Least Favorable Additive Noise under a Divergence Constraint

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Abstract — **An uncertainty class of additive noise pdfs satisfying a constraint on divergence from a prescribed nominal is analyzed in the context of antipodal communication. From within this class, the noise pdf that maximizes detection error probability is determined for both zero-threshold and maximum-likelihood detection strategies. An optional additional constraint on SNR is also considered, and asymptotic behavior is studied for vanishing divergence tolerance.**

I. Introduction

Consider the hypothesis test for the normalized binary-input additive noise channel, given by

$$H_0 : Y = -1 + N \quad \text{versus} \quad H_1 : Y = +1 + N$$

where N is a random variable with symmetric probability density function (pdf) f_N. The zero-threshold (ZT) detector (hard-limiter) exhibits error probability

$$P_{ZT}(f_N) = \int_1^\infty f_N(x)dx$$

while the maximum-likelihood (ML) detector achieves minimum error probability

$$P_{ML}(f_N) = \frac{1}{2}\int_{-\infty}^{\infty} \min\{f_N(x), f_N(x+2)\}dx \ ;$$

these quantities agree for symmetric unimodal pdfs. Often, the actual noise pdf is only known to resemble a given nominal pdf, leading us to define an additive noise uncertainty class by specifying a divergence neighborhood about a fixed pdf f_N, given by

$$\{\text{pdfs } f : \int_{-\infty}^{\infty} f(x) \log \frac{f(x)}{f_N(x)} dx \le \delta\} \ .$$

We find the members of the uncertainty class that maximize P_{ZT} and P_{ML}. This formulation is of intrinsic interest in the analysis of AWGN channels subject to jamming, interference or other distortions. The results also represent the optimum strategy for a jammer who wishes to avoid detection, since discernibility between ambient noise and jammer-degraded noise is directly governed by their mutual divergence.

II. Zero-Threshold Detection

The least favorable ZT pdf f_N^{ZT} is obtained from a prescribed nominal f_N through an area-preserving scalar weighting of the tails chosen to satisfy the divergence constraint. In the special case of small divergence tolerance δ, we obtain the relation

$$P_{ZT}(f_N^{ZT}) = P_{ZT}(f_N) + \sqrt{P_{ZT}(f_N)(1-2P_{ZT}(f_N))\delta} + o(\sqrt{\delta}) \ .$$

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III. Maximum-Likelihood Detection

Least favorable ML noise represents the maxmin solution to the detection error probability game of divergence-constrained noise versus unconstrained receiver. For symmetric continuous unimodal nominals f_N satisfying a local log-concavity condition (in cluding Gaussians) the least favorable ML pdf f_N^{ML} is obtained from f_N by weighting the tails around a 2-periodic interval made up of weighted mod-2 geometric means. The weightings are mutually dependent through demonstrated continuity of f_N^{ML}, with the remaining degree of freedom chosen to preserve area. The size of the 2-periodic interval is chosen to satisfy the divergence constraint.

IV. Additional Power Constraint

In both ZT and ML cases, imposing the further constraint that noise power not exceed the nominal power leads to a least favorable pdf obtained by performing the described transformations on a variance-scaled nominal pdf

$$f_{\tilde{N}}(x) = C f_N(x) \exp\{-cx^2\} \ .$$

An upper bound on detection error probability for least favorable ML power- and divergence-constrained noise is realized by least favorable ML power-constrained noise [1] as depicted in Figure 1; under an appropriate c ondition on the support of f_N, the former converges in distribution to the latter for growing divergence tolerance.

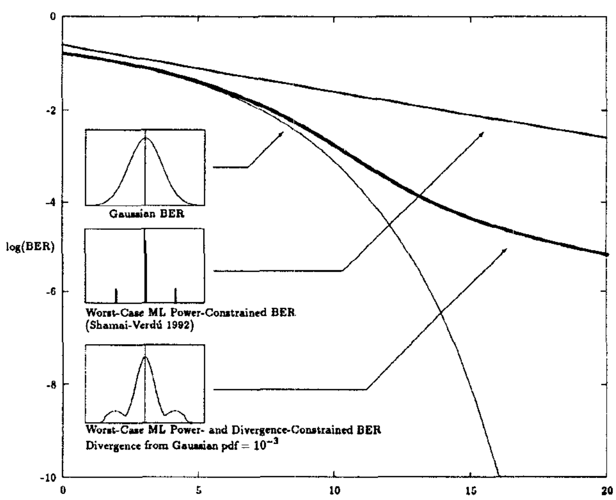

Figure 1: ML Bit-Error Rate under Various Constraints

References

[1] S. Shamai (Shitz) and S. Verdú, "Worst-case power-constrained noise for binary-input channels," *IEEE Transactions on Information Theory*, vol. 38, no. 5, pp. 1494–1511, Sept. 1992.