \leq \mathcal{S}_1 \leq \frac{2\theta}{1+\theta},\ 0 \leq \mathcal{S}_2 \leq \frac{2}{1+\theta}\}.$$

and the optimum broadcast slope region (achieved by superposition) is:

$$\{(\mathcal{S}_1, \mathcal{S}_2):\ 0 \leq \mathcal{S}_1 \leq \frac{2\theta\,(\theta + \sigma_2^2/\sigma_1^2)}{\theta^2 + 2\theta + \sigma_2^2/\sigma_1^2},\quad 0 \leq \mathcal{S}_2 \leq \frac{2\,(\theta + \sigma_2^2/\sigma_1^2)}{\theta^2 + 2\theta + \sigma_2^2/\sigma_1^2}\}.$$

For example, if $R_1 = 3R_2$ and $\sigma_2^2 = 10\sigma_1^2$, TDMA more than doubles the required bandwidth.

References

[1] S. Verdú, "Spectral efficiency in the wideband regime," *IEEE Trans. Information Theory*, June 2002.

[2] R. McEliece and L. Swanson, "A note on the wide-band Gaussian broadcast channel," *IEEE Trans. Communications*, p.452-453, Apr. 1987

[3] A. Lapidoth, E. Telatar, and R. Urbanke, "On wide-band broadcast channels," *submitted to IEEE Trans. Information Theory*, 2001.

0-7803-7501-7/02/$17.00©2002 IEEE.